Problem of the Month
The Wheel Shop

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used school wide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The POM is designed with a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially as the key challenge for grades kindergarten and one. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *The Wheel Shop,* students use algebraic thinking to solve problems involving unknowns, equations, and simultaneous constraints. The mathematical topics that underlie this POM are variables, inverse operations, equations, equalities, inequalities, and simultaneous systems.

In the first levels of the POM, students are presented with the task of considering a tricycle shop with 18 wheels and determining how many tricycles there are. Their task involves finding an unknown and "undoing" the straightforward question of how many wheels 6 tricycles have in all. As one continues through the levels,

students are presented with situations that involve components of bicycles, tricycles, tandem bicycles, and go-carts. These situations can be translated into systems of constraints with equal numbers of unknowns. In Level D, students are given a situation that can be translated into a system of three equations with four unknowns. Students are asked to define the relationship between two unknowns. In the final level, students are presented with a logic situation that involves using rational numbers, inequalities, and a set of constraints. Students are asked to find the number of bikes in the shop and the range of repairs that need to be made.

Problem of the Month

The Wheel Shop

Level A

You go to a shop that sells tricycles. There are 18 wheels in the Wheel Shop.

How many tricycles are in the shop?

Explain how you know.

Level B

The Wheel Shop sells other kinds of vehicles. There are bicycles and go-carts in a different room of the shop. Each bicycle has only one seat and each go-cart has only one seat. There are a total of 21 seats and 54 wheels in that room.

How many are bicycles and how many are go-carts?

Explain how you figured it out.

Level C

Three months later some vehicles have sold and new models have been brought into the Wheel Shop. Now, there are a different number of bicycles, tandem bicycles, and tricycles in the shop. There are a total of 135 seats, 118 front handlebars (that steer the bike), and 269 wheels.



How many bicycles, tandem bicycles and tricycles are there in the Wheel Shop?

Level D

In the back stockroom at the Wheel Shop, the number of seats and horns equals the number of wheels. The number of seats and handlebars equals the number of horns. Twice the number of wheels is equal to three times the number of handlebars. Determine the relationship of horns to seats.

Level E

The repair department of the bicycle shop repairs three things - flat tires, bent handlebars and ripped seats. Today in the repair department, 25% of the bikes had flat tires only, 5% had bent handlebars only, and 10% had ripped seats only. Just 1/12th of the bikes had all three repairs to do - flat tires, bent handlebars and ripped seats. No bikes were completely fixed and there are a total of 101 repairs to be made. How many bikes are in the repair department? How many bikes need two repairs? If less than half of all the bikes have a ripped seat, what is the range of bikes that need both the tires and handlebars repaired without needing to fix the seat?

Problem of the Month

The Wheel Shop

Primary Version Level A

Materials: A tricycle and/or a picture of a tricycle for each student, counters or round chips (20 per group).

Discussion on the rug: The teacher shows the tricycle. **"What do we call this? How many wheels does a tricycle have? Who can show me how they know there are three wheels? Who else can show me? What if I have two tricycles - how many wheels would I have?"** The teacher continues to ask children to explain how many wheels and how they know for sure, perhaps eliciting three examples before going to small groups.

In small groups: Each group has approximately 20 round counters to model as wheels.

Teacher asks the following questions, only going on to the next question if students have success.

1. **Show me with counters how many wheels you would have with three tricycles. How can you check to know for sure?**

Continue until you think students understand a process. If 10 is too large, use a different number.

2. **If you had 10 tricycles, how many wheels would you have? How do you know?**

At the end of the investigation have students either draw a picture to represent their solution or dictate a response to this summary question.

Tricycle

Problem of the Month

The Wheel Shop

Task Description – Level A

This task challenges students to use the relationship between multiplication and division to find the number of tricycles that have a total of 18 wheels.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Work with equal groups of objects to gain foundations for multiplication.

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and 5 columns, write an equation to express the total as a sum of equal addends.

Represent and solve problems involving multiplication and division.

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

3.OA.2 Interpret whole-number quotients of whole numbers, e.g. interpret 56 ÷ 8 as the number of objects in each share when 56 objects are portioned equally into 8 shares, or as a number of shares when 56 objects are portioned into equal shares of 8 objects each.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month
The Wheel Shop
Task Description – Level B
This task challenges students to reason about a situation with two variables - seats and wheels. Students can use guess-and-check or set up simultaneous equations to solve for the number of bicycles and go-carts.
Common Core State Standards Math - Content Standards
Operations and Algebraic Thinking **Understand properties of multiplication and the relationship between multiplication and division.** 3.OA.5 Apply properties of operations as strategies to multiply and divide. 3.OA.6 Understand division as an unknown-factor problem. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 3.OA.9 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Use the four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Expressions and Equations** **Analyze and solve linear equations and pairs of simultaneous linear equations.** 8.EE.8 Analyze and solve pairs of simultaneous linear equations.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

The Wheel Shop

Task Description – Level C

This task challenges students to use systems of equations with 3 unknowns to find the number of bicycles, tandem bicycles, and tricycles in a shop. Students must deal with 3 different constraints – wheels, seats, and handlebars – to reason about the task.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Expressions and Equations

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

High School – Algebra – Creating Equations

Create equations that describe numbers or relationships.

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context

High School – Algebra – Reasoning with Equations and Inequalities

Solve systems of equations.

A-REI.6 Solve systems of linear equations exactly and approximately (e.g. with graphs) focusing on pairs of linear equations in two variables.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

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MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

The Wheel Shop

Task Description – Level D

This task challenges students to analyze relationships with four variables and three equations. Students can use balance beams or equations and substitution to find the relationship between two of the variables.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Expressions and Equations

Use properties of operations to generate equivalent expressions.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

High School – Algebra – Creating Equations

Create equations that describe numbers or relationships.

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context

High School – Algebra – Reasoning with Equations and Inequalities

Solve systems of equations.

A-REI.6 Solve systems of linear equations exactly and approximately (e.g. with graphs) focusing on pairs of linear equations in two variables.

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Problem of the Month

The Wheel Shop

Task Description – Level E

This task challenges students to use logic in a situation that involves using rational numbers, inequalities, and a set of constraints. Students may use Venn Diagrams as a logic tool and systems of inequalities to reason out the solution.

Common Core State Standards Math - Content Standards

Expressions and Equations

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

High School – Algebra – Creating Equations

Create equations that describe numbers or relationships.

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context

High School – Algebra – Reasoning with Equations and Inequalities

Solve systems of equations.

A-REI.6 Solve systems of linear equations exactly and approximately (e.g. with graphs) focusing on pairs of linear equations in two variables.

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Problem of the Month

The Wheel Shop

Task Description – Primary Level

This task challenges students to think about groups of three, using counters to think about the number of wheels for 1,2,3, . . . 10 tricycles. Students must be able to draw or use numbers to record their results.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

K.CC. 3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects.)

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular arrays, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20 count out that many objects.

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions or equations.

Represent and solve problems involving addition and subtraction.

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Work with equal groups of objects to gain foundations for multiplication.

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Common Core State Standards Math – Standards of Mathematical Practice

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