## The Baker

This problem gives you the chance to:

- choose and perform number operations in a practical context

The baker uses boxes of different sizes to carry her goods.


## Cookie boxes hold 12 cookies. <br> Donut boxes hold 4 donuts. <br> Muffin boxes hold 2 muffins. <br> Bagel boxes hold 6 bagels. <br> Bagel boxes hold 4

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces. cookie boxes $\qquad$ donut boxes $\qquad$ muffin boxes $\qquad$ bagel boxes $\qquad$
2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make? $\qquad$
Show your calculations.
3. On Wednesday she baked 42 cookies.

How many boxes did she fill? $\qquad$
How many cookies were left over? $\qquad$
Explain how you figured this out.
$\qquad$
$\qquad$
4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday? $\qquad$
Show how you figured this out.

| Task 2: The Baker | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - choose and perform number operations in a practical context <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answers: | 1 x 4 | 4 |
| 2. Gives correct answer: 42 <br> Shows $6 \times 7=42$. Accept repeated addition. | $1$ | 2 |
| 3. Gives correct answers: 3 $6$ <br> Gives a correct explanation such as: <br> She filled 3 complete boxes: $3 \times 12=36$ and $42-36=6$. <br> This means that 6 were left over <br> or <br> Shows $42 \div 12=3$, remainder 6 . | 1 <br> 1 | 2 |
| 4. Gives correct answer: donuts <br> Shows work such as: $4 \times 8=32$ <br> Accept diagrams. | 1 <br> 1 | 2 |
| Total Points |  | 10 |

## TheBaker

Work the task. Look at the rubric. How does this task assess the big ideas about undestanding the operation of multiplication and division?

Look at student work on part 1.

- How many of your students undestood that this was a division situation?
- How many of your students just repeated the information in table $(12,4,2,6)$ ?
- How many multiplied by 24 ? $(288,48,96,144)$ ?
- Used addition? $(36,28,26,30)$ ?
- Reversed answers for muffins and donus (working down instead of across)? What other types of errors did you notice?
How often do students get oppotunities to work with division in context? How does context help them develop an undestanding of theoperation of division and its pupose? What modds do you use with students to help them undestand the operation of multiplication and division?

Now look at work for part 3.
How many of your students could solve the problem correctly? What strategies did they use and what explanationsdid you find of value? How could you use some of these explanations as modds with the class?
How many of your students thought 4 boxes would befilled with 6 remaining?
Think aboutthe action of the story - 42 cookies being distributed into boxes. Why does "48-6" notmake sense? The answer is 42 , so where is the error? What is the important mathematical idea that students are missing? What kind of activity could you design or discussion question could you pose to help them think more deeply aboutthe situation and examine the error in their computations?
What were some of theother answers that did notfit the action of the story? Why do you think students did notrecognize this as a division situation? How do students learn the meaning of operationsso that they can apply them appropriately rather than just randomly guessing an operation? What kinds of discussionshelp develop the desired sense-making of operations?

## Looking at Student Work on The Baker

Student A is comfortable with the operation of division. The student shows work for all the parts of the task. The explanation in part 3 just gives a set of procedures, butdoes not explain why it makes sense. While this get full marks, what types of explanations would you like students to strive for?
Student A


1. On Monday she baked 24 of everything.
How many boxes did she need? Fill in the empty spaces.

muffin boxes

$\qquad$ 44
On Tuesday she baked just bagels. She filled 7 boxes.
How many bagels did she make?

Show your calculations.

2. On Wednesday she baked 42 cookies.
How many boxes did she fill?
How many cookies were left over?

Student $B$ is starting to make sense of dimensions anal ysis and has clear labels for the work in pat 2 and 4 . While the student does computationsthat yield an answer of 42 in part 3, the strategy does not make sense in this context. How could you help this student to understand her mistake? What question would you pose? How could you facilitate a class discussion around this solution to bring out the big mathematical ideas?

## Student B

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

donut boxes

bagel boxes $\frac{4 \text { boxes }}{6 \times 4=24}$
$\qquad$
2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make?


Show your calculations.

3. On Wednesday she baked 42 cookies.

How many boxes did she fill?
How many cookies were left over?
Explain how you figured this out.
 then the answer was 48 . I subtracted 48 with 1 his and I got $42 . V$
4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday?


Show how you figured this out.



Now look at the explanation for part 3 of Student C. Notice how the student clearly describes the purpose of the cal culations and how they relate to the situation. Although the student uses multiplication to solve the problems, the student's grasp on the situation and underlying meaning of multiplication/division is evident.

## Student C

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

donut boxes

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make?
Show your calculations.
3. On Wednesday she baked 42 cookies.

How many boxes did she fill?
How many cookies were left over?
Explain how you figured this out.


4. On Thursday she baked 32 of just one item and she filled 8 boxes. What did she bake on Thursday? Show how you figured this out.


Student D does notunderstand division. The student uses multiplication in part 1. The student did a drawing and counting strategy for part 3. In part 4 the student has to use repeated addition to find the solution. How does context help students to build an understanding of operation? How mightlabels help the student to make sense of the situation? What question mightyou pose?

## Student D

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.
$\frac{x_{2}^{12}}{48}=$ cookie boxes $\quad 288 \lambda$
$\frac{1240}{288}=$ muffin boxes $48 \quad$
$\frac{r_{2}^{24}}{482 \text { 2. On Tuesday she baked just bagels. Sher }}$
How many bagels did she make?
Show your calculations.
$\frac{\times \frac{6}{4}}{\frac{6}{42}} 6 \times 7=4$
3. On Wednesday she baked 42 cookies.

How many boxes did she fill?
How many cookies were left over?


Explain how you figured this out.
 60 xes and lef 2 cookir $60 x e s$ half way $x$
4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday? $\qquad$
Show how you figured this out.
$4,8,1216,20,24,28,325$. I counted 64,4 to 10
 Resource Service. All rights reserved. $6 a \%$ ed on Thursday.

Student E uses an alternate division al gorithm in part 3 and 4, which would have yielded the correct solution except for a subtraction slip. Can you understand this method? Could you use it to solve another problem? How does it compare to the standard algorithm?

## Student E

11

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.
muffin boxes $\qquad$ donut boxes $\frac{6 \text { bones }}{4 \text { bootes boxes }}$ 44
2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make?


Show your calculations.

3. On Wednesday she baked 42 cookies.

How many boxes did she fill?


Explain how you figured this out.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday?


Show how you figured this out.


Students, who were not comfortable with division, had other strategies to make sense of the situation. See the work of Student F.

## Student F

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

2. On Tuesday she baked just bagels. She filled 7 boxes. 11
How many bagels did she make?
Show your calculations.


On Wednesday she baked 42 cook
How many boxes did she fill?
How many cookies were left over?
Explain how you figured this out.
On Wednesday she baked 42 c
How many boxes did she fill?
How many cookies were left
Explain how you figured this
$42-36=6$

$\qquad$
4. On Thursday she baked 32 of just one item and she filled 8 boxes.


## Some students did notrecognze filling boxes as a distributing or division action. Student G addsall the amounts together and finds that they total 24.

## Student G

- The baker uses boxes of different sizes to carry her goods.


Cookie boxes hold 12 cookies.
Donut boxes hold 4 donuts.
Muffin boxes hold 2 muffins.
Bagel boxes hold 6 bagels.

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.
cookie boxes

donut boxes

muffin boxes $\qquad$ bagel boxes $\qquad$

Student H is still not comfortable with multiplication and division and uses repeated addition for everything. How do we help students move to more efficient strategies? Notice that in part 1 the student does not understand the operation needed and repeats the information in the table. In part 3 the student has a strategy that yields the correct solution, butthe student doesn't recognize which part represents the remainder. What mightbegoodnext steps for this student?

## Student H

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make?


Show your calculations.

3. On Wednesday she baked 42 cookies.

How many boxes did she fill?
How many cookies were left over?


Explain how you figured this out.
$12+12+12+6=42$ So $42 \div 12=3 \quad 0$
So 3 are left over. I multiplyed to
get the answer for How many Dox-e
4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday?
Show how you figured this out.



Student I undestandsthat the context is a division operation. It is undear aboutwhether the student can do the arithmetic of division. The technology of the cal culator obscures some of the information of the task. The student doesn't understand that the 0.5 means half of the group of twelve or 6 . The mathematical understanding necessary to use a calculator in this task mightbemore complicated than understanding the arithmetic.

## Student I

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.
muffin boxes $\qquad$

2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make?
Show your calculations.

3. On Wednesday she baked 42 cookies.

How many boxes did she fill?


How many cookies were left over?


Explain how you figured this out.

4. On Thursday she baked 32 of just one item and she fitted $\delta$ boxes.

What did she bake on Thursday?


Show how you figured this out.


While Student received 6 points, there are some alarming ideas in the work. Notice that in part one the student was undear aboutoperations and tried a variety of strategies to get the answer. In part two the student did a drawing and counting strategy. Do you think this was an attempt to please the teacher (show the work) or do you think the student needed to think aboutwhat the answer would be? The student doenn't recognize the operations in part 3. Fill means add and left means subtract. Notice that the answer in part 4 is the logic of elimination rather than computation.

## Student J

1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.

2. On Tuesday she baked just bagels. ${ }^{\text {S }}$ he filed 7 boxes.

How many bagels did she make?
Show your calculations.

3. On Wednesday she baked 42 cookies. How many boxes did she fill?
How many cookies were left over? $\frac{42}{30}$


Explain how you figured this out.

4. On Thursday she baked 32 of just one item and she filled 8 boxes. What did she bake on Thursday?
Show how you figured this out.

```
Cause it did not
talk obout Ponuts,


Student K al so struggles with operations. Thestudent doesn't understand division in part 2. Why doesn't 28 make sense in this situation? Where do you think the 28 comes from? In part 4 the student interprets fill as multiply and left as subtract. What questions would you like to ask the student?

\section*{Student K}
1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.
muffin boxes \(\qquad\) 
donut boxes

bagel boxes

2. On Tuesday she baked just bagels. She filled 7 boxes. How many bagels did she make?


Show your calculations.

3. On Wednesday she baked 42 cookies.

How many boxes did she fill?
How many cookies were left over?
Explain how you figured this out.


4. On Thursday she baked 32 of just one item and she filled 8 boxes. What did she bake on Thursday? Show how you figured this out.


Now look at the work of Student L. Notice that in part onethestudent multiplies. In part 2 the students attempts a drawing strategy, but doesn't even make the amountthe same in each box. Understanding "equad-size groups" is fundamental to all work in multiplication, division, and proportional reasoning. In pat 4 the student multiplies and then subtracts. Where does the "-201" come from? This student is obviously missing some very important background bits in working with numbers, althoughthe student can do multiplication and seems to know multiplication facts (the carry the one indicates the student wasn't using a cal culator). How do we help students understand the meaning of operations? How is this different from learning facts and algorithms?

\section*{Student L}
1. On Monday she baked 24 of everything.

How many boxes did she need? Fill in the empty spaces.
cookie boxes \(\frac{96 X}{48 X}\)
muffin boxes 48
donut boxes \(\qquad\) 0
bagel boxes \(\qquad\)
2. On Tuesday she baked just bagels. She filled 7 boxes.

How many bagels did she make?


Show your calculations.

3. On Wednesday she baked 42 cookies.

How many boxes did she fill?
How many cookies were left over?


Explain how you figured this out.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.

What did she bake on Thursday? \(\qquad\)
Show how you figured this out.

\begin{tabular}{|c|c|}
\hline \(4^{\text {th }}\) Grade & Task 2 The Baker \\
\hline Student Task & Choos and perform number operationsin a practical context. Use multiplicative thinking and grouping to make sense of a context. \\
\hline \begin{tabular}{l}
CoreIdea 2 \\
Number Operations
\end{tabular} & \begin{tabular}{l}
Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently. \\
- Understand division as the inverse operation of multiplication, the operation of sharing (patitive), patitioning (measurement), repeated subtraction, and an operation to determine rates. \\
- Develop fluency with basic number combinationsfor multiplication and division and use these combinations to mentally compute related problems. \\
- Develop fluency in multiplying whole numbers.
\end{tabular} \\
\hline
\end{tabular}

Based on teacher observation, this is what fourth graders knowand are able to do:
- Use division to find the number the number of boxes for 24 of something
- Multiply to find thenumber of bagels needed to fill 7 boxes

Areas of difficulty for fourth graders:
- Interpreting division with a remainder
- Undestanding that while \((4 \times 12)-6=(3 \times 12)+6\), the meaning in context is not the same
- Choosing appropriate operations

Strategies used by successful students:
- Using more than onedivision algorithm
- Using the relationship between multiplication and division to figure outnumber of groups
- Using repeated addition for multiplication and division
- Using labeds to make sense of cal culations

\section*{MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 4}

Task 2 - The Baker
Mean: \(6.83 \quad\) StdDev: 3.04
Table 21: Frequency Distribution of MARS Test Task 2, Grade 4
\begin{tabular}{|c|r|r|c|}
\hline \begin{tabular}{c} 
Task 2 \\
Scores
\end{tabular} & \begin{tabular}{c} 
Student \\
Count
\end{tabular} & \begin{tabular}{c} 
\% at or \\
below
\end{tabular} & \begin{tabular}{c} 
\% at or \\
above
\end{tabular} \\
\hline 0 & 413 & \(5.5 \%\) & \(100.0 \%\) \\
\hline 1 & 226 & \(8.6 \%\) & \(94.5 \%\) \\
\hline 2 & 330 & \(13.0 \%\) & \(91.4 \%\) \\
\hline 3 & 252 & \(16.4 \%\) & \(87.0 \%\) \\
\hline 4 & 496 & \(23.0 \%\) & \(83.6 \%\) \\
\hline 5 & 440 & \(28.9 \%\) & \(77.0 \%\) \\
\hline 6 & 610 & \(37.1 \%\) & \(71.1 \%\) \\
\hline 7 & 583 & \(45.0 \%\) & \(62.9 \%\) \\
\hline 8 & 1373 & \(63.4 \%\) & \(55.0 \%\) \\
\hline 9 & 914 & \(75.7 \%\) & \(36.6 \%\) \\
\hline 10 & 1814 & \(100.0 \%\) & \(24.3 \%\) \\
\hline
\end{tabular}

Figure 30: Bar Graph of MARS Test Task 2 Raw Scores, Grade 4


The maximum score available on this task is 10 points.
The minimum score for a level 3 response, meeting standards, is 5 points.
Most students, \(90 \%\), could use multiplication to find the number of bagels needed to fill 7 boxes. Many students, \(84 \%\), could find the number of bagels to fill 7 boxes, and could use division to find 32 of what items would fill 8 boxes. \(77 \%\) could find out how many boxes of cookies, donuts, and muffins 24 would fill. More than half the students could solve all parts of the task except part 3, involving division with a remainder. \(24 \%\) could meet all the demands of the task. Almost \(6 \%\) scored no points on the task. All the students in the sample with this score attempted the task.

The Baker
\begin{tabular}{|c|c|c|}
\hline Points & Understandings & M isunderstandings \\
\hline 0 & All the students in the sample attempted the task. & Students had difficulty choosing opeations Many used subtraction or addition instead of multiplication and division. \\
\hline 1 & Students could solve part 4, finding that it would take 32 donut to fill 8 boxes. & 10.5\% of the students chose donut, but showed no work. 4\% thouglt the answer was muffins. \\
\hline 2 & Students could find the number of bages needed to fill 7 boxes and show their work. & 4\% thoughtthere would be \(14.2 \%\) thought 13 were needed. \(7 \%\) had answers higher than 50 . \\
\hline 4 & Students could solve part 2 and 4 of the task and show their work. & Students had trouble interp reting part 1. \(10.5 \%\) just repeated the given information ( 12 cookies). \(9 \%\) multiplied ( \(4 \times 24\) ) instead of dividing (24/4). 4\% added (24 +12 ). \\
\hline 5 & Students with this score could usually do 3 of the items in part 1 and find the answer with no work in part 4. & \\
\hline 6 & & M ost students with this score missed all of pat 1 , dividing 24 to find the number of boxes. \\
\hline 8 & & M ost students with this score missed all of part 3, interpreting division with a remainde. \(14 \%\) of the students thoughtthat 42 cookies could fill 4 boxes with 6 cookies left over ( \(4 \times 12-6\) ). \\
\hline 10 & Students could recognize multiplication and division contexts and solve problems, including interpreting remaindes in division. Students with this score seemed to undestand the relationship between operations and could use multiplication, division, or repeted addition to get ther answers. & \\
\hline
\end{tabular}

\section*{Implications for Instruction}

Students need to do work with division in context to help them understand the meaning of division, making and counting equad size groups In this task, many students could write number sentences that could yield a correct solution, ( \(4 \times 12\) ) - \(6=42\), butwhich did notfit the action of the story or yield the needed information of 3 groupsof 12 with 6 remaining. Context provides for the backdrop for struggling with sense-making: What do I know? What do I need to find out? In this situation, how do I interpret the remainder? These are important ideas around division that are notfocused on when just practicing procedures.

Modeds are useful tools to help students with understanding the meaning of operation. Division has two distinct meanings a partitive mode - sharing thingsor meting things out into equad groupsand a measurement modd - measuring how many equad size groups can be measured or madefrom a given quantity. The bar modeds are hel pful tools to make students visual ize the division situations.
\begin{tabular}{|c|c|c|}
\hline Interpretation & Interpretive question & Diagram \\
\hline Partitive division: & 20 is 4 groups of what unit? &  \\
\hline Measurement division: & 20 is how many groups of 4 ? &  \\
\hline
\end{tabular}

Can you think of a word problem that would go with each of the modds above? The division in this task is making sense of measurement division how many groupsor boxes of a certain item. Students are generally most familiar with partitive division. I have so many candies to share equally with my friends. Students need to becomfortable with a variety of types of division problems.

Below are some modes for interpreting remainders:

40 eggs is how many dozen?


3 groups with 4 left over.

How many feet in 40 inches?


Three feet with 4 inches left.
\(40 \div 12=3 \mathrm{R} 4\)

I deas for Action Research - Exploring M odels with Students Think about problems that use different types of division. For example, consider these two problems:

5 cases of peaches weigh 450 lb . How much does each case way?
J ason's mom made 210 cupcakes for the school bake sale. She puts them into boxes of 10 each. How many boxes of cupcakes were there?

What would the bar modds for each of these look like? How are the modeds different?

Introduce your students to the bar modds and see what students think of the modds. Do they find them interesting? How quickly do they make sense of modds? What evidence do you see of changes in student thinking after being introduced to the bar modeds?

Thebar modds help students in a variety of ways for learning multiplication and division; and, the set the stage for thinking aboutproportional reasoning and understanding variable at later gradelevels. For more example of bar modds, see Singapore math books 3A, 4A, and 5A (available from www.Singaporemath.com).
\begin{tabular}{|c|}
\hline Performance Assessment Task \\
\hline The Baker \\
Grade 4 \\
\hline
\end{tabular}

The task challenges a student to demonstrate understanding of the concepts involved in multiplication and division. A student must make sense of the relationship between multiplication and division. A student must understand division as the inverse of multiplication, the operation of sharing (partitive), partitioning (measurement), and repeated subtraction. A student must be able to recognize and use the factors of a given quantity. A student must determine a product and a quotient. A student must make sense of a division contextual problem with a remainder and without a remainder.

\section*{Common Core State Standards Math - Content Standards}

\section*{Numbers and Operations in Base Ten}

Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT. 5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
4.NBT. 6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

\section*{Operations and Algebraic Thinking}

Use the four operations with whole numbers to solve problems.
4.0A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

\section*{Common Core State Standards Math - Standards of Mathematical Practice} MP. 1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

\section*{MP. 6 Attend to precision.}

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of
precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

\section*{Assessment Results}

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.
\begin{tabular}{|c|c|c|c|c|}
\hline Grade Level & Year & Total Points & Core Points & \% At Standard \\
\hline 4 & 2007 & 10 & 5 & \(77 \%\) \\
\hline
\end{tabular}```

