Problem of the Month
Surrounded and Covered

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

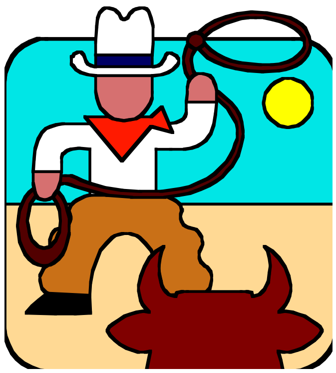
In the Problem of the Month *Surrounded and Covered,* students use one- and two-dimensional measurement to solve problems involving area and perimeter. The mathematical topics that underlie this POM are the attributes of linear measurement, square measurement, two-dimensional geometry, perimeter, area, functions, fractal geometry, recursion, and infinity.

The problem asks students to explore the relationship of area and perimeter in various problem situations. In the first level of the POM, students are presented with 12 squares that make a 2 by 6 rectangular patio. The students are asked to rearrange patio tiles to make rectangular figures with different perimeters. They are also asked whether they can make a square patio from the 12 square tiles. In Level B, students are presented with irregular shapes on grid paper and are asked to determine the figure with the largest area and the largest perimeter. In Level C, students explain the results of changing the area of a figure when the linear dimensions are changed by a scale factor. Students must make sense of why the price per square foot of sod must change at a different rate than linear scale factor when enlarging a geometric shape. In Level D, the students explore the concepts of maximizing area given a fixed perimeter. The students grapple with what quadrilateral will produce the largest area. Then the students explore other polygons and finally are confronted with finding the geometric shape that will produce the largest area given a fixed length boundary. In Level E, students explore area and perimeter in terms of fractal geometry. Students are introduced to Sierpinski's Triangle and are asked to determine the area and perimeter of the fractal at each stage of the fractal's development.

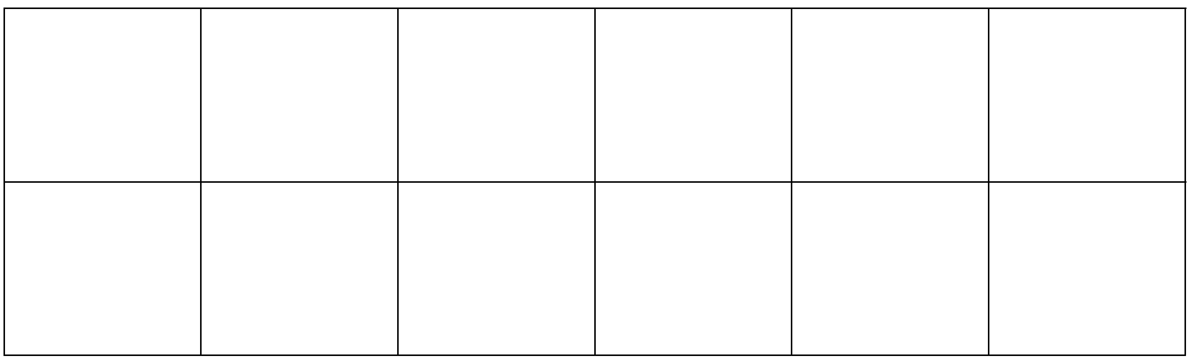
<u>Problem of the Month</u>

Surrounded and Covered

Level A:



Tex has a home on the range (where the deer and the antelope play). He has a patio made out of square tiles. The tiles are a foot long and a foot wide. How many tiles are in his patio?



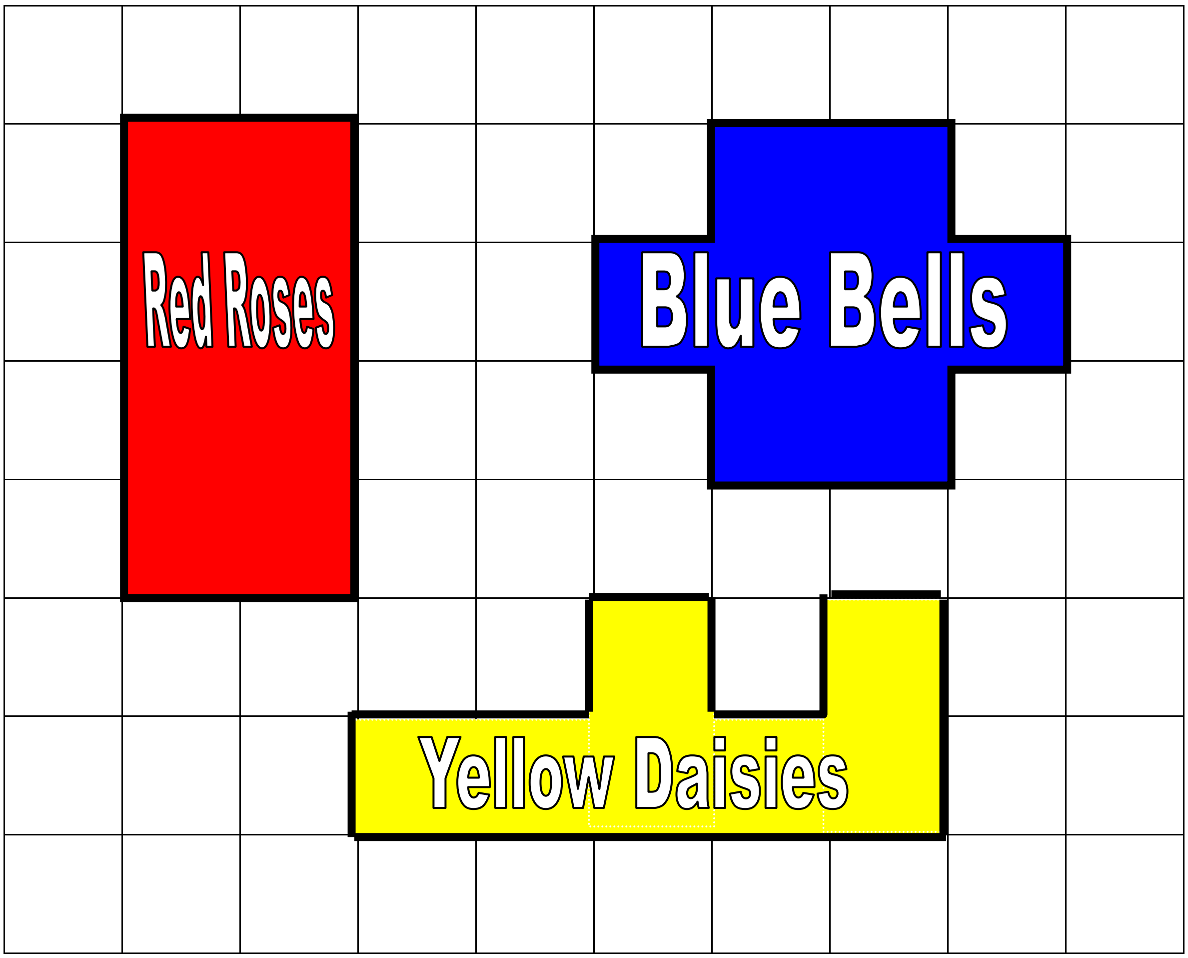
Arrange the tiles so that they make a different shaped rectangle. Draw a picture of the new rectangle.

Can you arrange the tiles so that you make a square? Why or why not?

How many more tiles do you need to make a big square patio? Draw the new patio and tell how many tiles there are in all.

Level B:

Tex decides to build his patio with three flowerbeds. The flowerbeds are different shapes. In the first he plants red roses. In the second he plants blue bells and in the third he plants yellow daisies. Which flowerbed has more space to plant the flowers? Explain how you found your answer.



Tex is going to build a little white fence around his flowerbeds. Measure around each flowerbed to see how long the fence must be. How long around is each fence? Explain how you found your answers.

Level C:

Tex wants to make a lawn in the front of his house. He buys enough sod to make a 10-foot by 30-foot rectangular section of sod for $200 to see how well it grows and how durable it is when played on. After a week he decides he needs more sod, so he doubles the dimensions to 20 feet by 60 feet and sends in a check for $400. The sod company calls and says he still owes money for the second order of sod. Explain why he still owes money and how much he owes.

Tex wants his lawn to be 100 feet by 120 feet. How much more sod does he need and how much more does he need to pay? Explain how you figured it out.

Level D:

Tex has a home on the range (where the deer and antelope play). He wants to build a rectangular corral for his horses. He only has 170 feet of fencing. What size of corral should be built to make sure the horses have the most room? List the dimensions and area. Justify how you know the corral is as large as possible. Explain how you figured it out.

Tex gets a better idea. He has a barn on his ranch that is 80 feet by 120 feet. He decides to build the corral using one side of the barn and the 170 feet of fencing. What should be the dimensions of the corral now? What is the area? Explain your mathematical reasoning.

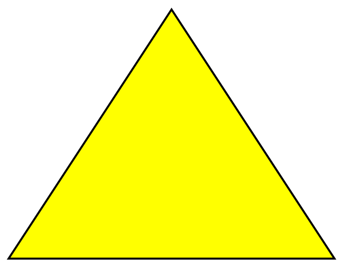
A year later, Tex needs a second corral. This time he has 240 feet of fencing. He doesn't want to use the other side of the barn, because it is near a small pond. He picks out a new location and realizes that he does not need to make the corral a rectangle. He designs a corral in the shape of a hexagon. What are the lengths of the sides and what is the area of the corral? Explain how you found your answer.

Tex thinks that maybe another shape would make an even larger area for his corral. Determine what the shape should be and its area and dimensions. Justify your answer using mathematical reasoning.

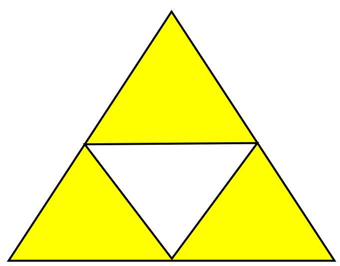
Level E:

A fractal is a geometric figure that has self-similarity, that is created using a recursive process, and that is infinite in structure. A fractal is an object whose detail is not lost as it is magnified. In fact, the structure looks the same as the original. In contrast, a circle appears to become straight as a portion of it is magnified.

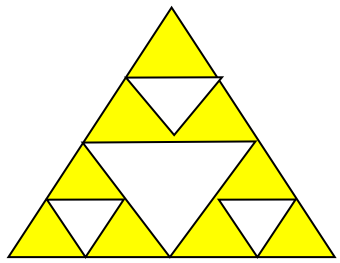
Sierpinski's Triangle is a fractal. This fractal is generated through a recursive process. The fractal can be viewed and analyzed at each stage of development. In its original stage the fractal is an equilateral triangle. For our purposes we will call this initial shape stage 0. For measurement purposes we will say it has a perimeter of three linear units and one area unit.



During the first iteration of the recursive process, a triangle is cut out of the center of the original equilateral triangle. The smaller cut-out triangle is similar to the original but with a length of 1/2 the stage 0 length. Therefore the perimeter, defined as the distance bordering the area has increased by 3/2 linear units, making the perimeter 4 1/2 in length. The area has been decreased by 1/4, making the new area 3/4 area units in size. Below is Sierpinski's Triangle at stage 1.



The process is repeated in stage 2, with three smaller triangle cut-outs of the remaining three sections of the original triangle. What is the total perimeter at stage 2? What is the total area at stage 2? Explain your calculations.



Sierpinski's Triangle is generated following this recursive process. Draw a picture of stage 3. Determine the area and perimeter of the triangle at this stage. Explain the mathematics.

Sierpinski's Triangle is created using the recursive process indefinitely. The fractal has an infinite structure. Find the area and perimeter of Sierpinski's Triangle for any stage n. Justify the mathematics.

Find the actual area and perimeter of Sierpinski's Triangle. Justify your solution.

Problem of the Month

Surrounded and Covered

Primary Version Level A

Materials: Some type of square tile, twelve manipulative tiles per group, picture of the patio tiles (2 by 6) for each student

Discussion on the rug: Teacher holds up a picture of the patio. **"This is a picture of a patio. What is a patio?"** Teacher solicits answers from students. **"This patio is made out of tiles. Each square in this picture is one of these tiles."** Teacher holds up a tile. **"What shape is this patio?"** Teacher might have students draw the outside perimeter to help students see the outside shape is a rectangle. **"How many tiles does it take to make this patio?"** Teacher solicits answers from students and has several students share how they counted them to find answer.

In small groups: Each group has twelve tiles and a picture of the patio. **"Check to see how many tiles you have."** The teacher solicits the counts from each student. The teacher lists the counts. **"Please make a copy of the patio with your tiles. Please draw a picture of your patio."**

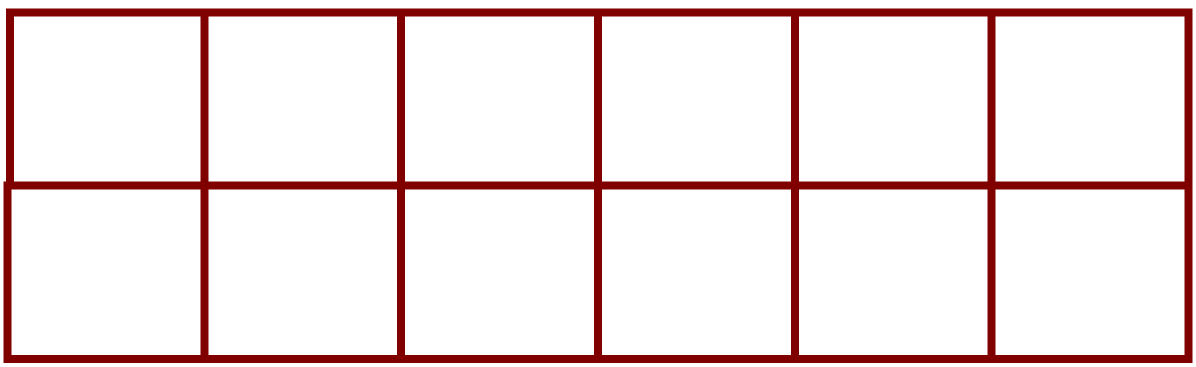
The teacher asks the following questions:
"Can you re-arrange the tiles to make a patio of a different shape? How many tiles do you have now? Please draw it on a piece of paper."

"Can you re-arrange the tiles to make a rectangle that is a different shape? How many tiles do you have now? Please draw it on a piece a paper."

"Can you re-arrange the tiles to make a square with all the tiles?"

Teacher questions whether it is really a square. **"Explain why you think it is or is not a square."** At the end of the investigation have students either discuss or dictate a response to these summary questions.

The Patio



Problem of the Month
Surrounded and Covered
Task Description – Level A
This task challenges a student to make and record rectangular patios from a set of 12 tiles. Students are challenged to make a square from the tiles and explain why it's not possible.
Common Core State Standards Math - Content Standards
<u>Measurement and Data</u> **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. 3.MD.3.a A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. 3.MD.3.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. 3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). 3.MD.7 Relate area to the operations of multiplication and addition. **<u>Geometry</u>** **Reason with shapes and their attributes.** 2.G.2 Partition a rectangle into rows and columns of the same-size squares and count to find the total number of them.
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.7 Look for and make use of structure.** Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Problem of the Month
Surrounded and Covered
Task Description – Level B
This task challenges a student to calculate area and perimeter for shapes on a grid. Students must be able to imagine the rows and columns inside the shape.
Common Core State Standards Math - Content Standards
<u>Measurement and Data</u> **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. 3.MD.5.a A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. 3.MD.5.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. 3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). 3.MD.7 Relate area to the operations of multiplication and addition. **<u>Geometry</u>** **Reason with shapes and their attributes.** 2.G.2 Partition a rectangle into rows and columns of the same-size squares and count to find the total number of them.
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.7 Look for and make use of structure.** Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Problem of the Month

Surrounded and Covered

Task Description – Level C

This task challenges a student to think about proportional relationships in the context area and enlarging shapes. Students need to investigate how a scale factor for the dimensions is different from the scale factor for the area of the shape. Students need to use proportional relationships to find the cost of sod for various areas.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.5.a A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

3.MD.5.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

3MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Geometry

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide

Problem of the Month

Surrounded and Covered

Task Description – Level D

This task challenges a student to explore the concepts of maximizing area given a fixed perimeter. Students grapple with what quadrilateral will produce the largest area. Then students explore area and perimeter with other quadrilaterals and polygons still trying to maximize area for a fixed perimeter.

Common Core State Standards Math - Content Standards

<u>Measurement and Data</u>

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

<u>Geometry</u>

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.

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Problem of the Month
Surrounded and Covered
Task Description – Level E
This task challenges a student to use area and perimeter in the context of fractal geometry and Sierpinski's Triangle.
Common Core State Standards Math - Content Standards
<u>**Geometry**</u> **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** 7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the fist by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. <u>**Expressions and Equations**</u> **Solve real-life and mathematical problems using numerical and algebraic expressions equations.** 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. 7.EE.4 Use variable to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. <u>**Functions**</u> **Define, evaluate, and compare functions.** 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. 8.F.3 Interpret the equation y=mx+b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. **Use functions to model relationships between quantities.** 8.F.4 Construct a function to model a linear relationship between two quantities Determine the rate of change and the initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph; interpret the rate of change and initial value of a linear function in terms of the situation it models, and in forms of the graph or table of values. 8.F.5 Describe qualitatively the function relationship between two quantities by analyzing a graph (e.g. where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. <u>**High School – Algebra – Creating Equations**</u> **Create equations that describe members or relationships.** A-CED.1 Create equations and inequalities in one variable and use them to solve problems, include equations arising from linear and quadratic functions, and simple rational and exponential functions. A-CED.2 Create equations in two or more variables to represent relationships between quantities, graph equations on coordinate axes with labels and scales. A-CED.3 Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

High School – Geometry – Similarity, Right Triangles, and Trigonometry
Understand similarity in terms of similarity transformations.
G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.

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MP.3 Construct viable arguments and critique the reasoning of others.
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Task Description – Primary Level
This task challenges a student to identify geometric shapes and make and draw shapes using square tiles. Students investigate whether it is possible to make other rectangles and make a square. Students can record their work by drawing the perimeters.
Common Core State Standards Math - Content Standards
<u>Counting and Cardinality</u> **Know number names and the count sequence.** K.CC.1 Count to 100 by ones and by tens. K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects. **Count to tell the number of objects.** K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20 count out that many objects. **<u>Operations and Algebraic Thinking</u>** **Represent and solve problems involving addition and subtraction.** 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions. **Add and subtract within 20.** 1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, make ten, decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums. **<u>Geometry</u>** **Analyze, compare, create, and compose shapes.** K.G.5 Model shapes in the world by building shapes from components and drawing shapes. K.G.6 Compose simple shapes to form larger shapes. **Reason with shapes and their attributes.** 2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and

previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.