Problem of the Month
Squirreling It Away

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *Squirreling It Away,* students use number operations, organized lists, and counting methods to solve problems. The mathematical topics that underlie this POM are knowledge of number sense, comparison subtraction, division, factors and divisibility, counting principles, systematic charting, and closed-form equations. The mathematics that includes counting principles and systemic charting is often referred to as discrete mathematics.

In the first level of the POM, students are presented with a situation that involves making sense of totals and comparison differences. The task involves making a number story about giving acorns to two different-sized groups of squirrels and then determining how many were left over from the original total acorns. In Level B, students start to examine how the acorns can be partitioned into different sets. The students are told that different squirrels can carry different amounts of acorns on given trips. Students are asked to find the number of trips it takes for each type of squirrel to carry the acorns. In Level C, students are asked how many ways three different types of squirrels can carry away 24 acorns. In Level D, the students determine the number of ways 24 acorns can be divided between three specific squirrels. In Level E, students are asked to find and justify a closed-form equation that will determine the number of ways that n acorns can be divided between three squirrels.

Problem of the Month

Squirreling It Away

Level A:

Austin had a bag of 17 acorns. Eight squirrels came up to him. He gave each squirrel an acorn. Then five more squirrels came up to him and he gave away one acorn to each of them. How many more squirrels could he still feed?

Show how you figured it out?

How do you know you have the right answer?

Level B:

Austin likes to watch squirrels find and store acorns for the winter. Brown Squirrels can carry two acorns at a time. Gray Squirrels can carry three acorns at a time and Black Squirrels can carry five acorns at a time. There is a pile of 24 acorns.

How many trips would a Brown Squirrel need to make to store all of the acorns in the pile?

How many trips would a Gray Squirrel need to make to store all of the acorns in the pile?

How many trips would a Black Squirrel need to make to store all of the acorns in the pile?

If all three squirrels worked together to store the acorns how many trips would the squirrels need to make to store all of the acorns?

Explain your solution.

Level C:

Brown Squirrels can carry 2 acorns at a time. Gray Squirrels can carry 3 acorns at a time. Black Squirrels can carry 5 acorns at a time.

Suppose the three squirrels all wanted to store acorns for the winter. Depending on how motivated each squirrel was, they would end up with different amounts. For instance suppose the Brown Squirrel took 4 trips, the Gray Squirrel took 2 trips and the Black Squirrel took 2 trips. The Brown Squirrel would end up with 8 acorns, the Gray Squirrel would have 6 acorns and the Black Squirrel would have 10. Between them they took every one of the 24 acorns.

How many different ways could the three Squirrels divide up the 24 acorns and not leave any left over? Each Squirrel must carry his maximum load on each trip.

How do you know that you have found all of the ways?

Level D:

The squirrels are rather smart. They realize that they can carry less than their maximum loads. How many different ways could the squirrels divide up the 24 acorns?

Explain your solution.

Level E:

Suppose there are a different number of acorns than 24. Determine a generalization for finding how 3 squirrels can divide up any given number of acorns.

Explain your solutions.

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Squirreling It Away

Primary Version

Materials: A set of acorns or cubes (1-10) for each pair of students, paper and pencil, crayons, or markers

Discussion on the rug: "Here are some acorns. What animal likes to eat acorns?" Teacher continues to ask children to name animals who like acorns. Teacher holds five acorns in her hand. "Suppose I had five acorns and one squirrel came up to me and I gave it an acorn How many would I have left?" The teacher encourages the students to find answers for different amounts of acorns and to ask the students to explain how they know.

In small groups: Each group has a set of acorns or cubes.
Teacher asks the following questions and goes on to the next question when students have success.

1. "You have 10 acorns. Four squirrels come to you. You give each squirrel one acorn. How many acorns did you give? How many are left? Now two more squirrels come to you and you give them each an acorn. How many did you give now? How many are left? How many more squirrels can you feed?"

2. Select a set of numbers that is reasonable for your class. "You have __ acorns. __ squirrels come to you. You give the squirrel each one acorn. Now __ more squirrels come to you and you give them each an acorn. How many more squirrels can you feed?" At the end of the investigation, have students either draw a picture or dictate to you to represent their solutions.

Problem of the Month
Squirreling It Away
Task Description – Level A
This task challenges a student to reason about repeated subtraction in a word problem to find the number of acorns left after giving set amounts to two different sets of squirrels.
Common Core State Standards Math - Content Standards
<u>Operations and Algebraic Thinking</u> **Represent and solve problems involving addition and subtraction.** 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problems.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.2 Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Problem of the Month
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Task Description – Level B
This task challenges students to solve problems involving subdividing a whole amount into sub-sets. Students need to think about equal groups or maximum-sized groups and to interpret remainders in context.
Common Core State Standards Math - Content Standards
<u>Operations and Algebraic Thinking</u> **Represent and solve problems involving multiplication and division.** 3.OA.1. Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. 3.OA.2 Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares,, or as a number of objects in each share when 56 objects are partitioned into equal shares of 8 objects each. 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and mathematical quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problems. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Ass the reasonableness of answers using mental computation and estimation strategies including rounding. **Use four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.2 Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Problem of the Month
Squirreling It Away
Task Description – Level C
This task challenges a student to use division and equal-sized groups to find all the possible ways the 3 squirrels can carry away exactly 24 acorns if they take the maximum amount every trip. Students are challenged to organize their thinking to develop a convincing argument about how they know they have found all the possibilities.
Common Core State Standards Math - Content Standards
<u>Operations and Algebraic Thinking</u> **Represent and solve problems involving multiplication and division.** 3.OA.1. Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. 3.OA.2 Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares,, or as a number of objects in each share when 56 objects are partitioned into equal shares of 8 objects each. 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and mathematical quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problems. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Ass the reasonableness of answers using mental computation and estimation strategies including rounding. **Use four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Gain familiarity with factors and multiples.** 4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of the factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is a prime or composite. **<u>Statistics and Probability</u>** **Investigate chance processes and develop, use, and evaluate probability models.** 7.SP.8 find probabilities of compound events using organized lists, tables, tree diagrams and simulation.
Common Core State Standards Math – Standards of Mathematical Practice
MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.8 Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
Squirreling It Away
Task Description – Level D
This task challenges a student to determine the number of ways 24 acorns can be divided between three specific squirrels, if they don't have to carry the maximum on every trip. Students must develop a convincing argument to explain their strategy and how they know they have every possibility.
Common Core State Standards Math - Content Standards
<u>Operations and Algebraic Thinking</u> **Represent and solve problems involving multiplication and division.** 3.OA.1 Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. 3.OA.2 Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares,, or as a number of objects in each share when 56 objects are partitioned into equal shares of 8 objects each. 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and mathematical quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problems. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Ass the reasonableness of answers using mental computation and estimation strategies including rounding. **Use four operations with whole numbers to solve problems.** 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **<u>Statistics and Probability</u>** **Investigate chance processes and develop, use, and evaluate probability models.** 7.SP.8 find probabilities of compound events using organized lists, tables, tree diagrams and simulation.
Common Core State Standards Math – Standards of Mathematical Practice
MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.8 Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
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Task Description – Level E
This task challenges a student to find and justify a closed form equation that will determine the number of ways that n acorns can be divided between three squirrels.
Common Core State Standards Math - Content Standards
Operations and Algebraic Thinking **Analyze patterns and relationships.** 5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. **Expressions and Equations** **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **Statistics and Probability** **Investigate chance processes and develop, use, and evaluate probability models.** 7.SP.8 find probabilities of compound events using organized lists, tables, tree diagrams and simulation. **High School – Algebra – Creating Equations** **Create equations that describe numbers or relationships.** A-CED.1 Create equations and inequalities in one variable and use them to solve problems, include equations arising from linear and quadratic functions, and simple rational and exponential functions.
Common Core State Standards Math – Standards of Mathematical Practice
MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.8 Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation (y -2)/(x-1) = 3. Noticing the regularity in the way terms cancel when expanding (x-1)(x+1), (x-1)(x^2 + x + 1), and (x-1)(x^3 + x^2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
Squirreling It Away
Task Description – Primary Level
This task challenges a student to think about subtraction and taking away. Students use cubes to help them act out the situation and record their ideas with paper and markers.
Common Core State Standards Math - Content Standards
<u>Counting and Cardinality</u> **Know number names and the count sequence.** K.CC.1 Count to 100 by ones and by tens. **Count to tell the number of objects.** K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20 count out that many objects. **<u>Operations and Algebraic Thinking</u>** **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations verbal explanations, expressions or equations. **Represent and solve problems involving addition and subtraction.** 1.OA.2 Solve word problems that call for addition of three whole numbers, whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.