Problem of the Month: *Sorting the Mix*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used school wide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month, *Sorting the Mix,* students use mathematical concepts of statistics including average, consistency, and optimization. The mathematical topics that underlie this POM are measures of center, measures of dispersion, the distribution of data, and the interpretation of data sets. The later levels involve mixture problems that require students to understand rate as a measure. Mixture problems involve understanding a set of unknowns and a set of constraints. Students use equations to determine the values of unknowns.

In Level A of the POM, students examine and make sense of a data set around the measures of center and the range. In Level B, the students are given two sets of temperatures from cities in the coldest parts of the lower 48 United States. The data includes sign numbers of temperatures reading in Celsius. Students are to make arguments why one city is colder than the other city. They then have to use the same data to make the reverse argument. In Level C, students investigate a problem involving the statistical measure of the mean. They use their conceptual understanding of that measure to solve problems. In Level D, the problem is posed that an all girls' school invites a busload of boys from an all boys' school to a dance, then the bus takes a same-size load of students, boys and girls, back to the boys' school dance. The question is which school has a greater ratio of one gender to the other. In Level E, the students are given four problems that involve mixtures. The students are to solve the problems and explain the solutions so that the characters in the problem conceptually understand the mathematics.

Mathematical Concepts

Statistical measures are used to find the center, middle or typical value of a set of data. Often this value is referred to as the average of the data. In statistics, there are several measures that may be used to represent the *average* of a set of data. The most common of these are called *measures of center* and include the mean, the median, the mode, and the midrange.

Problem of the Month

Sorting the Mix

Level A

Ms. Gavin asked her class to make a chart of the people who lived with them, including the students themselves. The nineteen students each recorded their number in the following chart.

8	4	2	10	4	5	6
6	3	6	8	7	3	5
2	6	8	6	4		

What is the most common number of people who live together (mode)? Explain.

What is the middle number if all numbers were arranged in order (median)? Show how you figured it out.

Suppose two new students joined the class. They each add the number of people who live in their homes to the chart. The middle number now switched to a different number. What numbers could have been added? Explain.

Not only did the middle number change, but now the most common number also changed. Which numbers could have been added?

You find out that the middle number changed to a smaller number; what is the new middle number? What is the new, most common number? What numbers were added? How do you know for sure?

Level B

Two different cities in the lower 48 states of America claim they are the coldest cities. The cities are Rogers Pass, Montana, and Peter's Sink, Utah. Below are tables that indicate the minimum and maximum temperature for each month last year. All temperatures are measured in Celsius.

Rogers Pass, Montana	Minimum Temperature	Maximum Temperature
Jan	-12	0
Feb	-9	3
Mar	-6	7
Apr	-1	12
May	3	17
Jun	7	22
Jul	10	26
Aug	9	26
Sep	4	20
Oct	0	14
Nov	-6	5
Dec	-10	1

Peter's Sink, Utah	Minimum Temperature	Maximum Temperature
Jan	-13	1
Feb	-7	4
Mar	-6	6
Apr	-2	10
May	4	18
Jun	6	21
Jul	12	28
Aug	7	26
Sep	4	19
Oct	-1	16
Nov	-7	4
Dec	-9	3

Analyze the temperatures in the tables. Compare the two cities. Which city would you say was the coldest in the lower forty-eight states? Use mathematical data to support your claim and explain your reasoning.

Suppose there was a contest to be named the coldest city in the lower forty-eight. If you were the mayor of Rogers Pass, what argument would you make to convince the judges your city was the coldest?

If you were the mayor of Peter's Sink, what argument would you make to convince the judges your city was the coldest?

Level C

A survey of 14 middle school students showed the mean number of minutes students talked on the cell phones in a week was 133. What is the total number of minutes those students talked in a week?

What was the typical amount of time a student talked in a day? Explain.

Another survey asked the same 14 students plus 5 other students how long they talked for the same week. This survey said the mean number of minutes all the students talked was 171 minutes.

What was the mean number of minutes the additional 5 other students talked? Show your solution.

What was the typical amount of time one of the 5 students talked in a day? Explain.

Level D

There are two schools and both schools have the same number of students. Hillary High is an all-girl school. Barack Academy is an all-boy school. Each school is holding a dance. A bus is completely filled with boys from the Academy and the bus takes the boys over to Hillary High to attend the dance. After an hour, students from the dance including both girls and boys decide to go over to the dance that is being held at Barack Academy. The same bus is filled with a combination of boys and girls. They travel back over to Barack Academy to attend that dance.

At that time, does Hillary High have more boys on campus than Barack Academy have girls on campus, or is it the other way around? Explain.

Level E

You work for a food distributor. You are the mathematician at the headquarters. You come in the morning and you get these four emails. Please answer each email and explain your reasoning, so that in the future they can figure these things out on their own and you won't continually be bothered by these requests.

Email from Poncho Bueno

Help!! Two hundred liters of a punch that contain 35% fruit juice was mixed with 300 liters of an unknown punch. We found the mixture is now 20% fruit juice. Can you please tell us what percent of fruit juice was in the 300 liters of unknown punch? We need to know so this mistake won't be made again.

Email from Mel K. Snax

Hope you can answer this question, because we need it ASAP in order to get our packaging labeled correctly. We added an additional ten grams of sugar to a 40 gram serving of our Flakey Oats breakfast cereal. That cereal comes naturally with 30% sugar. What is the percent concentration of sugar in the cereal now? We need that percentage for the label on the box.

Email from Ed A. Bull

Hey, we have a pricing problem in produce. We need to create a vegetable medley of lima beans and corn that costs 65 cents per pound. We have already added 16 pounds of corn that costs 50 cents per pound. We have lima beans that cost 90 cents a pound. How many pounds of lima beans should we add to make the price right? Please let us know quickly as these vegetables perish quickly.

Email from O. J. Englass

They added a new worker to our force. Without knowing it, he mixed an 800-liter container of the orange soda. But he made it so that it only has 15% concentration of orange juice. As you know, our orange soda must have exactly 20% concentration of orange juice. How much of the 100% concentration of pure orange juice do we need to add to his mixture to make it the 20% concentration required by our company? Please let us know soon, before he gets us all fired!!!

Problem of the Month

Sorting the Mix

Primary Version Level A

Materials: Post-it notes, class line-plot, individual line plot, square counters, paper and pencil.

Discussion on the rug: Teacher tells the class, **"We are going to make a special math chart to learn about our class. We call it a line plot, because it looks like a number line."** Teacher shows line plot – see example. Teacher says, **"I want you to think about how many brothers or sisters each of you have. Some of us don't have brothers or sisters. How many would that person have as a total?"** Students respond and teacher re-voices and writes 'zero' on a post-it note, then puts the note on the line plot above zero. **"This is how the student would show us on the line-plot."** The teacher asks and demonstrates other examples of calculating the total number of siblings, then makes a post-it [with words on post-its not numbers] and finally places the post-its on the line-plot. The teacher demonstrates how to put one post-it above another post-it if students have the same total. The teacher uses the language 'total of brothers and sisters' to distinguish between the frequency number of post-its on a particular total. Finally, the teacher clears the line plot and asks students to record the actual data with each student placing their post-it on the line-plot.

In small groups: Each student has access to square counters and a line-plot. The students are told to make an exact copy of their class' line-plot. Teacher checks and provides help. The teacher asks the students the following questions:

“Look at our line-plot, when there is no post-it on a total brothers and sisters, what does that mean?”

“Which totals are like that on our line-plot?”

“Which totals of brothers and sisters happen only once in our class? How do you know?”

“Which total of brothers and sisters is most common in our class? How do we know?”

“How many of our friends have that same total?”

“Can we count to make sure of that number?”

At the end of the investigation have students either discuss or dictate a response to these prompts:

“Which total of brothers and sisters happened only once?”

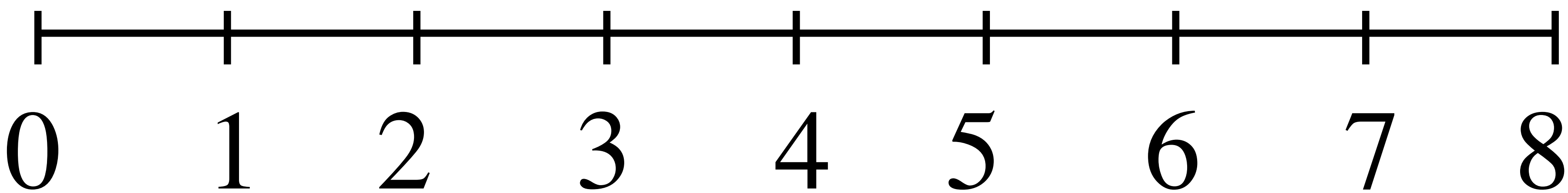
“Which total of brothers and sisters happened two times?”

“How many of our friends have a total of three brothers or sisters?”

“How can we tell by the line-plot?”

“Which total of brothers and sisters is most common in our class?” “How do we know?”

“How many of our friends have that same total?”


0
1
2
3
4
5
6
7
8

Problem of the Month
Sorting the Mix
Task Description – Level A
This task challenges students to use a data set and solve problems involving the measures of center. The students are presented data from a class about the number of people who live with them. Students examine and make sense using the mean, median and mode.
Common Core State Standards Math - Content Standards
Measurement and Data 1.MD **Represent and interpret data.** 4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.2 Reason abstractly and quantitatively**. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month
Sorting the Mix
Task Description – Level B
This task challenges a student to use sign numbers and measures of center to solve problems involving temperatures. The students are presented with a family tree. The students are given two sets of temperatures from cities in the coldest parts of the lower 48 United States. The data includes sign numbers of temperatures reading in Celsius. Students are to make arguments why one city is colder than the other city. They then have to use the same data to make the reverse argument.
Common Core State Standards Math - Content Standards
Operations and Algebraic Thinking 3.OA **Solve problems involving the four operations, and identify and explain patterns in arithmetic**. 8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Statistics and Probability 6.SP** **Develop understanding of statistical variability.** 3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **The Number System 6.NS** **Apply and extend previous understandings of numbers to the system of rational numbers.** 5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. 7. Understand ordering and absolute value of rational numbers.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month
Sorting the Mix
Task Description – Level C
This task challenges a student to investigate a problem involving the mean of a data set. The students investigate a problem involving the statistical measure of the mean. They use their conceptual understanding of that measure to solve problems. The students have to re-evaluate the mean when more students join the group.
Common Core State Standards Math - Content Standards
Statistics and Probability 6.SP **Develop understanding of statistical variability.** 3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **Summarize and describe distributions.** 5. Summarize numerical data sets in relation to their context, such as by: c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month:
Sorting the Mix
Task Description – Level D
This task challenges a student to investigate a problem involving mixture. The problem is posed that an all girls' school invites a busload of boys from an all boys' school to a dance, then the bus takes a same-size load of students, boys and girls, back to the boys' school dance. The question is which school has a greater ratio of one gender to the other.
Common Core State Standards Math - Content Standards
Statistics and Probability 7.SP **Use random sampling to draw inferences about a population.** 1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month
Sorting the Mix
Task Description – Level E
This task challenges a student to investigate real-life problems involving mixtures and rates. The students are given four problems that involve mixtures. The students are to solve the problems and explain the solutions so that the characters in the problem conceptually understand the mathematics.
Common Core State Standards Math - Content Standards
<u>High School – Algebra</u> **Creating Equations★ A -CED** **Create equations that describe numbers or relationships** 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Sorting the Mix

Task Description – Primary Level

This task challenges students to represent, determine and compare quantities. The students, as a class, construct a line plot of the number of brothers and sisters they all have. The students copy the line plot and then answer questions about the quantities on the graph.

Common Core State Standards Math - Content Standards

Measurement and Data K.MD

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

Measurement and Data 1.MD

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Measurement and Data 2.MD

Represent and interpret data.

10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems4 using information presented in a bar graph.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.