Problem of the Month
Piece it Together

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *Piece it Together,* students use two and three-dimensional geometry to solve problems involving polygons and polyhedra. The mathematical topics that underlie this POM are the attributes of polygons, linear measurement, angular measurement, spatial visualization, and geometric solids.

In the first level of the POM, students are presented with the task of examining and comparing attributes of different pattern blocks. Their task involves making a list of attributes and finding area relationships between different pattern blocks (triangle, blue rhombus, hexagon, and trapezoid). As one continues through the levels,

students are presented with situations that involve covering a two dimensional design with different pattern blocks. The students are asked to find the ways the design can be covered with just one pattern block and the ways it can be covered using multiple blocks. In Level C students are given three different perspectives of a soccerball. Using spatial visualization the student must determine the number and type of each polygon that makes up the surface area of the ball. Two different soccer ball designs are presented. In Level D, the student is told that a major league soccer ball is between 27 and 28 inches in circumference. The student is asked to determine the dimensions of each of the surface area polygons. In Level E, students are asked to design their own soccerball and describe the dimensions of the surface area polygons and the attributes of the polyhedra they created.

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Level A:

You have 6 different pattern blocks of different colors. Organize the pattern blocks and draw the large face of each of them.

Determine the mathematical name of the large face of each of the blocks. List the attributes of the figures including length of sides and size of angles.

Compare the length and area of the figures. Make a systematic list indicating their relative sizes.

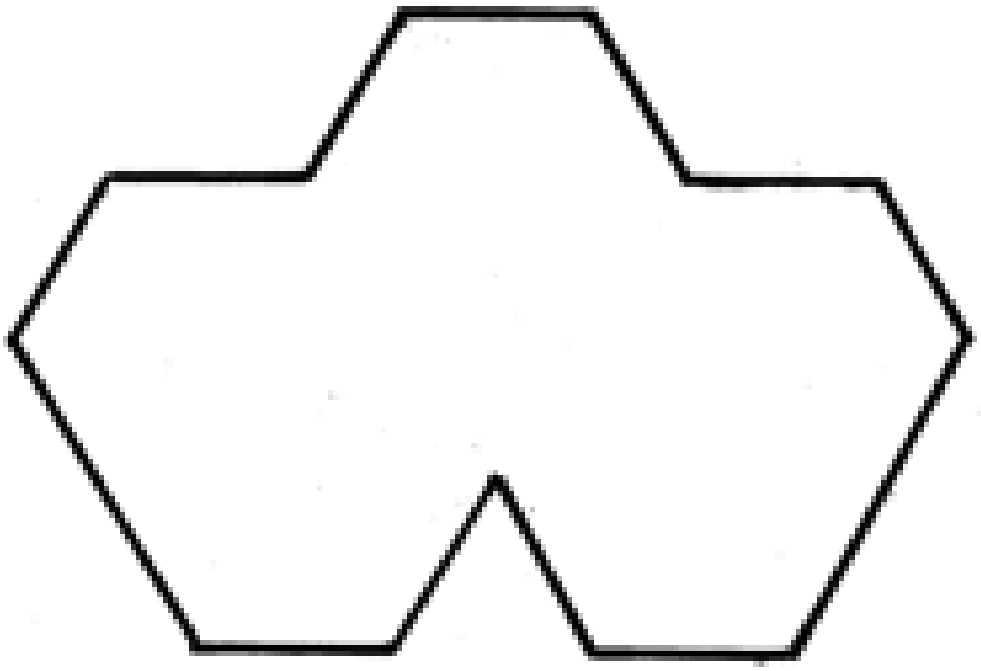
If the green triangle is one area unit in size, how does it compare with the trapezoid, blue rhombus and the hexagon?

List the possible ways that a hexagon can be covered with the other pattern blocks.

Level B:

Consider the outline of the figure below. Investigate how to cover up that outline by arranging different blocks to fit into the space exactly.

If you were to use only one type of block, which block can be used to cover the outline?



Explain how you know for sure.

If you could use more than one type of block, which combination of blocks could be used to cover the outline? Explain or draw your solutions.

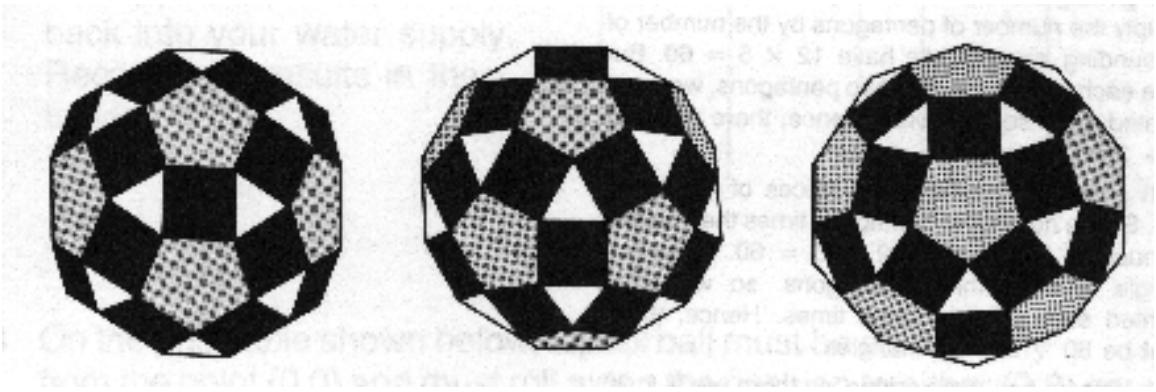
Which pattern block(s) cannot be used at all? Explain why some pattern blocks work and others do not.

Level C:

You work for a sporting goods manufacturer and you are finding that soccer is becoming more and more popular.

The top-selling soccerball is made by **Everkick**. The design pattern of this ball is comprised of pentagons and hexagons. Below are three views of the **Everkick** soccerball. Analyze the design pattern and determine the number of pentagons and hexagons it takes to make a ball. Explain your conclusion using mathematical reasoning.

The **GoalScorer** ball is second on the list of best-selling soccerballs. It has a different design pattern. The design pattern of this ball is comprised of pentagons, triangles and squares. Below are three views of the **GoalScorer** soccer ball. Analyze the design pattern and determine the number of pentagons, triangles and squares it takes to make a ball. Explain your conclusion using mathematical reasoning.



Level D:

Soccerballs are made in different sizes for age-group soccer leagues. There are three standard balls, a size 5, a size 4 and a size 3 soccerball. Major League Soccer and the World Cup use a size 5 soccerball. According to the rulebook, the circumference (around the equator of a sphere) of a size 5 soccerball when fully inflated must be between 27 and 28 inches. Using your knowledge of the diameter (across the center of a sphere), volume and surface area of a sphere, determine those measurements of the size 5 soccerball.

Using geometry, determine the size (including the length of sides and area) of a pentagon and hexagon from a size 5 **Everkick** soccerball. Explain the process you use to determine the dimensions and area.

Using geometry, determine the size (including the length of sides and area) of a pentagon, square and triangle from a size 5 **GoalScorer** ball. Explain the process you used to determine the dimensions and area.

Level E:

The executive board has assigned you the task of creating a new design pattern for the company's new soccer ball. Use various polygons to create a spherical-shaped object. Build a design model using construction paper and tape. Draw a blueprint design of the ball showing at least three different views of the soccer ball. List the types of polygons used in the design. For each polygon state the quantity needed to construct a ball.

Prepare a design specification report for a size 5 soccer ball. State the volume, diameter, circumference and surface area. Also, list the dimensions of each polygon used in your design, including the length of the sides, the measure of interior angles, the perimeter and the area of the polygon.

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Primary Version Level A

Materials: A set of pattern blocks for each pair, paper and pencil to write or draw, color crayons, markers or pencils

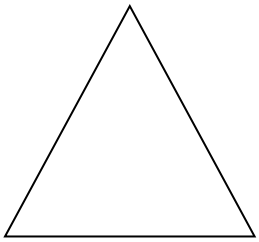
Discussion on the rug: Students have sets of pattern blocks. **"Here are pattern blocks. What do you notice about them? What else do you notice about them?"** Teacher continues to ask children to notice that they are different colors, shapes, sizes and different lengths. The teacher encourages the students to play with them and make different things.

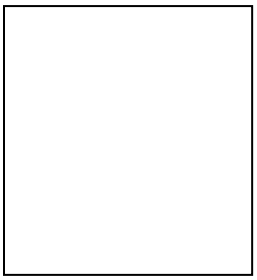
In small groups: Each group has a set of pattern blocks.
Teacher asks the following questions, only going on to the next question when students have success.

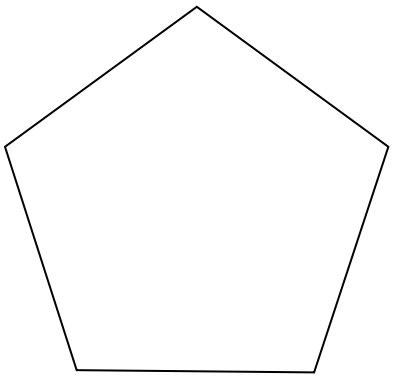
1. **"Which of these blocks do we know? What is its name? How many sides does it have? How many corners does it have?"**
Introduce the name of a block, if the students demonstrate knowledge of its attributes.

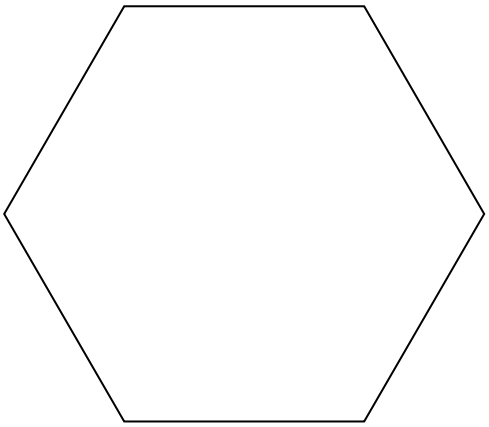
2. **"Which blocks fit on top of each other? How many green triangles fit on a blue rhombus? A red triangle? A yellow hexagon?"** Have students write a number next to each block drawn.

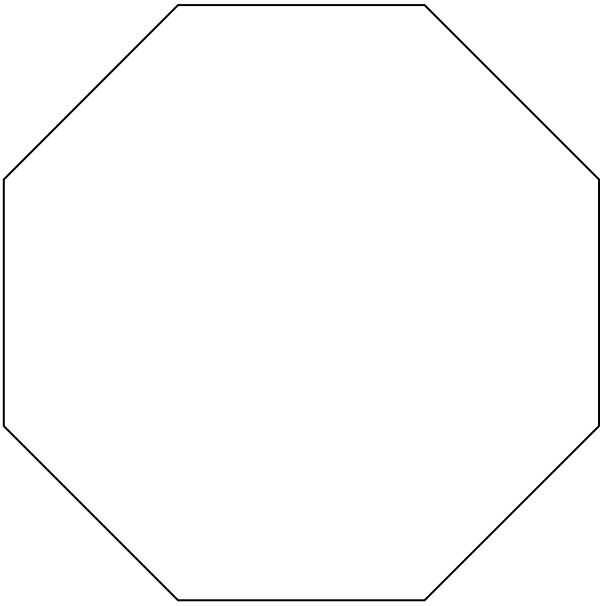
3. **"How many different ways can you cover the hexagon with other blocks?"**
At the end of the investigation have students either draw a picture or dictate a response to represent their solution.











Problem of the Month

Piece It Together

Task Description – Level A

This task challenges a student to draw shapes using pattern blocks and to list their attributes. Students are also asked to compare side lengths and areas of the blocks by making a list. Students are also challenged to find the area of some blocks using other blocks as the "unit." Students also need to find all the ways that a hexagon can be covered using combinations of the other blocks.

Common Core State Standards Math - Content Standards

Measurement and Data

Measure lengths indirectly and by iterating length units.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end: understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.6 Measure areas by counting unit squares.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of a specified measure.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.MD.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Geometry

Reason with shapes and their attributes.

1.G.1 Distinguish between defining attributes versus non-defining attributes; build and draw shapes to possess defining attributes.

2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

3.G.1 Understand that shapes in different categories (e.g. rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Statistics and Probability

Investigate chance process and develop, use, and evaluate probability models.

7.SP.8 Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams.

Common Core State Standards Math – Standards of Mathematical Practice

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of

varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Problem of the Month

Piece It Together

Task Description – Level B

This task challenges students to reason how a space can be covered with two-dimensional shapes. Students explore shapes or combinations of shapes that will cover a space. Students must investigate and construct convincing arguments about why some shapes will not cover a space.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.6 Measure areas by counting unit squares.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of a specified measure.

Geometry

Reason about shapes and their attributes.

1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter=circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

Statistics and Probability

Investigate chance process and develop, use, and evaluate probability models.

7.SP.8 Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Problem of the Month

Piece It Together

Task Description – Level C

This task challenges a student to reason about shapes composing a three-dimensional form looking at different views of the objects. Students need to identify the type of shapes and the number of each shape, and to justify their conclusions.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of a specified measure.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.MD.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Geometry

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.

High School – Geometry - Modeling and Geometry

Apply geometric concepts in modeling situations.

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects.

G-MG.3 Apply geometric methods to solve design problems.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Piece It Together

Task Description – Level D

This task challenges a student to work from the measure of the diameter of a sphere to find the dimensions of the composing pieces of a soccerball. Students need to apply ideas about volume and surface area, circumference, and angle measurement to solve the problem.

Common Core State Standards Math - Content Standards

Geometry

Draw construct, and describe geometrical figures and describe the relationships between them.

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

High School – Geometry - Geometric Measurement and Dimension

Visualize relationships between two-dimensional and three-dimensional objects.

G-GMD.4 Identify the shapes of two-dimensional cross sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.

High School – Geometry - Modeling with Geometry

Apply geometric concepts in modeling situations.

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects.

G-MG.3 Apply geometric methods to solve design problems.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

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Problem of the Month

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Task Description – Level E

This task challenges a student to design a soccerball using polygons to create a spherical shape. Students must sketch different views of the ball and give dimensions of pieces including angles, area, surface area, and volume of the ball. Students must meet the constraint of the ball having a diameter of approximately 28 inches.

Common Core State Standards Math - Content Standards

Geometry

Draw construct, and describe geometrical figures and describe the relationships between them.

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

High School – Geometry - Geometric Measurement and Dimension

Visualize relationships between two-dimensional and three-dimensional objects.

G-GMD.4 Identify the shapes of two-dimensional cross sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.

High School – Geometry - Modeling with Geometry

Apply geometric concepts in modeling situations.

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G-MG.3 Apply geometric methods to solve design problems.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Piece It Together

Task Description – Primary Level

This task challenges a student to recognize shapes in pattern blocks and see which shapes can cover other shapes. Students need to find and record different ways to cover a hexagon.

Common Core State Standards Math - Content Standards

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Analyze, compare, create, and compose shapes.

K.G.6 Compose simple shapes to form larger shapes.

Reason with shapes and their attributes.

1.G.1 Distinguish between defining attributes versus non-defining attributes; build and draw shapes to possess defining attributes.

2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

Measurement and Data

Measure lengths indirectly and by iterating length units.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end: understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.