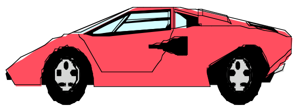
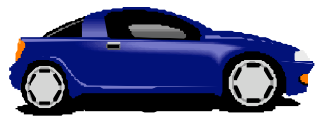
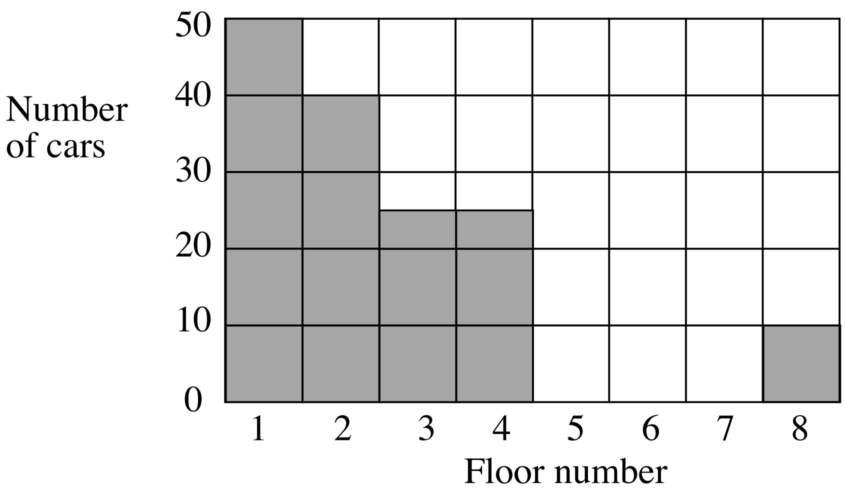
# Parking Cars

This problem gives you the chance to:
• interpret a bar graph



This bar chart shows the number of cars parked in the multi-level parking garage at a shopping center today. Each floor holds up to 50 cars.




1. On which floor are the most cars parked? ____________

   How many cars are there on this floor? ____________

2. On which two floors are the same number of cars parked?

   Floors __________ and __________

3. How many more cars are parked on Floor 1 than on Floor 8? __________

4. How many cars, in all, are parked on the parking garage? __________
Show how you figured this out.

5. Fifteen more cars come into the parking garage.
Show these cars on the graph, parking them in the lowest floors.

Explain why you parked these cars in this way.

______________________________________________________________

______________________________________________________________

7

| **Task 2: Parking Cars** | | Rubric |
|---|---|---|
| The core elements of performance required by this task are:<br>• interpret a bar chart<br>Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answers: Floor **1**<br>and **50** | 1 | 1 |
| 2. Gives correct answer: **3 and 4** | 1 | 1 |
| 3 Gives correct answer: **40** | 1 | 1 |
| 4. Gives correct answer: **150**<br>Shows work such as: 50 + 40 + 25 + 25 + 10 = 150<br>Accept "I counted them" if student has given 150. | 1<br>1 | 2 |
| 5. Draws 15 more cars on the block graph.<br>10 cars on Floor number 2 and 5 cars on Floor number 3.<br>Gives a correct reason for their positioning.<br>*Partial credit*<br>Places 15 cars in garage. | 1<br>1<br>(1) | 2 |
| **Total Points** | | **7** |

## Parking Cars

Work the task and look at the rubric. What are the big mathematical ideas a student needs to be successful on this task? ________________________________________

________________________________________

________________________________________

________________________________________

Think about what it takes to understand and interpret scale on a graph. What kinds of understanding allow students to think about the differences between the actual numbers and the units for the numbers? Look at student work for part 3 of the task. How many of your students put:

| 40 | 50 | 10 | 42 | 1 | 4 | Other |
|---|---|---|---|---|---|---|
| | | | | | | |

Can you think how a student might have arrived at each of the incorrect solutions above? What was the misconception of each student?

While you are still thinking about the complications of scale, look at student responses for the total in part 4. How many of your students put:

| 150 | 140 | 141/142 | 135 | 15 | 16 | Answer larger than 200 | Other |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

Can you reconstruct the thinking that would lead to some of these wrong answers? How is each student interpreting the number of cars in floor 3?

Now look at the difference between interpreting someone else's scale and trying to add data to a graph and make sense of the existing scale. Look at student work in part 5. Did you students:

| Add 15 to the lowest floors | Add only 15 cars somewhere | Add 15 cars to 2 or more columns | Shade in 15 boxes or partial boxes | Draw pictures of cars | Make the total cars on floor 8 = 15 cars | Other |
|---|---|---|---|---|---|---|
| | | | | | | |

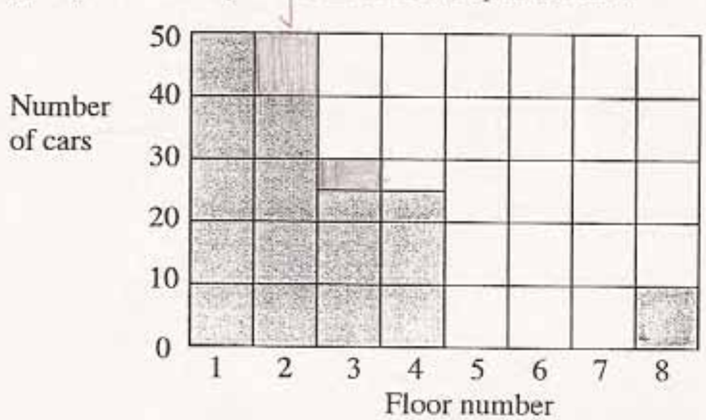
Did any of your students add cars making a column have more than 50 cars?
How is the cognitive demand of adding information to a graph different from that needed to just interpret information already on a graph?
How might you adjust questioning strategies or classroom activities to make student thinking about reading and interpreting scale more explicit?
What types of activities might help students become clearer about the meaning of scale?
Do students in your class have enough opportunities to make their own graphs and decide on an appropriate scale?
Are you convinced that your students understand the scale as an equal size unit?

## Looking at Student Work on Parking Cars

Student A shows a thorough understanding of reading and interpreting a graph by identifying the most, the same, and using comparison subtraction. Notice the clear use of labels to explain thinking with the computations. The student seems very comfortable with using scale. The explanation of parking the 15 cars is very complete.

**Student A**

This bar chart shows the number of cars parked in the multi-level parking garage at a shopping center today. Each floor holds up to 50 cars.




1. On which floor are the most cars parked? Floor 1 ✓

   How many cars are there on this floor? 50 cars

2. On which two floors are the same number of cars parked?

   Floors 3 and 4 ✓

3. How many more cars are parked on Floor 1 than on Floor 8? 40 more cars ✓

   50 cars on Floor 1
   −10 cars on Floor 8
   40 more cars

**Student A, part 2**

4. How many cars, in all, are parked on the parking garage? 150 cars ✓
Show how you figured this out.

Floor 1 = 50 cars
Floor 2 = 40 cars
Floor 3 = 25 cars
Floor 4 = 25 cars
Floor 5 = 0 cars
Floor 6 = 0 cars
Floor 7 = 0 cars
Floor 8 = 10 cars

$$\begin{array}{r} 50 \\ +40 \\ \hline 90 \\ +25 \\ \hline 115 \\ +25 \\ \hline 140 \end{array} \qquad \begin{array}{r} 140 \\ +10 \\ \hline 150 \end{array}$$ cars ✓

5. Fifteen more cars come into the parking garage.
Show these cars on the graph, parking them in the lowest floors.

Explain why you parked these cars in this way.

I parked it in this way because for floor 1, There were no more cars to fit. So I went to the next floor. It had 10 more spaces. So I put 10 cars there. Then I went to the next floor. It had lots of space. I put 5 cars in there.

Student B also has a good explanation for how the cars were parked.

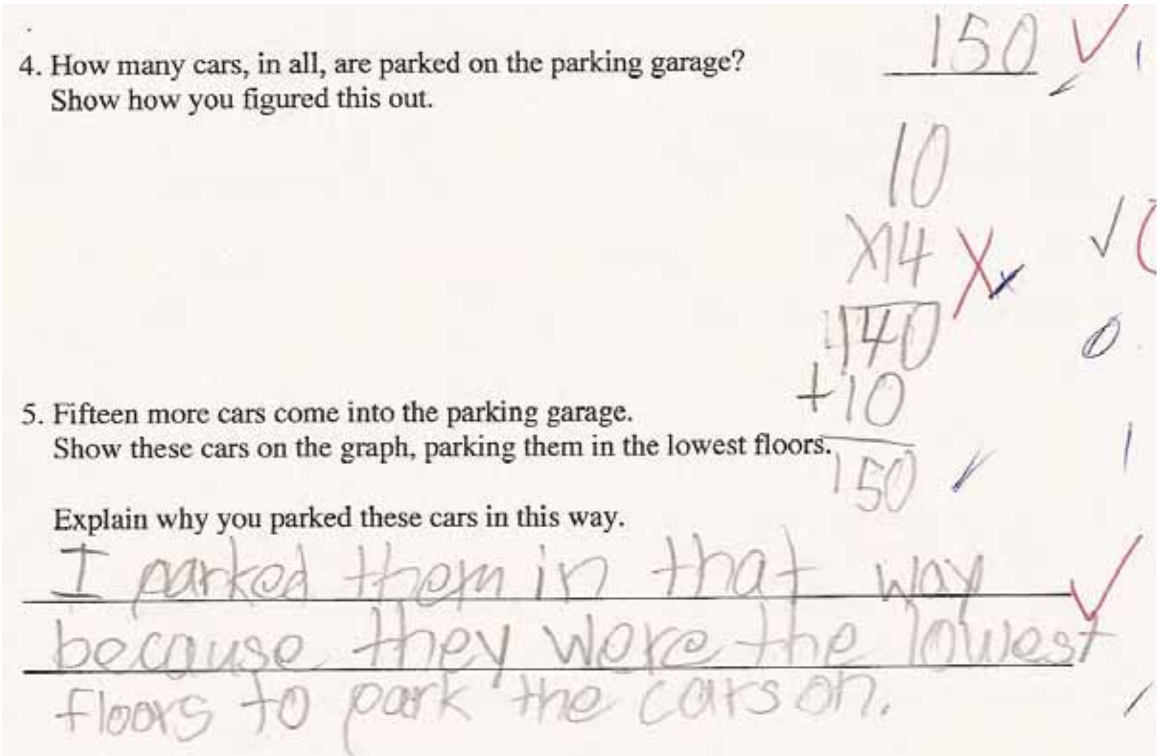
**Student B**

Explain why you parked these cars in this way.

I parked 10 cars on Floor 2 and 5 cars on floor 3 because they were the lowest floors with room left to park cars on. ✓

Student C has a very thorough understanding of scale and of unit. However the scorer did not understand the strategy in part 4. *Can you think why this strategy is not just a convenient number sentence but actually uses the idea of scale? What understanding underlies this strategy?*

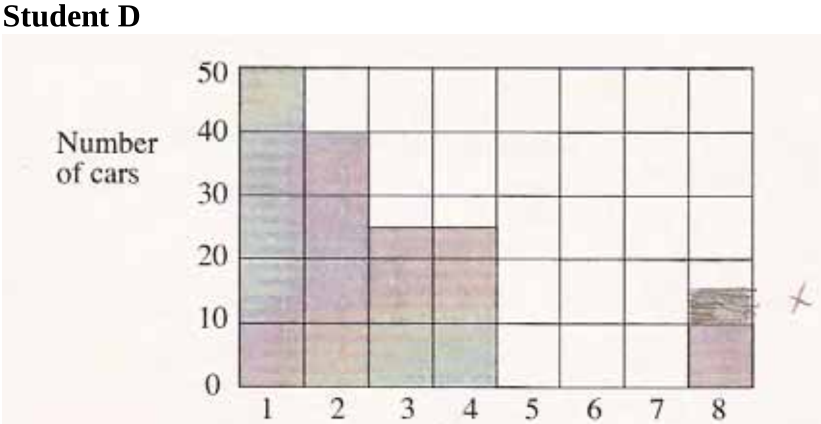
**Student C**



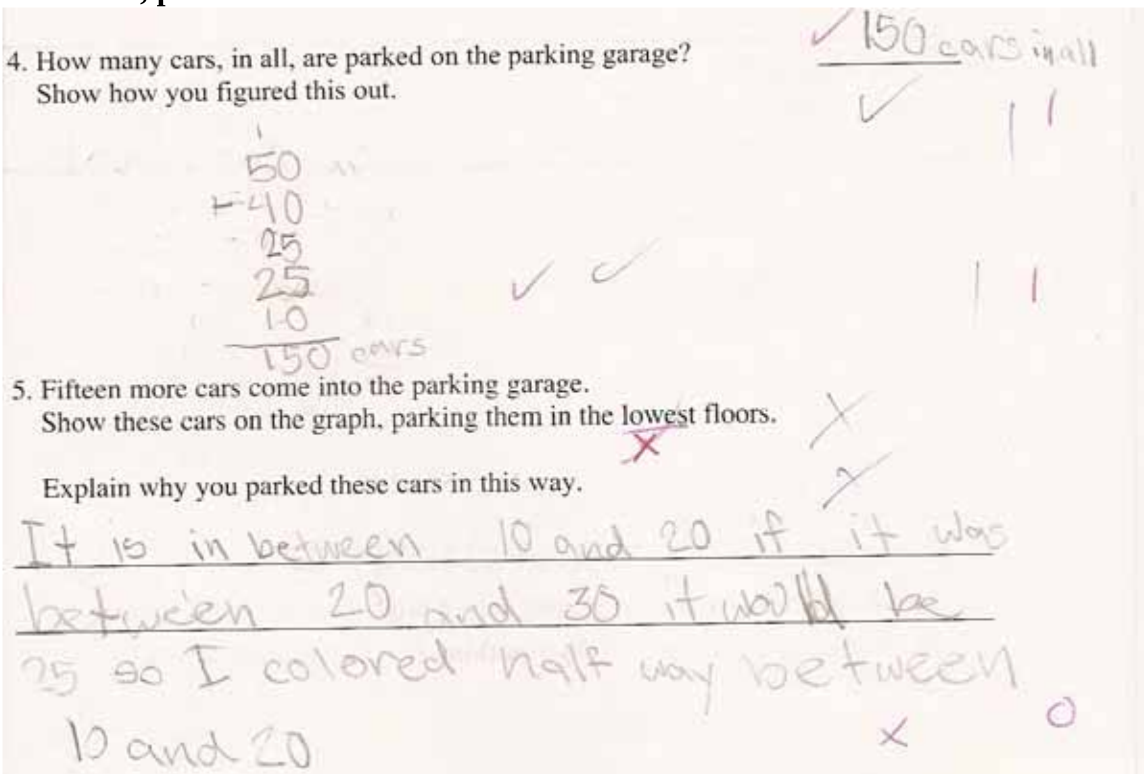

## Issues of Scale and Issues of Mathematical Literacy

Student D has a good understanding of scale, as evidenced by the work in part 4. The student even explains how 15 cars would be marked with a bar. The student struggles with a mathematically literacy issue, understanding the difference between the contextual meaning of adding 15 more cars to the total already represented in the graph and having a bar with a height representing 15 cars. (adding 15, an operation, or height 15, absolute value)

**Student D**



**Student D, part 2**


4. How many cars, in all, are parked on the parking garage? 150 cars in all
Show how you figured this out.

50
+40
25
25
10
150 cars

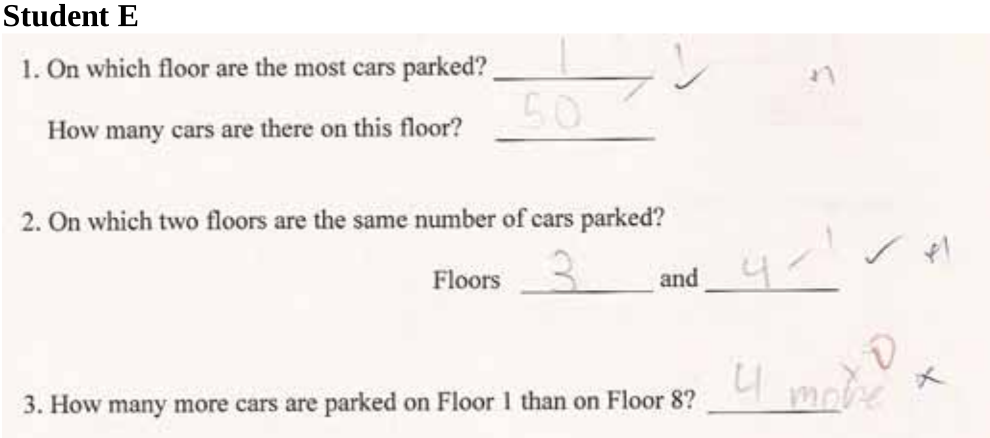
5. Fifteen more cars come into the parking garage.
Show these cars on the graph, parking them in the lowest floors.

Explain why you parked these cars in this way.

It is in between 10 and 20 if it was between 20 and 30 it would be 25 so I colored half way between 10 and 20

When learning a new idea, an understanding may drift in and out of consciousness. It doesn't hold for all occasions. Look at the work of Student E. The student seems to understand the scale in answering part 1 and in the readings to find the total in part 4. However when making the comparison between cars in floor 1 and 8, the student reverts back to counting squares rather than using a subtraction of the values represented. The student seems to think of floors 3 and 4 as one single bar instead to two bars with the same height. There is literacy issue. The student, instead of adding 15 more cars to the graph, simply recreates the original graph. Instead of explaining how to put the additional cars on the graph, the student explains again how she figured out the total number of cars. *How is the level of detail needed for reading a math problem different from the level of detail for reading a story?*

**Student E**


1. On which floor are the most cars parked? 1

How many cars are there on this floor? 50

2. On which two floors are the same number of cars parked?

Floors 3 and 4

3. How many more cars are parked on Floor 1 than on Floor 8? 4 more

## Student E, part 2

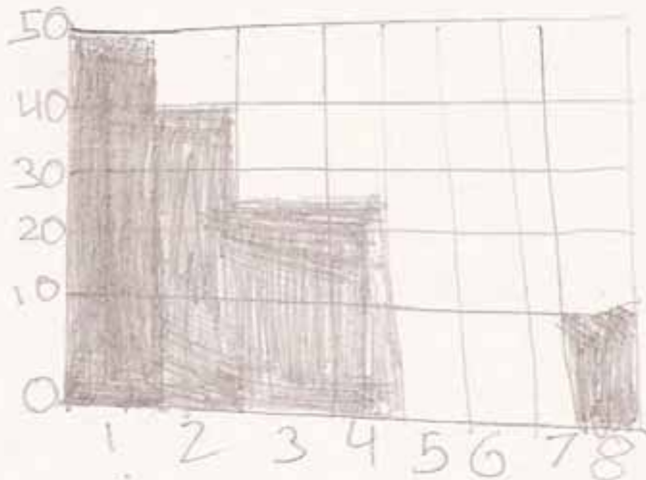
. How many cars, in all, are parked on the parking garage? 125
Show how you figured this out.

50
40
25
+ 10
125

5. Fifteen more cars come into the parking garage.
Show these cars on the graph, parking them in the lowest floors.

Explain why you parked these cars in this way.

I add them all together

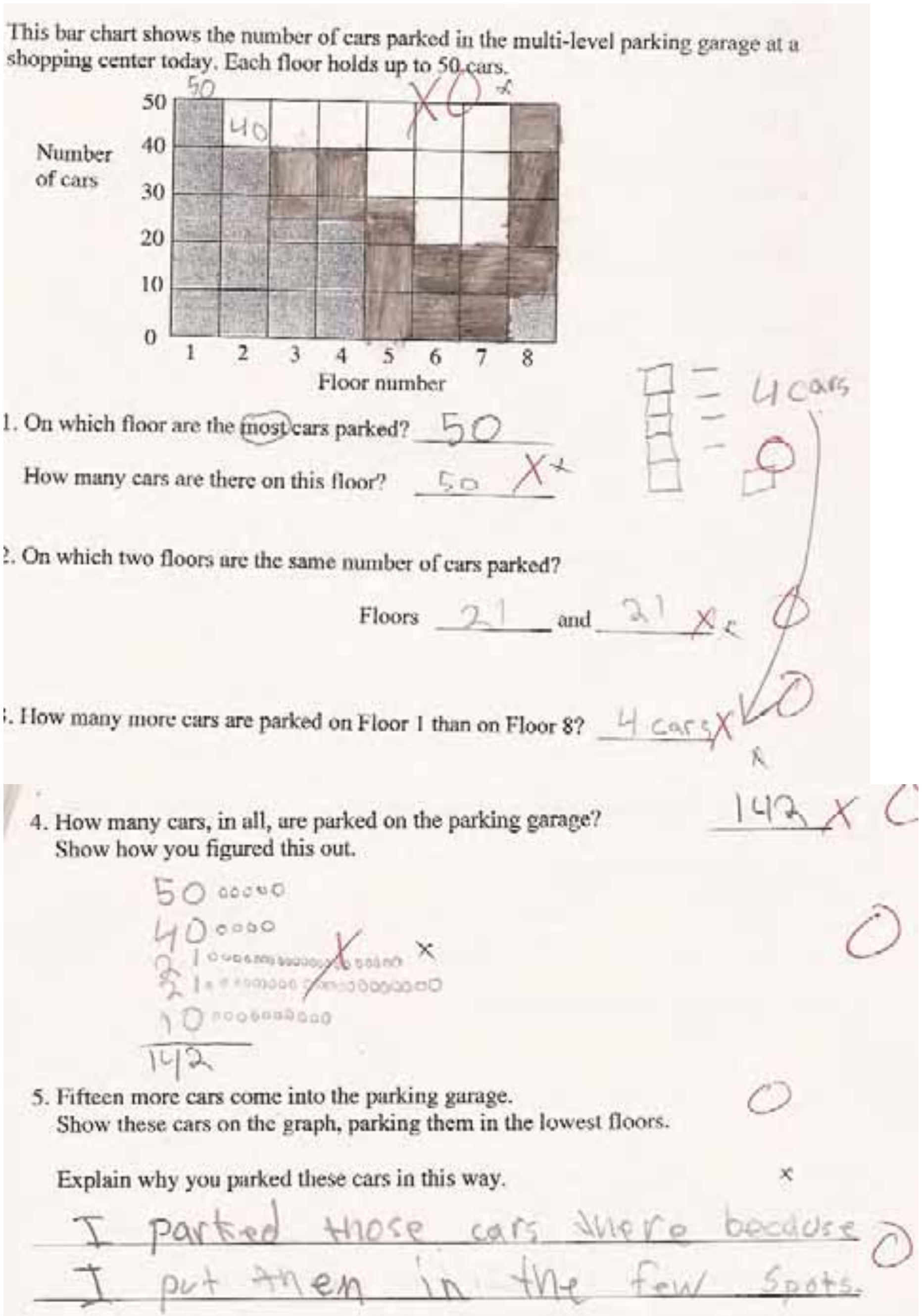



50
40
25
+10
125

7

Student F again illustrates the idea about an inconstant understanding of scale. In questions one and two the student looks at the value of the columns height without connection to the context of floors. *Is this a literacy issue of just reading numbers off a graph versus thinking about context?* The students seems to understand the idea a square =10 cars except when doing comparison, again the idea of thinking for counting comparison versus comparison subtraction is well-illustrated by the child's drawing to the right of part 1. While the student knows that a whole square is 10, the student doesn't know what a half square represents. *Do you think the student may have only worked with scales of one and 2?*

**Student F**



This bar chart shows the number of cars parked in the multi-level parking garage at a shopping center today. Each floor holds up to 50 cars.



1. On which floor are the most cars parked? 50

   How many cars are there on this floor? 50

2. On which two floors are the same number of cars parked?

   Floors 21 and 21

3. How many more cars are parked on Floor 1 than on Floor 8? 4 cars

4. How many cars, in all, are parked on the parking garage? 142

   Show how you figured this out.

   50
   40
   21
   21
   10
   142

5. Fifteen more cars come into the parking garage.
   Show these cars on the graph, parking them in the lowest floors.

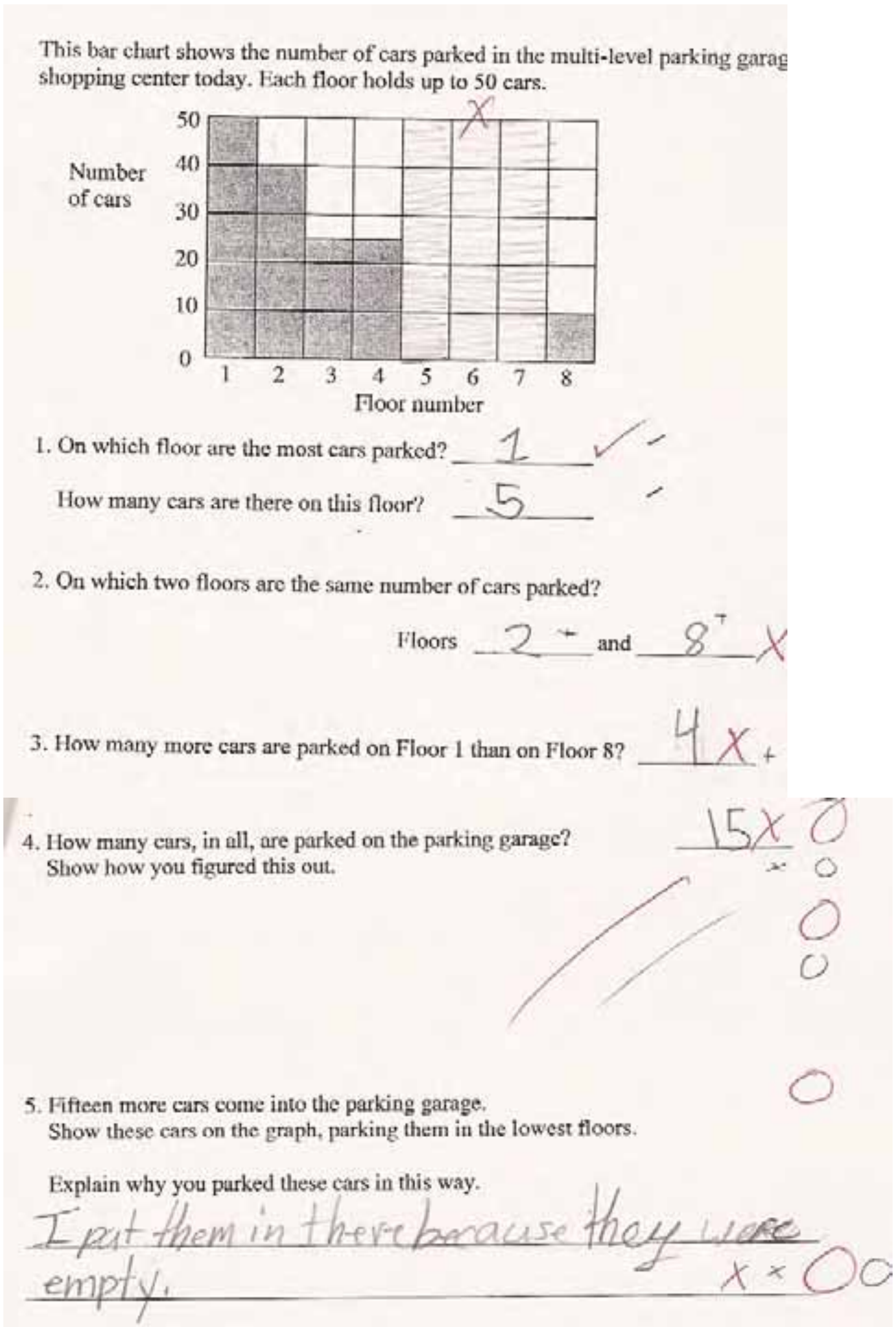
   Explain why you parked these cars in this way.

   I parked those cars there because I put then in the few spots.

3rd grade 2007

Student G only understands the square as representing one unit. The student does have the sophistication to combine the two half squares to make a whole in part 4. *Why might the student have put floor 2 and 8 together? What type of experience can help the student move to thinking about a scale larger than 1? Do you think making a graph from collected data, where there isn't enough room would help? Do you think a class discussion of several answers to part 4 would help the student rethink her position? What would be your next move?*

**Student G**



This bar chart shows the number of cars parked in the multi-level parking garag
shopping center today. Each floor holds up to 50 cars.



1. On which floor are the most cars parked? 1

   How many cars are there on this floor? 5

2. On which two floors are the same number of cars parked?

   Floors 2 and 8

3. How many more cars are parked on Floor 1 than on Floor 8? 4

4. How many cars, in all, are parked on the parking garage? 15
   Show how you figured this out.

5. Fifteen more cars come into the parking garage.
   Show these cars on the graph, parking them in the lowest floors.

   Explain why you parked these cars in this way.

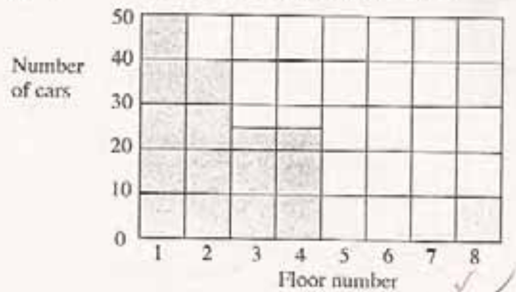
   I put them in there because they were empty.

3rd grade 2007

Student H makes several errors around understanding scale. On the original for part 1, 50 is erased and replaced with 15. *How many different issues of scale can you find? What are the different ways the student makes sense of the half-squares? How do you think the student got an answer of 42 for part 3?*

**Student H**

This bar chart shows the number of cars parked in the multi-level parking garage at a shopping center today. Each floor holds up to 50 cars.




1. On which floor are the most cars parked? 1 x

   How many cars are there on this floor? 15

2. On which two floors are the same number of cars parked?

   Floors 3 and 4 and 20½

3. How many more cars are parked on Floor 1 than on Floor 8? 42 x

4. How many cars, in all, are parked on the parking garage? 65 x 0 0

   Show how you figured this out.

   x 0 0

5. Fifteen more cars come into the parking garage. 0 0

   Show these cars on the graph, parking them in the lowest floors.

   Explain why you parked these cars in this way.

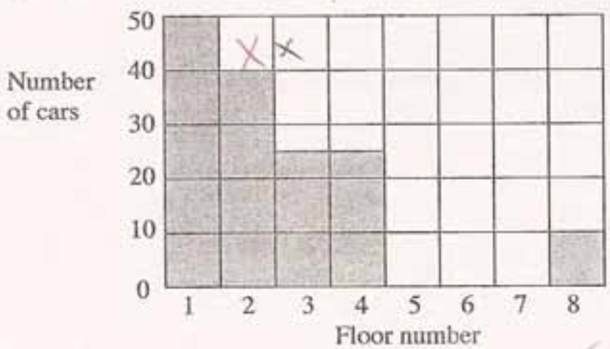
   I made more on 3 and 4

   and the number is 30 x x 0 0


3rd grade 2007

Student I may be having trouble with mathematical literacy. In part 3 the student seems to be thinking about how many cars versus how many more. Each action describes a different operation. *Can you think of two reasons why the student may have added 50 +8 in part 4?*

**Student I**




1. On which floor are the most cars parked? 1

   How many cars are there on this floor? 50

2. On which two floors are the same number of cars parked?

   Floors Third and foorth

3. How many more cars are parked on Floor 1 than on Floor 8? 60

4. How many cars, in all, are parked on the parking garage? 58
   Show how you figured this out.

   50+8=58

5. Fifteen more cars come into the parking garage.
   Show these cars on the graph, parking them in the lowest floors.

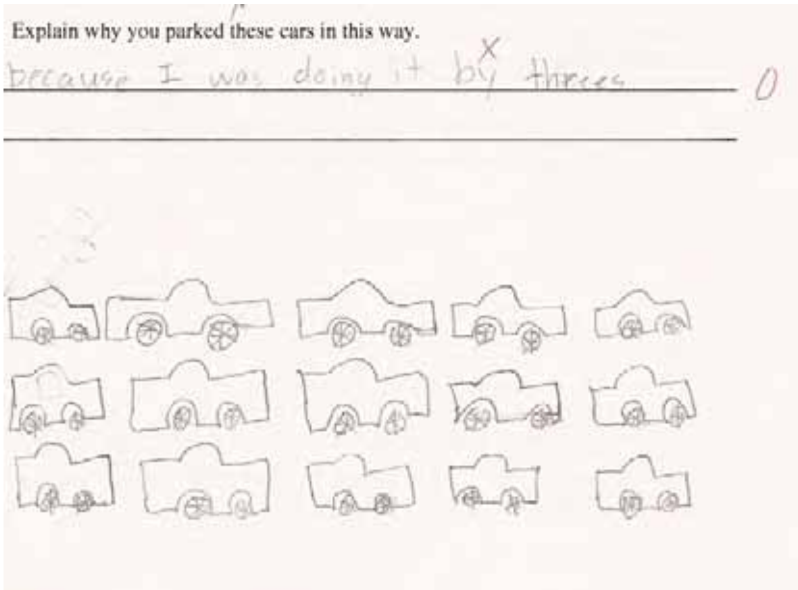
   Explain why you parked these cars in this way.

   I parked the car this way becouse its has more room

3rd grade 2007

Student J is interesting, because the student has all 5 points preceding part 5 of the task. Then the student reverts to drawing an array of 15 instead of adding 15 cars to the graph.

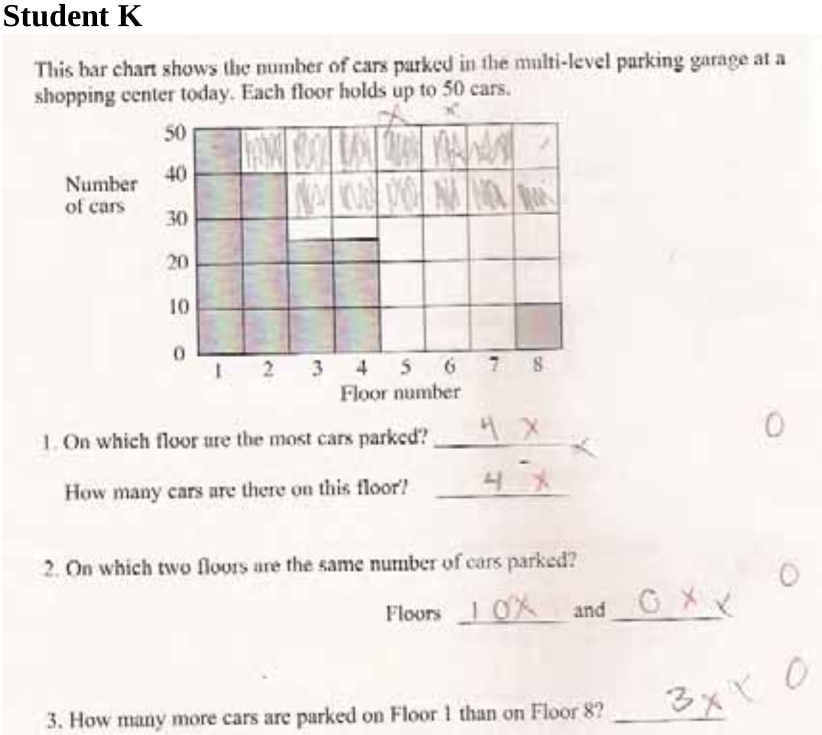
**Student J**


Explain why you parked these cars in this way.

because I was doing it by threes 0

Student K does not distinguish between the numbers representing frequency or scale (number of cars parked) and numbers as a category (floor 1, floor 2). Notice that the student adds to the bars at 50 and 40 to get a total of 15 squares for those two rows. There is some logic, although incorrect, in most of the student's responses. *Why might the student think floors 10 and 0 have the same number of cars? Where might the 3 have come from in part 3?*

**Student K**


This bar chart shows the number of cars parked in the multi-level parking garage at a shopping center today. Each floor holds up to 50 cars.



1. On which floor are the most cars parked? 4 0

How many cars are there on this floor? 4

2. On which two floors are the same number of cars parked? 0

Floors 10 and 0

3. How many more cars are parked on Floor 1 than on Floor 8? 3 0

**Student K, part 2**

5. Fifteen more cars come into the parking garage.
   Show these cars on the graph, parking them in the lowest floors.

   Explain why you parked these cars in this way.

   Because if 1 is shaded than just add 6 more. Then if 2 is shaed add 6.

0

6

## 3rd Grade Task 2 Parking Cars

| | |
|---|---|
| **Student Task** | Represent data using a bar graph and draw conclusions from the data. Interpret a scale going up by 10's. Use information on a graph to find totals and make comparisons. Reason about a problem in context and make a justification for where to park additional cars, given a constraint about lowest levels. |
| **Core Idea 5 Data Analysis** | **Collect, organize, display, and interpret data about themselves and their surroundings.**<br>• Describe important features of a set of data (maximum, minimum, increasing, decreasing, most, least, and comparison). |

*Mathematics in this task:*
- Ability to read and interpret a graph
- Ability to use scale of 10 in a graphing setting
- Ability to distinguish between categorical and numerical data
- Ability to add data to a graph
- Ability to identify and use constraints when solving a problem
- Ability to do comparison subtraction

*Based on teacher observations, this is what third graders knew and were able to do:*
- Identify most and same size on a graph
- Use interpret a scale of 10 and do comparison subtraction
- Add and subtract 2-digit numbers

*Areas of difficulty for third graders:*
- Interpreting what number is half-way between when using a scale of 10
- Adding information to a graph, using a scale to make a graph seems to be different or more complicated than reading a graph with the same scale
- Interpreting the meaning of lowest floors
- Confusing how many and how many more

MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 3

## Task 2 - Parking Cars

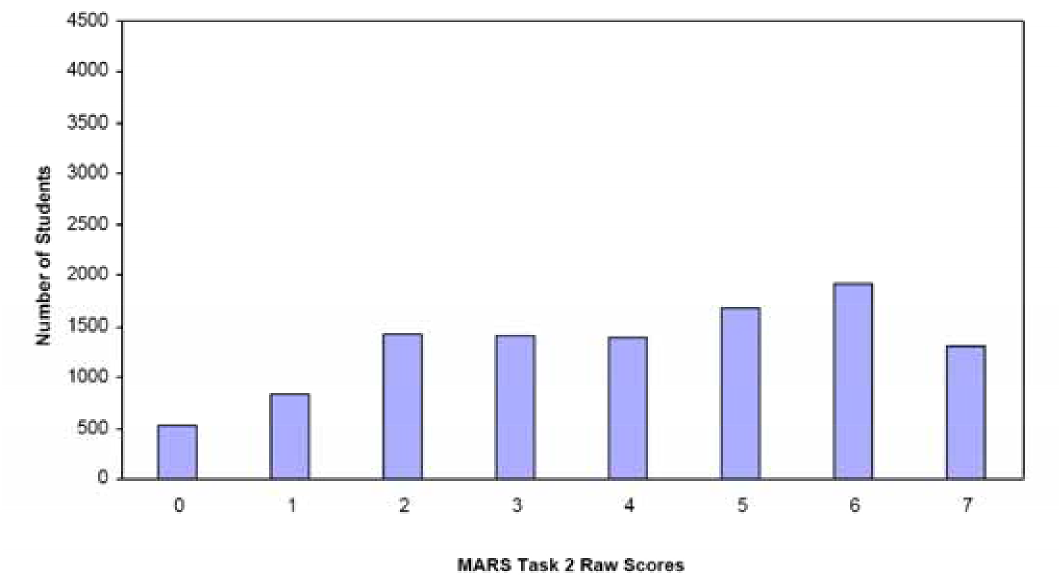
Mean: 4.05 StdDev: 2.05

Table 16: Frequency Distribution of MARS Test Task 2, Grade 3

| Task 2 Scores | Student Count | % at or below | % at or above |
|---|---|---|---|
| 0 | 528 | 5.0% | 100.0% |
| 1 | 833 | 13.0% | 95.0% |
| 2 | 1427 | 26.6% | 87.0% |
| 3 | 1405 | 40.0% | 73.4% |
| 4 | 1397 | 53.3% | 60.0% |
| 5 | 1674 | 69.3% | 46.7% |
| 6 | 1920 | 87.6% | 30.7% |
| 7 | 1302 | 100.0% | 12.4% |

Figure 25: Bar Graph of MARS Test Task 2 Raw Scores, Grade 3

Figure 25: Bar Graph of MARS Test Task 2 Raw Scores, Grade 3




*The maximum score available on this task is 7 points.*
*The minimum score for a level 3 response is 3 points.*

*Most students, 87%, could identify the floors with the most number of cars and the same number of cars. Many student, 73%, could also do comparison subtraction with numbers from the graph. More than half, 60%, could also add 15 cars to the graph, but not necessarily in the desired location. 12% could meet all the demands of the task including finding the total number of cars represented on the graph and putting 15 cars on the lowest floors with an appropriate explanation. 5% scored no points on this task. All the students in the sample with this score attempted the task.*

## Parking Cars

| Points | Understandings | Misunderstandings |
|---|---|---|
| **0** | All the students in the sample attempted the task. | Students had difficulty understanding scale. 3% thought there were 5 cars on floor 1. 7.5% gave answers of 50 or more. |
| **2** | Students could read a graph with a scale of 10 and identify most and same on the graph. | While many students could read the graph and think about scale, it was more difficult to interpret scale for comparison purposes. 10% thought the difference was only 4 cars, 3% picked 10 cars, 3% 50 cars, 3% 42 cars, and 3% 5 cars. |
| **3** | Students could read and interpret a graph for finding most, same, and making comparisons. | The thinking involved for putting data on a graph for scale was difficult for students. 16% did not attempt to add cars. 8% added 15 cars to each of the empty floors. Many filled in 15 squares and 4% drew 15 cars. |
| **4** | Students could read and interpret the graph, do comparison subtraction, and fill in 15 cars on the graph. | 22% did not put the 15 cars in the correct locations to meet the constraints of the task. Students also struggled with finding the total number of cars represented on the graph. They struggled with how to account for the half-squares with a scale of 10. 4% ignored the half-squares (140). 4% gave an answer of 3. 4% thought the total was 15, counting each square as 1 and the half-squares as 1/2. 3% had answers of 135, thinking the bar for floors 3 and 4 was only one bar. |
| **7** | Students read and interpret a graph with a scale of 10 and use the scale for comparison and addition. Students could also add data to a graph to meet given constraints of the lowest floor. | |

## Implications for Instruction

Students need opportunities to think about scales other than 1. They need to be able to think about the value of bars that fall in between the grid marks and start to see that half way should not be rounded and is different from 1/2. It is half of the amount on the scale. This is a difficult concept and students need to encounter it several times for the idea to take hold.

In designing lessons, teachers need to think about the layers of learning a big idea, such as scale. Being able to read and interpret a number on the graph is different from being able to use the numbers on a graph for computation purposes. Comparison subtraction is especially difficult because of the way that students think about the process. For those students who see “finding the difference” or “how much more” as the operation of subtraction, then they just need to plug the values from the graph into the subtraction problem and compute the answer. However, for students who are looking at the difference in the size of the bars it is easy to forget about the scale and just look at how many squares higher one bar is than another. How can you help them think about changing that difference from squares to squares times the value of the square? What does each square represent? Classroom discussions, that help students confront this misstep or misconception and try and resolve different answers, allows students to clarify their thinking.

In planning learning trajectories or units around a big idea, like understanding scale, there aren’t short cuts. The type of thinking required to add data to a graph has a higher cognitive demand than just reading information from a graph. The ability to think between squares and what the squares represent is challenging. Further, the cognitive demands for planning a scale and putting equal-size units on a graph is different from using a graph with a scale already provided. Planning what size units will allow all the data to be represented and then skip-counting by that amount is one part. Understanding that the scale is different from just a listing of the different data points is another. Classroom activities and discussions need to help students build these layers of understanding.

Students also need to understand the difference between numbers on a graph used as labels and numbers used as frequency. In this task, numbers on the horizontal scale are used to describe floor numbers and also to order the floors from lowest to highest. Numbers on the vertical scale represent frequency or number of cars parked on that floor. It doesn’t make sense to take 50 cars parked on floor one and subtract floor #8 to get 42. What would that 42 represent? While dimensional analysis will be a critical mathematical idea at later grades, students should have the opportunity to think about and discuss units at early grades. Many successful students are those who think in units and can identify what the numbers in their calculations represent and what is found by each computation. Setting up situations for students to discuss and talk about labels and what they mean or don’t mean and seeing that certain computations don’t make sense in terms of labels helps other students develop this strategy for thinking and organizing their work.

**Ideas for Action Research**
**Assessing Understanding of Scale**

Think about an interesting graphing task from your textbook or some other source that has a scale other than 1. How could you redesign the task to look at different layers of cognitive demand, such as using scale for computation and making a scale to fit the numbers? Then try out different versions of the task with different groups of students to see how the thinking and misunderstandings vary between the groups.

Take for example this task from a third grade book:

2. This bar graph shows Sulian's savings for four months.


Use the graph to answer the following questions.

(a) How much did Sulian save in May?

(b) How much more did she save in June than in May?

(c) In which month did she save $15?

(d) In which month did she save the most?

(e) In which month did she save twice as much as in August?

(f) What was her total savings for the four months?

- For one group of students, change part c to read, “How much did she save in July?” How does this assess a slightly different understanding than the original question?
- Give one group of students a graph with enough space to add a bar for September. Explain that Sulian saves $50 in September, please add that information to the graph. Ask a different group to graph $55 for September.
- Give a third group a blank graph with the labels and scales provided and information about Sulian’s savings for the 5 months.
- Try giving a fourth group a blank graph with no labels or scales. You might suggest that the savings should go up $10 each time, if you feel this scaffolding would help. Then give them the data about the savings for Sulian.

Compare the data from the different groups.

- Can all groups read the scale? Interpret numbers lying between grid marks?
- Are there more errors with computation particularly with comparison subtraction in some groups than others?
- Are you students able to think about multiplicative relationships, twice as much, in a graphing situation?
- What surprised you when you looked at the work of students who had to make their own graph?

## Performance Assessment Task

## Parking Cars
## Grade 3

This task challenges a student to use their understanding of scale to read and interpret data in a bar graph. A student must be able to use knowledge of scale to add data to a graph. A student must be able to identify and describe important features of a set of data on a graph, such as maximum, minimum, most, least, and to be able to make comparisons.

## Common Core State Standards Math - Content Standards

**Measurement and Data**
**Represent and interpret data.**
3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in a scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

## Common Core State Standards Math – Standards of Mathematical Practice

**MP.2 Reason abstractly and quantitatively.**
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**MP.6 Attend to precision.**
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

| Grade Level | Year | Total Points | Core Points | % At Standard |
|---|---|---|---|---|
| 3 | 2007 | 7 | 3 | 73% |