## OUR PETS

Sam asked his classmates: "What kind of pets do you have?" He made a tally table to show the answers.

	OUR PETS		
BIRDS			
CATS			
DOGS			
FISH	JH1 I		
NO PETS	Ĩ		

1. Use the information in the tally table to fill in the bar graph below.



## **Kinds of Pets**

2. Which kind of pet do the fewest children have?

3. How many dogs and cats do these children have in all?\_\_\_\_\_ Show how you know that your answer is correct.

4. How many more children have dogs than fish? \_\_\_\_\_\_ Show how you know that your answer is correct.

5. Using the same data, Sam found out how many of the children in his class have pets. He wasn't able to finish the table. Finish it for him.

6. What is one thing you can learn from the bar graph that you can't learn from the table above?

10

## **Our Pets Mathematics Assessment Collaborative**

	Our Pets: Grade 2	Points	Section Points
	<ul> <li>The core elements of the performance required by this task are:</li> <li>Represent and interpret data using graphs or other representation</li> <li>Describe and compare data using qualitative and quantitative measures</li> <li>Represent the same data in more than one way.</li> <li>Communicate reasoning using words, numbers or pictures</li> </ul>		
	Based on these credit for specific aspects of performance should be assigned as follow:		
1	Completes graph correctly Bar Heights: <b>4-8-14-6-5</b> Completes 3-4 graph columns correctly	2 (1)	
2	Gives correct answer as: <b>birds</b> , accept <b>4</b>	1ft	2
3	Gives correct answer: 22	1ft	1
	Shows work such as: $14 + 8 = 22$	1ft	2
4	Gives correct answer: 8	1ft	
	Shows work such as: $14 - 6 = 8$	1ft	2
5	Correctly finishes tally table with <b>32</b> have pets	1ft	
	Correctly finishes tally table with <b>5</b> who do not have pets	1ft	2
6	Explains one thing learned such as:		
	That there were 6 fish. You can learn the kinds of animals. How many people have cats.	1	1
Total			10

## 2nd grade – Task 2: Pets

Work the task and examine the rubric. What do you think are the key mathematics the task is trying to assess?

Looking at the student work, determine which students were able to accurately transfer the data from the tally table to the bar graph in Part 1. As you look at the student work in other parts of the task, think about how the accuracy of this bar graph affects the accuracy of all other calculations and statements students are required to complete to be successful at this task.

- For students who made mistakes in creating the bar graph, were these caused by confusion on how to read tallies? Inaccuracies in visual discrimination, such as moving data over one bar for each pet when transferring? Misunderstandings in how to make a bar graph, including starting from the top to shade, leaving the first bar blank, or using tallies in the bar graph?
- What learning experiences would benefit students who are struggling with this transfer of information?

Look at the student work for Part 3. How many students put:

22	32	37	6	Other

- For students who accurately summed to 22, which strategies did they use? How could those successful strategies be shared with other students?
- How might students arrive at the other answers of 32, 37, and 6?
- When you look at the student work for students who got an "other" answer, how many of them chose a correct process and strategy, but were using incorrect data because of an inaccurately constructed bar graph?

Look at the student work for Part 4. How many students put:

8	20	9	6	Other

- For students who accurately found a difference of 8, which strategies did they use? How could those successful strategies be shared with students?
- How might students arrive at the other answers of 20, 9, and 6? What does this tell us about student understanding?
- When you look at the student work for students who got an "other" answer, how many of them made an error in calculating? Which were based on an error in the data?
- What strategies did students use to find the difference between 14 and 6? Were some more accurate than others? More efficient than others?

When analyzing Parts 3 and 4, think about examples in the student work where students used particularly effective tools, strategies, or explanations of their thinking. In what ways can these successes be shared with other students? Are there models that you don't see evidence of, such as the use of open number lines or bar models or drawings, that you think would have been helpful in clarifying student thinking, improving their explanations, and increasing their efficiency and accuracy? How might these tools and strategies also be introduced in meaningful ways?

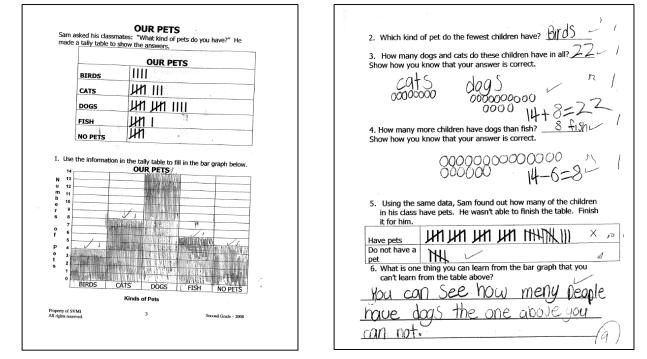
Looking at the statements in Part 6, can you identify students who:

- Offer judgments on usability of a graph, such as one being harder or prettier?
- Make a statement that could easily be applicable to both graphs, such as "I can count" or "There are 32 pets"?
- Make statements about an incomplete tally count in the table from Part 5, such as stating that there are 22 or 3?
- Make unclear or incomplete statements about one or both graphs?

What do these different answers tell you about the students' understanding of the relationship between the three representations of the data? What evidence is there that students understand that each of these is a representation of the same data? Or is there evidence that they see these as unrelated, separate tasks?

In order to be successful at this task, students needed to move accurately and flexibly between three representations of one data set, and perform qualitative and quantitative analysis on the information. **Student A** represents the 23.9% of students who performed at the highest level on this task. The student accurately transferred the data from the tally chart to the bar graph, was able to use that data to compare and combine quantities, and could explain at least one difference in the kinds of information available on two representations of the same data. A small error in representing the total number of pets with tally marks on the last chart did not diminish the student's understanding of the data. When performing calculations, the student could justify the work using symbols, drawings, and numbers.

#### Student A



**Student B** represents the 7.8% of students who performed at the lowest end on this task. The student loses points because they were not able to accurately transfer the data between the three representations, and moreover, they did not seem to understand that the charts and graph are all based on the same information. Although **Student B** had some good strategies for adding and subtracting, including using the graph to count both sets and to do a comparison by direct matching, all calculations were based on incorrect values in the bar graph, and the explanations provided were accurate, but incomplete.

a tally table	to show the	/hat kind of pe answers.	ets do you n	aver ne
		OUR PET	S	
BIRDS	111			
CATS	Ш	1		
DOGS	Ш	1 µ11 11		
FISH	Ш	11		
NO PETS	Ûł	1		
the informa	tion in the t	ally table to fi UR PETS	ll in the bar	graph below
the informa	ition in the t	ally table to fi	ll in the bar	graph below
the informa	tion in the t	ally table to fi	ll in the bar	graph below
the information of the informati		ally table to fi	I in the bar	

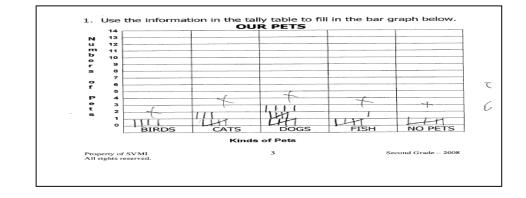


2. Which kind of pet do the fewest children have? $binds^{\vee}$
3. How many dogs and cats do these children have in all? $\frac{9}{2}$ show how you know that your answer is correct. The put of 0.95 and cat $\frac{1}{2}$
4. How many more children have dogs than fish? × &
I matched them togethere
5. Using the same data, Sam found out how many of the children in his class have pets. He wasn't able to finish the table. Finish it for him.
Do not have a pet y
6. What is one thing you can learn from the bar graph that you can't learn from the table above?
There's at 1:0 hours
hard a start of the
children hanne
0.875.

The next few pieces of student work for **Students C – H** demonstrate the variety of strategies students use to make sense of the data, and provide evidence of the ways that students see (or don't see) the connection between the different representations. Are your students using any of these counting and organizing strategies? Are your students unable to find correct solutions to qualitative and quantitative questions about the data because of errors in how they set up the data?

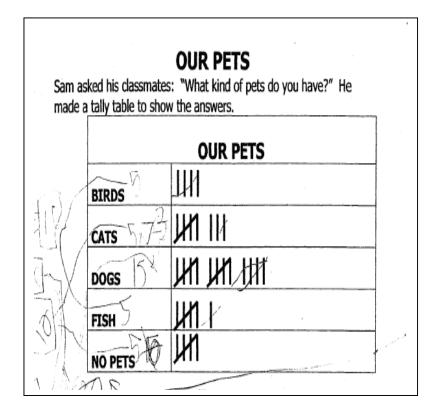
**Student C** struggled with the logistics of completing the bar graph, but seems to understand that the data should come from the tally chart.

#### Student C



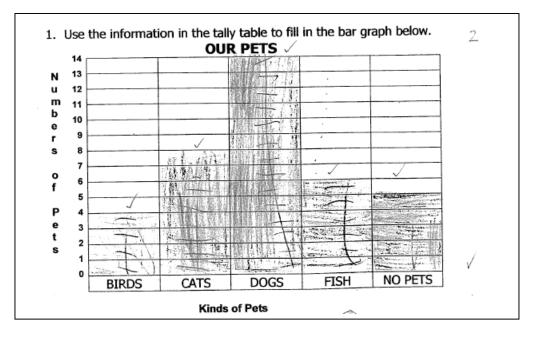
In this work, **Student D** uses a strategy that helps him or her to answer Part 6. In totaling the number of pets, this student uses the "extra" tally marks to complete groups of 5. They mark one tally off of the Cats and add it to Birds. They mark one tally off of fish and add it to Dogs. Now this student can count by fives, with one group of two extra tally marks with the Cats. *How many of your students attempted to complete the tally chart in Part 6 but made errors in the total number of tallies because of miscounting?* 

## Student D



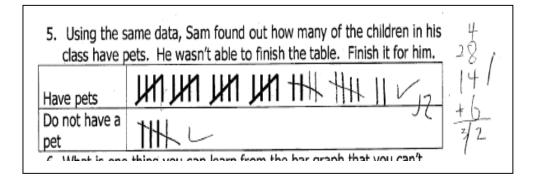
**Student E's** work shows evidence of how they are managing the data. The student marked each box, verifying that they had shaded the number of boxes that corresponded to the number of tally marks. In order to find the difference between the number of dogs and the number of fish, this student counts the blank boxes that match the remainder of the shaded boxes in the dog category. *Does this student even see the number labels on the vertical axis? If they were aware of this graph feature, would they need to count every box?* 

#### Student E



In the margin of **Student F's** paper, there is evidence of how they determined the total number of pets for the answer to Part 6. The numbers they are working with come from the tally chart in Part 1.

#### Student F



**Students G –J** solved Part 3 using a variety of strategies. Look at the strategies they've used. *Identify the evidence of student thinking. Are any of these strategies more efficient than others? Are any more accurate? How can student work be used to re-engage with the problem and begin to make connections between different strategies to deepen understanding?* 

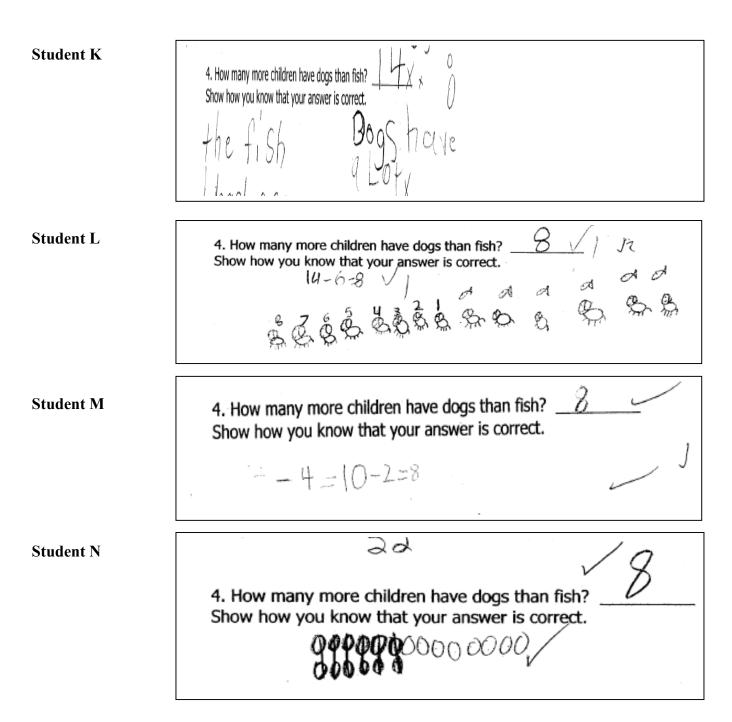
## Student G

## Student H

3. How many dogs and cats do these children have in all? 3. How many dogs and cats do these children have in all Show how you know that your answer is correct. Show how you know that your answer is correct. Student I 3. How many dogs and cats do these children have in all? Show how you know that your answer is correct 4. How many more children have dogs than fish? Show how you know that your answer is correct. 3. How many dogs and cats do these children have in all? Student J Show how you know that your answer is corr 4. How many more children have dogs than fish?

Show how you know that your answer is correct.

2<sup>nd</sup> Grade – 2008 Copyright ©2008 by Noyce Foundation All rights reserved. **Students K** – **N** solved Part 4. Look at their work. What strategies can you identify? What do they understand about comparing quantities?



#### **Re-Engagement**

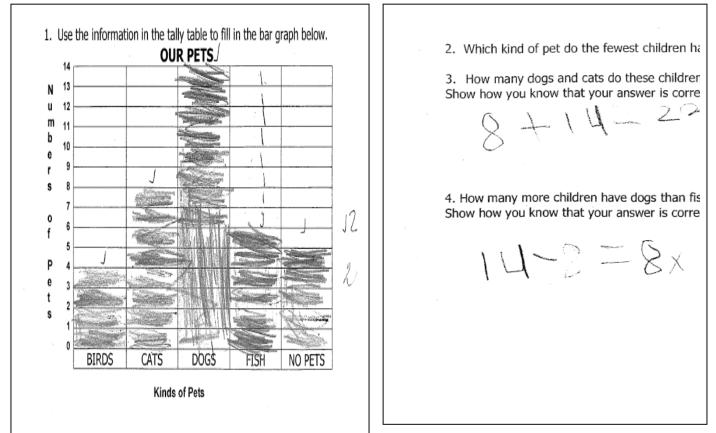
**Student O** and **Student P** both solved Part 4 using the graph to count the physical differences in the number of shaded boxes between the categories of dogs and fish. They both came to the correct answer. *Look at the number sentences these two students wrote to go with their work. How does thinking addition and then writing subtraction seem to cause confusion for Student P? Would it help Student P to verbally describe their process, as Student O did? Does connecting to the action help choose an operation?* 

#### Student O

V1F+ 4. How many more children have dogs than fish? Show how you know that your answer is correct. Fish 6+8=14

"I counted all the gray spots that are higher than the grey spots for fish.

#### Student P



 $2^{nd}$  Grade - 2008Copyright ©2008 by Noyce Foundation All rights reserved.

#### **Re-Engagement**

**Students Q and R** both used counting to solve the quantitative questions for this task. Only one of them, though, got credit for their explanations. *What can you identify in Student Q's explanation that is not part of Student R's explanation? Tell the class that they have to solve the problem based only on the instructions you read them. Read Student R's explanation and ask, could they solve this problem? Would they be able to identify what information was missing? Can they ask a question or two to get clarification? Do the same with Student Q's work. Can they clearly articulate what makes Student Q's explanation more mathematically explicit than Student R's?* 

#### Student Q

4. How many more children have dogs than fish? Show how you know that your answer is correct. I counted on from 6 to 14 My fingers. Witn E Licing the came data. Cam found out how many of the children

#### Student R

3. How many dogs and cats do these children have in all? 22Show how you know that your answer is correct. L knew this becuse I count 4. How many more children have dogs than fish? Show how you know that your answer is correct. Λ

• 2nd G	
Student Task	Complete a bar graph to represent data given in a tally table. Find numerical and categorical information and compare values. Explain their thinking in finding quantitative information about the graphs. Complete a tally table using summary information from the data. Make a statement that compares the kinds of information found on two different representations of the same data.
Core Idea 5:	Students organize, display, and interpret data about themselves and their
Data Analysis	<ul> <li>surroundings.</li> <li>Describe and compare data using qualitative and quantitative measures.</li> <li>Represent the same data in more than one way.</li> <li>Represent and interpret data using bar graphs, tally charts, and other representations.</li> <li>Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently.</li> </ul>
Core Idea 2:	<ul> <li>Demonstrate fluency in adding and subtracting whole numbers.</li> </ul>
Number	
Operations	

## Mathematics in this task:

- Complete a bar graph based on the information in a tally chart.
- Represent strategies of addition and subtraction for values based on the graphs.
- Use the same information to create a different interpretation of the data and record the information by completing a second tally table.
- Compare the type of information available on two representations of the same data.
- Represent and explain thinking and strategies using words, pictures, models, and/or symbols.

Based on teacher observations, this is what second graders know and are able to do:

- Use the tally marks in the first chart to make a bar graph.
- Identify and name the least value in the graph.
- Use a variety of strategies to add and subtract values from the data, including finding landmark numbers, algorithms, using missing addend addition to find a difference, using the bar graph to count the number of extra or missing bars when comparing.

## Areas of difficulty for second graders:

- Adding the two values instead of finding the difference when comparing.
- Using the standard algorithm to subtract accurately.
- Most students only completed the tallies for No Pets on the second tally table.
- Making a clear statement that compares two representations of the same data.

## Strategies used by successful students:

- Use tallies, drawings, or other models to represent values and strategies when adding and subtracting.
- Use a variety of strategies to find the difference between two numbers.
- Identify the relevant information needed to choose the values and operations.
- Move between representations to make sure the values are the same in each representation.

MAC 2008

MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 2

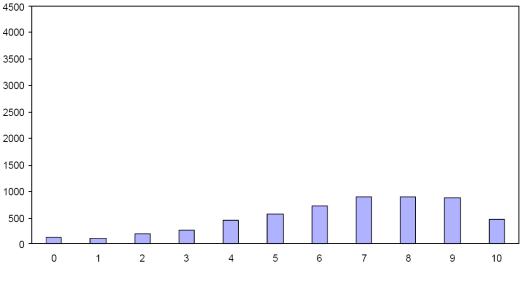
#### Task 2 - Our Pets

Mean: 6.49 StdDev: 2.46

Task 2 Scores	Student Count	% at or below	% at or above
0	138	2.5%	100.0%
1	99	4.2%	97.5%
2	199	7.8%	95.8%
3	273	12.6%	92.2%
4	459	20.8%	87.4%
5	577	31.1%	79.2%
6	730	44.1%	68.9%
7	895	60.1%	55.9%
8	895	76.1%	39.9%
9	875	91.7%	23.9%
10	468	100.0%	8.3%

Table 11: Frequency Distribution of MARS Test Task 2, Grade 2

Figure 20: Bar	Graph of MARS	Test Task 2 F	Raw Scores,	Grade 2
----------------	---------------	---------------	-------------	---------



MARS Task 2 Raw Scores

Points	Understandings	Misunderstandings
0 - 2	All the students in the sample with this score	85% of the students at this level were unable
0 - 2	attempted the task. Some students could identify	to accurately transfer data from the tally table
	the pet with the least tallies. Nearly half the	to the bar graph. Some students confused
	students described an accurate process to	the meaning of "least" to be "none" in Part
	answer the quantitative questions in Parts 3 and	2, as evidenced by answering with "No
	4, but based their calculations on incorrect data	Pets". Half the students made calculation
	from the bar graphs they had created.	errors in their addition and subtraction. 75%
		of the students made no attempt to complete
		the second tally table representation of the
		data.
3	More than half of these students received full	Students at this level were working with
	credit on creating the bar graph, and the rest	better data, but continued to be plagued by
	received partial credit. They could correctly	errors in calculations. Many of these
	identify the pet with the least tallies. 55% also	students incorrectly used the standard
	described an accurate process to answer the	algorithm for subtraction. When using a
	quantitative questions in Parts 3 and 4. About a	matching strategy, some students used a tally
	third attempted to complete the tally table in Part 5 for the category "No Pets".	mark model, which proved difficult for
	5 for the category into rets.	making a direct match for comparison purposes. Nearly half of these students
		were also using an inaccurate bar graph to do
		their calculations and comparisons.
4 5	Two-thirds of these students got full credit for	60% of these students were not able to
4 - 5	the bar graph they created. 40% of the students	accurately answer Part 3. When answering
	answered Part 3 correctly, and another 40%	Part 4, 80% gave an incorrect response, and a
	chose a correct process. Students used a variety	third of them added instead of finding the
	of strategies to find the difference between the	difference. These students were not able to
	two values in Part 4, including direct matching,	make a clear or accurate statement comparing
	counting up, and subtraction or counting back.	the information on the different
	60% of these students attempted to complete the	representations of the graph. Attempts to
	tally table in Part 5 for the category "No Pets"	complete the tally table in Part 5 indicate that
	and another third of them attempted to complete	they may not have seen the connection
	it for the category of "Have Pets".	between the representations.
	Students were able to transfer the data from the	Students continued to make calculation errors
6	tally table to the bar graph. In Part 3, 80% of	when subtracting, particularly with the
	the students were able to identify the relevant	standard algorithm. About 10% of these
	information and accurately sum two numbers.	students incorrectly added the two values
	Nearly 50% were able to accurately find the	they were supposed to be comparing in Part
	difference between two values in Part 4, and	4. At this level, students who were unable
	used a variety of strategies including direct	to complete the second tally table were also
	matching and counting up. More than 60% of	unable to write an accurate statement in Part
	these students attempted to complete the tally	6, and nearly 40 % of them didn't even
	graph in Part 5, and nearly half were able to	attempt to complete the first section of Part 5.
	make an accurate statement in Part 6.	
7 - 8	Students working at this level were able to	Some students continued to make small or
<i>i</i> = 0	accurately complete the bar graph. More than	isolated errors when creating the bar graph.
	80% were able to correctly write and solve	This small percentage of graphing errors led
	addition and subtraction equations to solve Parts	to some incorrect calculations in Parts 3 and
	3 and 4. These students all attempted to answer	4, but students still got process points for
	Part 5, and almost 40% were able to accurately	their explanations. Around 60% of these
	complete the tallies in both "Have Pets" and	students were not able to complete the "Have
	"No Pets". An additional 42% were able to	Pets" part of the tally table, though nearly all
	create the "No Pets" tallies. When using a	attempted to. Common errors included

	more accurate tally chart, two-thirds were able to make a clear and accurate statement in Part 6.	counting "No Pets" in for a total of 37 tally marks.
9 – 10	Students were able to accurately complete the bar graph. They could identify the relevant values in Parts 3 and 4 and use addition and subtraction to answer quantitative questions about the data. These students demonstrated an understanding of the relationship between the three representations of the data, and could create the second tally table and make a clear comparative statement about the type of data found in each representation.	

#### **Teaching Implications**

In this task, we saw that students working at all but the highest levels struggled with answering the question "How many more?" when comparing two values on the graph.

**Direct Modeling** In their book, *Children's Mathematics* (Heinemann, 1999), Thomas Carpenter and his colleagues reflect on their research with young children who have been given the opportunity to direct model their thinking and actions when operating on two values.

These students were given many opportunities to use cubes to directly model various actions they interpreted from math stories involving combining, separating, or comparing values. Looking at the curriculum and materials for your classroom, where can you provide experiences for them experience direct modeling?

**Counting Strategies** From these experiences, they invented counting strategies that were direct reflections of the actions they took with the cubes. From these counting strategies, they began to construct increasingly efficient counting strategies that involved groupings, doublings, etc. At each step along this progression, the students were engaged in constructing their own understandings and strategies. As you observe your own students, where can you guide them and use questioning techniques to help them identify and formalize the strategies they are using?

**Comparisons** In many elementary math classrooms, young children who are asked to solve a task that involves a comparison to find out "How many more" between two values are often instructed to look for those "key words" which indicate that they should then subtract to find the answer. Interestingly, when children construct their own meaning, they very often don't think of comparisons as subtractive in nature. In fact, when these children had stacked two sets of cubes (for example a tower of 5 and a tower of 7) to represent to values, they tended to solve the comparison by directly matching the cubes that were the same, and then *counting on* to the smaller value to find out the difference between the two. In our example, they would start with 5, and count up two cubes until they reached 7. Those two cubes represent the difference between the numbers.

**Reflection** If we insist that they use the definition "finding the difference" to mean "subtract", are we overwriting their intuition about how this relationship works? How could using the techniques of Cognitively Guided Instruction described in Carpenter's book help us to *build* on the intuition they bring to the mathematical situation? Making a connection between the two equations  $5 + \_\_ = 7$  and 7 - 5 =

\_\_\_\_\_\_ deepens the children's understanding of addition, subtraction, comparison, and inverse operations. It helps them understand the relationship between both operations, and between both values. When they understand this relationship, picking an operation becomes a trivial part of the mathematics, because they know the situation can be resolved using either addition *or* subtraction.

In what ways could this deeper understanding support students who continue to struggle with the standard algorithm and counting backward models of subtraction?

#### **Ideas for Action Research**

In this task, we also saw that students working at all but the highest levels struggled with understanding that the three representations in the task were all generated from the same data.

#### **Re-Engagement**

Have your students complete the task, *Pets*, and/or use this activity any time your students are working with data. Use the student work to re-engage with your students afterward.

Have them look at the three graphs that are part of the task. When students understand that these represent the same data, that they are in fact related, they can begin to reflect on the idea that different representations might be helpful to answer different questions. They can begin to imagine other ways they might represent the data.

In order to help students make connections between various representations of the same data, use a questioning strategy such as:

- Can students identify the question asked to make the original tally table in the task?
- Can they make the connection that the same question is answered with the bar graph?
- Can they identify the question asked to make the second tally table?
- Can they make a connection between that question and the data pool that answered that question?
- Where did this data come from?
- Who answered the question "how many students did not have pets"?

Once students have connected the data in the different representations, are they more successful in thinking about and explaining differences in information reflected *about* that data? Does thinking about the data this way help them to accurately make the second tally table? If not, what other experiences would help students explore these two questions about data sets and multiple representations of them?

Performance Assessment Task					
Our Pets					
Grade 2					
The task challenges a student to demonstrate understanding of concepts involved in representing					

and interpreting data. Students must make sense of representing the same data in more than one way. A student must understand how to interpret data from different representations. A student must understand how to describe and compare data and data representations using qualitative and quantitative measures.

#### Common Core State Standards Math - Content Standards

#### Measurement and Data

#### Represent and interpret data

2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put- together, take-apart, and compare problems using information presented in a bar graph.

# Common Core State Standards Math – Standards of Mathematical Practice **MP.2 Reason abstractly and quantitatively**.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
2	2008	10	6	69%