Incredible Equations

On March 16th, some of the students in Mrs. Daniels' class wrote equations equal to 16. During recess, Mrs. Daniels erased parts of each equation. Find the missing parts.

16

7 + ______ = 16	___ + 8 + ___= 16
16 = 3 + ______	27 - ______ = 16
100 - _____ = 16	16 = ______ + 10

______ - 4 = 16

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

11 + 5 = _____ + 8

Show how you know your answer is correct.

Incredible Equations
Mathematics Assessment Collaborative
Performance Assessment Rubric Grade 2

	Task 5: Incredible Equations: Grade 2:	Points	Section Points
	The core elements of the performance required by this task are: • Demonstrate fluency in adding and subtracting whole numbers • Use strategies to estimate and solve problems involving addition and subtraction • Communicate reasoning using words, numbers or pictures		
1	Gives correct answers, starting from top left to right: **9** **Any two addends equal to 8** **13** **11** **84** **6** **20**	**7 x 1**	**7**
2	Gives correct answer as: **8** Explains answer such as: 11 + 5 = 16 and 8 + 8 = 16	**1** **2**	**3**
	Total Points		**10**

2^{nd} grade – Task 5: Incredible Equations

Work the task and examine the rubric.
What do you think are the key mathematics the task is trying to assess? ______________

____
____
____
____

Look at student work for the first 7 equations. How many of your students were successful solving:

7 + 9 = 16	_ + 8 + _ = 16	27 – 11 = 16	16 = 6 + 10	16 = 3 + 13	100 – 84 = 16	20 – 4 = 16

- Which problems are easiest for your students to solve?
- What makes these "easier" problems to solve?
- What strategies are necessary to successfully solve each problem?
- What do the errors reveal about students' understanding of equality?
- What do the errors reveal about students' understanding of part-whole relationships?

Look at student work for the last equation. Chart the answers given:

8	16	24	16 and 24	Other

- What prompts a response of 16?
- What prompts a response of 24?
- Why do so many children have trouble balance equations in this format?

____
____
____

Looking at Student Work on Incredible Equations

The common correct and incorrect answers in the Incredible Equations can be useful to use for re-teaching number relationships and number operations. Each of these errors can open up rich discussions - *"Why does this work?" "Why doesn't this work?"*
Student A gives a clear and concrete explanation to why "8" + 8 will balance out the equation at the end of the task. *Why does this drawing make sense? How might your students respond to this equality statement?* This paper also includes two very common errors in the top equations. Either of these errors could be used effectively in a class discussion setting. Students might suggest why another student might think "12" was correct as well as suggest the correct answer and why.

Student A

Incredible Equations

On March 16th, some of the students in Mrs. Daniels' class wrote equations equal to 16. During recess, Mrs. Daniels erased parts of each equation. Find the missing parts.

16

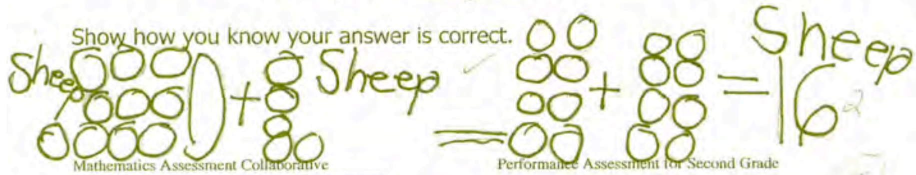
7 + 9 = 16	5 + 8 + 3 = 16
16 = 3 + 13	27 - 11 = 16
100 - 85 = 16	16 = 6 + 10

12 - 4 = 16

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

11 + 5 = 8 + 8

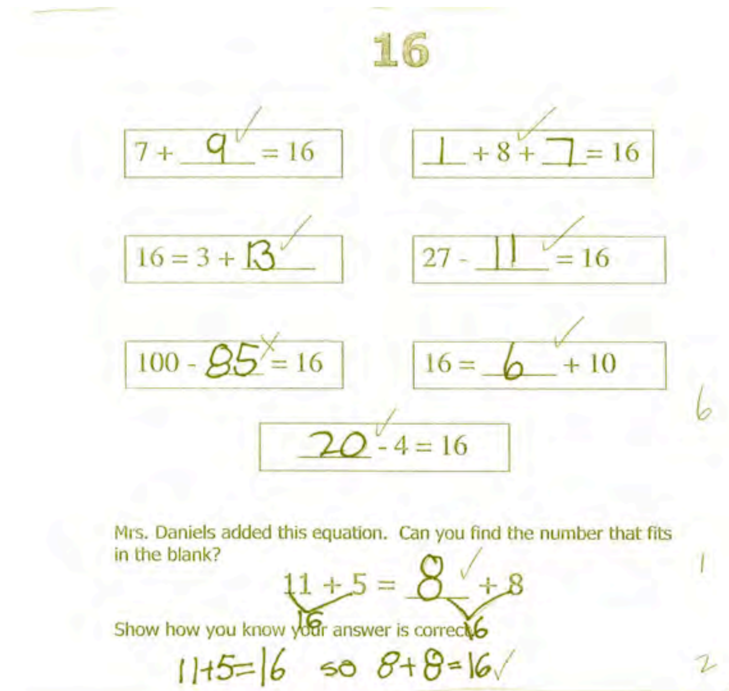
Show how you know your answer is correct.



Performance Assessment for Second Grade

Student B's notations are clear and illustrate the definition of the equals sign as a balance or as signifying "is the same number as".

Student B


16

7 + 9 = 16

1 + 8 + 7 = 16

16 = 3 + 13

27 - 11 = 16

100 - 85 = 16

16 = 6 + 10

6

20 - 4 = 16

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

11 + 5 = 8 + 8

1

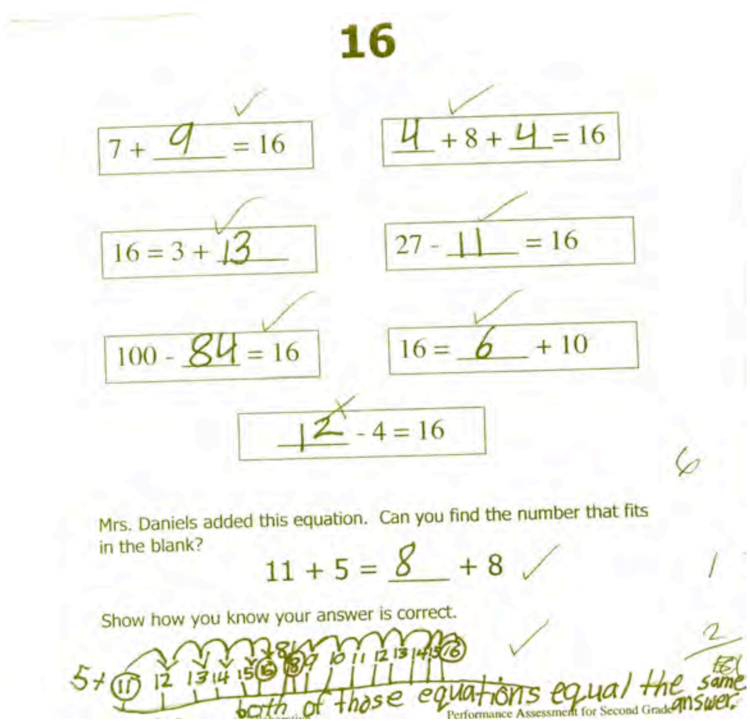
Show how you know your answer is correct.

11+5=16 so 8+8=16

2

Student C's unique use of the number line may not be clear at first glance. *What is the evidence of understanding in this work? What question would you ask of Student C, or any other student, regarding this number line? What might make this illustration clearer?*

Student C


16

7 + 9 = 16

4 + 8 + 4 = 16

16 = 3 + 13

27 - 11 = 16

100 - 84 = 16

16 = 6 + 10

12 - 4 = 16

6

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

11 + 5 = 8 + 8

1

Show how you know your answer is correct.

5+ 11 12 13 14 15 16 8 9 10 11 12 13 14 15 16

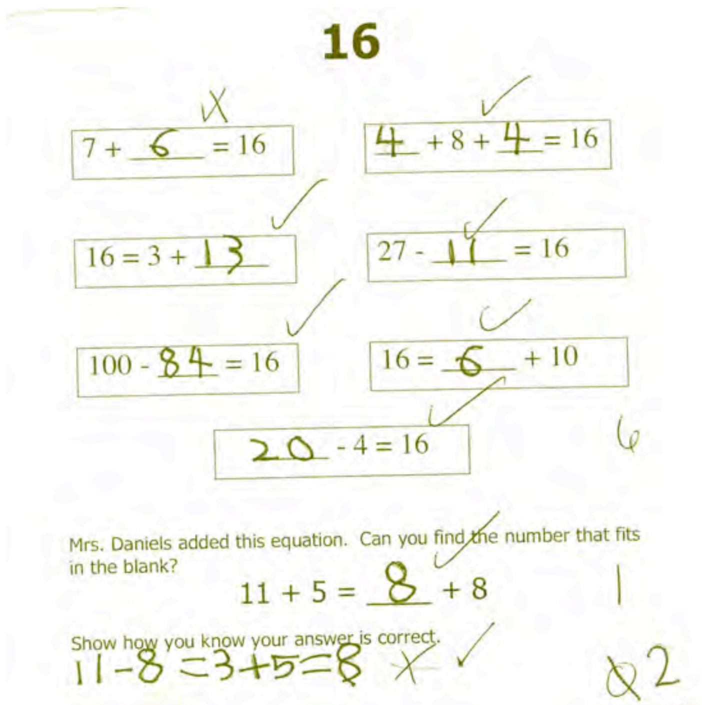
both of those equations equal the same answer.

2

Performance Assessment for Second Grade

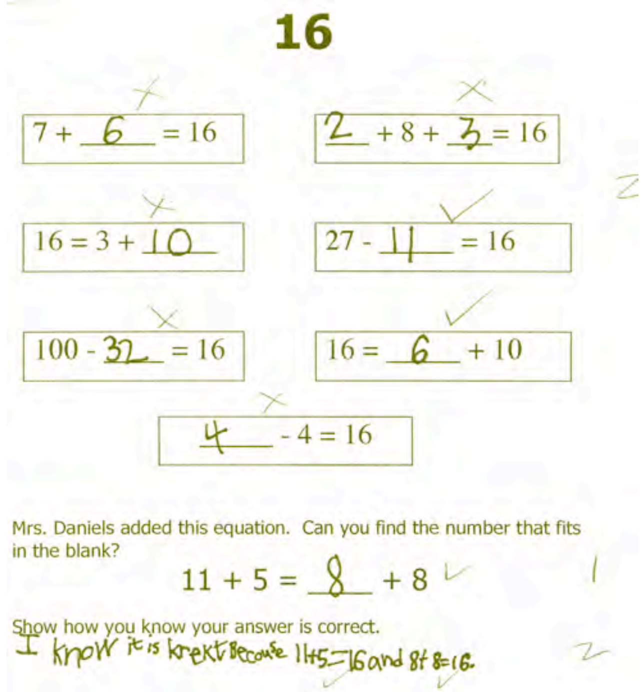
Student D's explanation for "8" in the last equation was originally scored as incorrect. *What thinking went into 11 – 8 = 3 and 3 + 5 = 8? Why does this work?*

Student D



Student E presents a clear, correct and concise explanation for the answer of "8" in the last equation. *What evidence leads to a questioning of this understanding? What evidence of understanding do you see? What evidence of misunderstanding do you see? What would be your first questions in an interview with this student?*

Student E



16

7 + 6 = 16 | 2 + 8 + 3 = 16

16 = 3 + 10 | 27 - 11 = 16

100 - 32 = 16 | 16 = 6 + 10

4 - 4 = 16

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

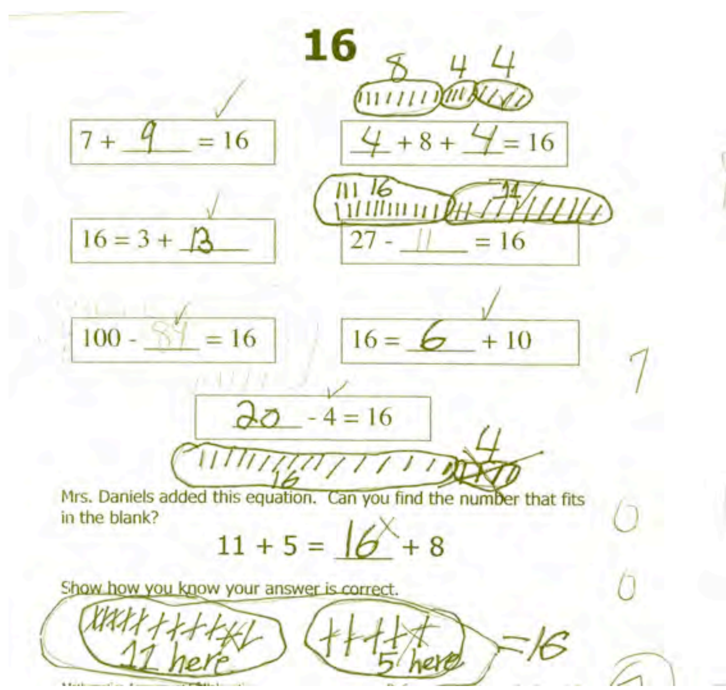
11 + 5 = 8 + 8

Show how you know your answer is correct.

I know it is krekt becoue 11+5=16 and 8+8=16.

Student F shows us some of the thinking involved in finding correct solutions. *What does this student know about part-whole relationships with regard to addition and subtraction? Can this student find the whole? The parts?* What you don't see is the similar work around 100 – 84 = 16! It is tedious but successful. Student F's paper reflects the most common error seen in the final equation. In total 35% of the sample papers answered with 11 + 5 = 16 + 8. It appears that for this student, and others, the equals sign signifies "and the answer is".

Student F



16

7 + 9 = 16 | 4 + 8 + 4 = 16

16 = 3 + 13 | 27 - 11 = 16

100 - 84 = 16 | 16 = 6 + 10

20 - 4 = 16

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

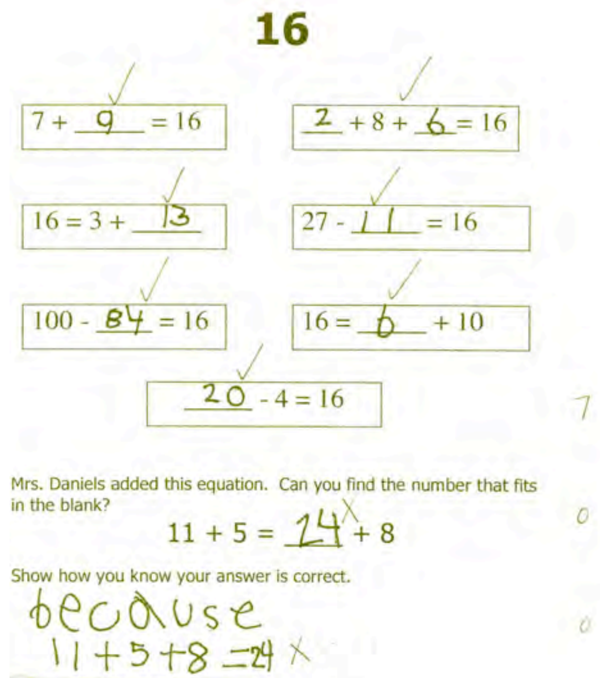
11 + 5 = 16 + 8

Show how you know your answer is correct.

11 here 5 here =16

As the work of Student G reflects, 7% of the papers reviewed added the three addends together to find 24 and place this answer in the blank.

Student G



Student H appears to be fluent in finding missing parts for most equations presented with the answer to the right. Several of the errors in this paper, however, are excellent resources for class discussions. *What do your students know about the size of the number necessary in "? – 4 = 20"? How might they reason around this problem and a number line or a hundred's chart? What might happen if they had cubes or objects to help their reasoning? Why might this student have answered 16 = 3 + 19? 16 = 26 + 10?* These two errors can open up a great deal of talk around different correct formats for equations.

Student H

16

7 + 9 = 16

5 + 8 + 3 = 16

16 = 3 + 19

27 - 19 = 16

3

100 - 84 = 16

16 = 26 + 10

? - 4 = 16

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

11 + 5 = 16 + 8

0

Show how you know your answer is correct.

0

Which of the errors in Student I's paper would you present to discuss with your class? Many of these may confuse rather than enlighten students. *What kind of thinking and flexibility might be discussed around 2 + 8 + 4 = 16? How might this error help to illustrate the meaning of the equals sign? How might changing the format of this equation help to clarify equality?*

Student I

16

7 + 9 = 16

2 + 8 + 4 = 16

16 = 3 + 5

27 - 44 = 16

100 - 15 = 16

16 = 10 + 10

1

8 8 - 4 = 16

Mrs. Daniels added this equation. Can you find the number that fits in the blank?

0

11 + 5 = 9 + 8

Show how you know your answer is correct.

0

because I Just Thike it is 9

1

Performance Assessment for Second Grade

Incredible Equations

2nd grade **Task 5** **Incredible Equations**

Student Task	Find the missing parts of incomplete equations equal to 16. Estimate and solve problems using addition and subtraction. Show understand of the equals sign as a balance point between the two sides of an equation.
Core Idea 2: Number Operations	**Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently.** • Demonstrate fluency in adding and subtracting whole numbers • Use strategies to estimate and judge the reasonableness of results
Core Idea 3: Patterns, Functions, and Algebra	**Use mathematical models to represent and to understand quantitative relationships.** • Use principles and properties of operations • Use concrete, pictorial, and verbal representations to show an understanding of symbolic notations

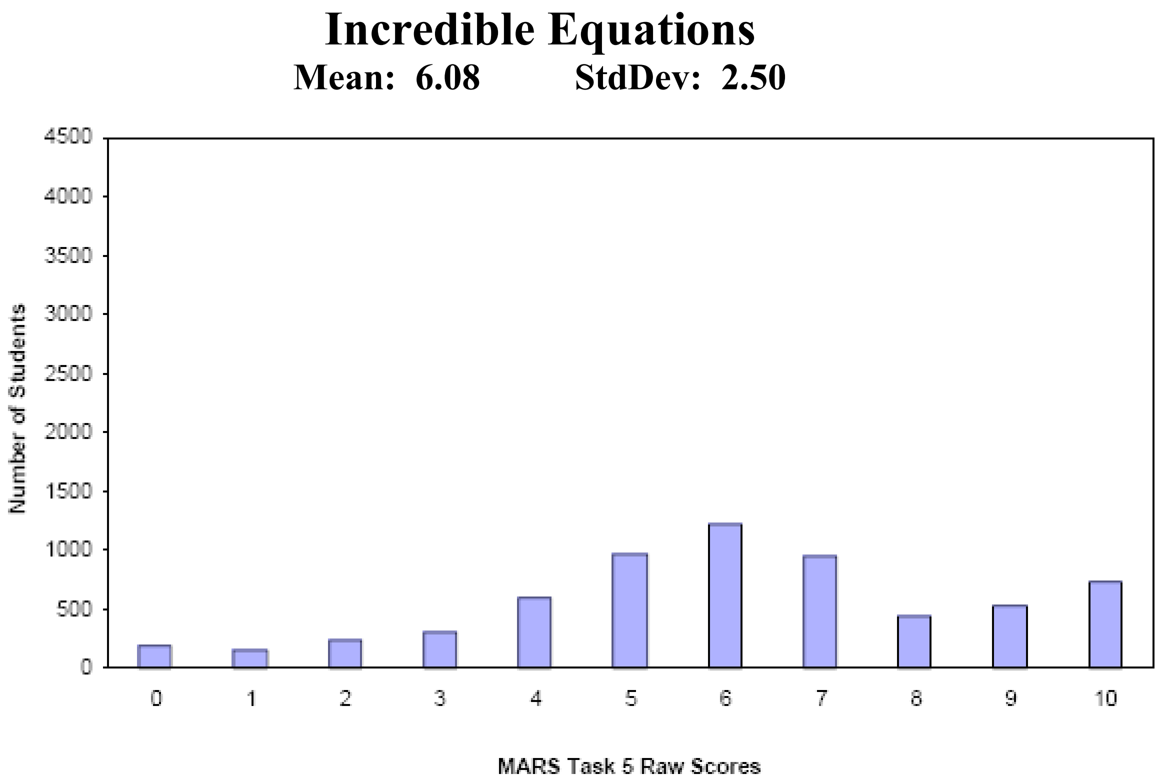
Mathematics of the task:
- Ability to fluently add and subtract whole numbers
- Ability to estimate and solve problems involving addition and subtraction
- Ability to use the relationship between addition and subtraction to solve problems
- Ability to show the meaning of the equal sign as a balance point between two values

Based on teacher observation, this is what second graders knew and were able to do:
- Find answers to addition problems when asked to join amounts
- Find the missing addend
- Find the missing subtrahend

Areas of difficulty for second graders:
- Finding the missing minuend ex: (__ - 4 = 16)
- Finding the complementary fact to make 100 ex: (100 - __ = 16)
- Finding the missing digit to make both sides of the equation equal
-

Strategies used by successful students:
- Showing their work
- Drawing circles or tallies to solve the equations
- Using addition and subtraction as inverse operations to solve missing addend or subtrahend problems
- Using slash marks, circles, or tally marks to show both sides of the equation would be equal to 16

Frequency Distribution for Task 5 – Grade 2 – Incredible Equations



Score	0	1	2	3	4	5	6	7	8	9	10
Student count	186	145	235	311	591	961	1222	954	449	524	727
% < =	3.0%	5.2%	9.0%	13.9%	23.3%	38.5%	57.9%	73.0%	80.2%	88.5%	100%
% > =	100%	97.0%	94.8%	91.0%	86.1%	76.7%	61.5%	42.1%	27.0%	19.8%	11.5%

There is a maximum of 10 points for this task
The cut score for a level 3 response, meeting standards, is 5 points.

Most of the students, 79%, were able to find the missing addends in equations written with the answer to the right. Approximately three-quarters of the students were able to meet the essential demands of the task. More than half of the students found the correct answers to at least four equations equal to 16. A little more than 11% were able to meet all the demands of the task including illustrating or explaining an equality statement. 3% of the students scored a zero on this task.

Incredible Equations

Points	Understandings	Misunderstandings
0		38% of students with a score of zero attempted the task. The answers given for equations showed little understanding of the effects of subtraction or addition.
1-2	Students were able to correctly answer those problems set up with the answers to the right: 7 + *9* = 16, _+ 8 + _ = 16 and 27 – *11* = 16.	The answers given for missing parts of the equation had little relationship to size or quantity.
3-4	In addition to the previously mentioned successes, these students were also able to answer 16 = *6* + 10 and 16 = 3 + *13*. 35% of these students could find *20* – 4 = 16.	59% of these students completed the last equation as follows: 11 + 5 = *16* + 8. 7% answered: 11 + 5 = *24* + 8. 10% left this equation blank.
5	10% correctly answered 11 + 5 = 8 + 8. 17% were able to find the missing addend to 100 – *84* = 16	100 – *86* = 16 was the most frequent error to this equation. 10% of these students left the final equation blank. 58% answered 11 + 5 = *16* + 8. 11% answered 24 for the same equation.
6-7	90.7% of students were successful in at least 6 of the 7 top equations. 32.8 % were able to at least complete 11 + 5 = *8* + 8. 5% could explain their reasoning.	100 – *84* = 16 proved the most challenging of the top 7 equations. In the equation: 11 + 5 = _+ 8, 45% of the students answered 16, 14% answered 24.
8-9	80% scored all three last points by answering correctly and showing how the answers were correct.	100 – *84* = 16 proved the most challenging of the top 7 equations. Incorrect and understandable responses included: 85, 86, 74. 94, 76, 96, 116.
10	Students showed how they knew that *8* correctly answered the last equation by drawing pictures of 16 and 16 objects, showing the two sides equal to facts for 16, and giving two separate equations equal to 16.	

Implications for Instruction and Action Research:

Students need help to develop appropriate understandings of equality. Children must come to understand that equality is a relationship that expresses the idea that two mathematical expressions hold the same value. It can be productive to challenge their existing understandings (that the equal sign is the "do it" sign) using the correct and incorrect responses they give to this task. It will help to engage your students in discussions of these responses in which ideas are shared, evaluated and clarified. This can be done in whole class or small group settings where students are encouraged to state their understanding around equality. You will learn a lot about what students are currently thinking and where your next challenges might go.

2nd grade students benefit from seeing correct answers and errors from this task and in being asked if the equations are true or false. If they state that they are true, ask "*How do you how?*" If they say false, ask *"What addend must appear to make the equation true?* How do you know?" Children can pick up critical ideas in the context of discussing all these problems and any other problems that come up in the course of your math class.

Children benefit from seeing several collections of problems that bring out different views of equality. The collection in this task might be arranged from easiest to most difficult as follows:

a. $7 + \underline{9} = 16$
b. $\_ + 8 + \_ = 16$
c. $16 = \underline{6} + 10$
d. $16 = 3 + \underline{13}$

a. $27 – \underline{11} = 16$
b. $100 – \underline{84} = 16$
c. $\underline{20} – 4 = 16$

Most children agree that a and b are true but c and d will start to cause disequilibrium to their view of equality unless they can consider the equals sign as meaning "the same number as". We are trying to help students understand that the equals sign signifies a relationship. It is useful to use words that express this directly – *"16 is the same number as 3 + 13"*.

Choosing which equations to discuss and when to do so is critical to developing deeper and more thorough understandings about the use of the equals sign. Listening to student discussions will tell you what you will need to confront with them next. It is equally important that students are productively engaged in mathematical discussions and arguments during math time. *Why do you think that? Is that always true?* Helping students state and clarify their view on equality will help develop and highlight the importance of justifying answers. We not only want to find out what answers are correct but to also come to understand how we negotiate and resolve differences in our knowledge bases. This is what it means to do mathematics.

Performance Assessment Task
Incredible Equations Grade 2
The task challenges a student to demonstrate understanding of concepts involved in addition and subtraction. A student must be able to understand addition and subtraction as inverse operations and apply this understanding to problems where the unknown is in different positions. A student must make sense of the equals sign as a balance point between the two sides of an equation in order to find a missing addend on one side of an equation. A student must be able to justify a solution.
Common Core State Standards Math - Content Standards
<u>**Operations and Algebraic Thinking**</u> **Represent and solve problems involving addition and subtraction.** 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **Add and subtract within 20.** 2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the

results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
2	2007	10	5	77%