High Horse

Horses have always been measured by using the hand span. Mom measures the horse using her hand span. Jean measures the horse using her hand span.



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2. If Mom measures the whole horse, how many hand spans will she use? Show how you figured it out.

_____hand spans

3. If Jean measures the whole horse, how many hand spans will she use? Show how you figured it out.

_____hand spans

4. Who will use more hand spans to measure the whole horse?

High Horse Mathematics Assessment Collaborative

	High Horse: Grade 2: 2009	Points	Section Points
	 The core elements of the performance required by this task are: Understand how to measure using non-standard units Correctly iterates the unit of measure Communicate reasoning using words, numbers or pictures 		
	Based on these credit for specific aspects of performance should be assigned as follow:		
1	Mom	1	1
2	8 hand spans Show work such as:	1	
	Drawing hand spans on the horse measure	2	3
3	12 hand spansShow work such as:	1	
	Drawing hand spans on the horse measure	2	3
4	Jean	1	1
			8

2nd Grade – Task 4: High Horse

Work the task and examine the rubric. What do you think are the key mathematics the task is trying to assess?

How many students were able to correctly answer Parts 1 and 4? Is there any additional information that might provide evidence of how they are thinking about unit size and it's effect on the number of units needed to measure a length?

Look at the student work for Part 2. Sort the student papers into two groups; those who were able to answer correctly, and those who did not. Now look at the student work for the students who were able to answer correctly. Be sure to look at the picture and the space given. How many students used the following strategies:

Draw Their Own Picture	Use the Picture Provided	Draw Hand Spans	Draw arrows, lines, or "jumps" for spans	Use Number Sentences or "count by"	Other

Looking at successful strategies:

- Which strategies did you value the most? Why?
- Which strategies were most successful for the students?
- What was the evidence that they were counting lines? Spaces?
- What did the students need to know or understand in order to use each of these strategies?
- How can these successful strategies be shared with the class?

(Repeat this process for Part 3. Did students use the same or different strategies when moving from Part 2 to Part 3? Were most students able to answer both correctly? Or was it common for one to be correct and another wrong? Why would one be more confusing than the other?)

Reflection: Read the following Big Ideas about measurement. In what ways does the High Horse task require the students to have made sense of these ideas? Where will you find evidence, in this task, that the student is using or thinking about the following ideas?

- Matching ~ the units being used to measure the length need to "match up" against the attribute being measured. In this case, do not overlap the units, and the units shouldn't "stop short" or "go over" at either end of the length being measured.
- Unit Consistency ~ the units being used to measure the length need to be the same size. The units shouldn't get larger or smaller as the measurement is executed.
- Iteration ~ in this task, the unit can be identified by the number of spaces *or* the number of lines covered by the hand span. Sometimes a single unit is iterated over and over to measure

an attribute (as opposed to a measuring tool that has multiple units in one place). If the student is making sense of the iteration based on the number of lines (rather than spaces) in the hand span, than they must take this into account and overlap the last line of one hand with the first line of the next hand being iterated.

Now look at the student work for Part 2 for the students who answered incorrectly. What strategies did these struggling students use?

Draw Their	Use the Picture	Draw Hand	Draw arrows,	Use Number	Other
Own Picture	Provided	Spans	lines, or	Sentences or	
			"jumps" for	"count by"	
			spans		

- Did they use the same, or a different set of strategies, from the more successful students?
- What evidence is there that they understand or were using the Big Ideas?
- Was a lack of understanding in these areas affecting their ability to use any of the strategies successfully?
- Did students use correct drawings or counting strategies, counted inaccurately?
- How can you encourage the students to transfer to the more successful strategies?

Student AA scored the maximum points on the task. The student identified the number of spaces or lines needed to span one unit, then accurately and completely iterated that unit across the attribute of the length. How might this drawing help other students to understand the relevant details s/he needed to include? What questions would you want to ask this student about the different attributes of their drawing, and where in the task they found the information to inform those decisions?

Student AA

 If Mom measures the whole horse, how many hand spans will she use? Show how you figured it out. 	
5	
	2
hand spans	S
3. If Jean measures the whole horse, how many hand spans will	
Show how you figured it out.	
	2
97	
E /	
hand span	S
4. Who will use more hand spans to measure the whole horse?	Jean
	8
Copyright © 2009 by Noyce Foundation Page 7	High Horse

 2^{nd} Grade - 2009 Copyright ©2009 by Noyce Foundation All rights reserved. **Students A and B** are using words and drawings to make sense of the effects of unit size on the number of iterations needed to measure a length. **Student A** may be estimating hand spans needed, but s/he is making the estimation smaller in Part 2 because "her hand is greater than Jean's hand" and larger in Part 3 because "Jean's hand is smaller than her mom's". **Student B** draws a picture of six of Mom's hands next to 6 of Jean's hands, to demonstrate that one is larger and one is smaller, and that the larger hand will go further in the same number of iterations.

Student A

r 2. If Mom measures the whole horse, how many hand spans will she use? Show how you figured it out. a su mamá mide el caballo entero, ¿quántos palmos usará? tra cómo lo averiguaste. hand spans 0 por que vo tombien te ise el Si Jean mide el caballo entero, ¿cuántos palmos usará? del cabille 3. If Jean measures the whole horse, how many hand spans will palmo she use? Muestra cómo lo averiguaste. Show how you figured it out. mas grande chiquito. ٧ mino su manos eran chiquititas. Pareve 13 palmos 0 \mathbb{R}^{\times} hand spans ¿Quién usará más palmos para medir el caballo entero? le an Who will use more hand spans to measure the whole horse? Te 8 8 Z

Student B

In Part 3, **Students C and D** are both making sense of the number of spaces needed to measure one of Jean's hands in the drawing. **Student C** explains in words that s/he counted by 2s. **Student D** uses a drawing. In this drawing, what do the circles represent? What do the two marks inside each circle represent?

3. If Jean measures the whole horse, how many hand spans will she use? show how you figured it out. I counted by 2's UP the graph -2. If Mom measures the whole horse, how many hand spans will she use? MID hand spans 4. Who will use more hand spans to measure the whole horse? Tean Jean because I uses 1235 Spaces 8 3. If Jean measures the whole horse, how many Capylight © 2008 by Noyce Fr Page 7 she use? Show how you figured it out. ean12 8 ١ 2 12_hand spans Who will use more hand spans to measure the whole horse? $\int QG\dot{\eta} v$ 8 (8

Student D

Student C

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Now look at the drawing provided by **Student E.** How is this student counting the spaces? This student answered "18" for both Part 2 and Part 3. How might this student benefit from a reengagement lesson that helps him/her make sense of the counting strategy used by **Students C and D**?

Student E

Students F to H represented a variety of strategies that students used to make sense of the number of hand spans they had to identify and count, and the *total* number of hand spans they needed to account for Parts 2 and 3. Where in **Student F's** work does s/he keep track of the 2 Mom hands provided as part of the task? The 3 Jean Hands?

Student F



How might **Student G**'s labeling strategy be helpful for students who weren't keeping track of the hands already provided in the drawing? What questions could you pose to **Student H** about the decision to change Part 2 from 6 to 8? How can this decision be connected back to the drawings ~ both the one s/he made, and the one provided with the task?





In Part 1, students were asked whose hands would have measured the furthest after 6 iterations. Some students, like **Student I**, made an identifying mark where those hands would end. This information was then used in Parts 2 and 3 as a starting point, where they "counted on from there" until they got to the top. How might it be helpful to not have to recount what they have already identified? Which labeling strategies might be helpful?

Student I



Students J and K drew in a set of hands. There is little evidence that they are attending to the amount of space each hand span would measure. What evidence is there that they do, or do not, understand the idea of "matching" in measurement? Do the hands drawn match the attribute being measured? How might these students benefit from other, more streamlined marking strategies used by other students in your classroom or this Tool Kit?

Student J





Students L to O all drew their own pictures, attempting to recreate the drawing provided for this task, with varying success. **Students L, M, and N** all drew the horse. What evidence is there that each student has, or has not, accurately recreated the drawing? How are **Student L** and **Student O** similar? How are they different? How does **Student O** know that they have drawn in the correct number of hands?

Looking at the work from both **Student L** and **Student M**, what would you want **Student M** to notice about the drawing made by **Student L**?

Student L

Student M



Looking at the student work for **Students N and O**, in what ways are the drawings and notations helpful for determining how the students are making sense of the task? Which strategies were successful? What characteristics do the successful strategies share? What questions would you like to ask these students about their work? How might it be helpful for **Student M** to be able to explain why a hand might represent four in Part 2 and three in Part 3?

Student N





Students P and Q are using two different labeling and counting techniques. **Student P** draws in hands. Is the student attending to how many spaces each hand should span? Is there any evidence that the hands are a consistent size? Is there any evidence of how the student is counting the number of hands to measure the height of the horse? **Student Q** is counting either the spaces or the lines that each hand will span, and is labeling the space for each hand with a number as s/he counts. Where does **Student Q** lose track of the spans? Where do the units become inconsistent in size?

Student P







Students R to T are using a variety of labeling strategies. **Student R** marks every third line on the left of the horse, and makes alternating marks across every two spaces on the right. What does each mark mean? How did the student decide where to make the marks? How does **Student S** decide where to write the names on the chart? How is this helping him/her to keep track of counting the iterations? How do these two strategies compare to the strategy used by **Student T**?

Student R





Student T interprets the iteration as two of Mom's hands, and three of Jean's hands. In Part 2, each iteration then is counted as two hands (with a counting mistake where 8 ought to be!) and in Part 3, each iteration is counted as three hands.





2 nd Grade	Task 2	High Horse			
Student Task	Make sense of the effects of u length. Iterate two different s find a total number of each un measurement using pictures, n	init size when measuring the attribute of ized units across the same distance, and hit needed to span the distance. Justify the number sentences, models, and/or words.			
Core Idea 4	Students will recognize and use characteristics, properties, and				
Geometry	relationships of shapes and	apply appropriate techniques to			
and	determine measurements.				
Measurement	• Understand how to measure	e using non-standard units.			
	• Select an appropriate unit a (length).	nd tool for the attribute being measured			

Mathematics of the task:

- Identify the unit that will measure the furthest in the same number of iterations
- Count the total number of units needed to measure a given length
- Keep track of counting and measurements using pictures, labels, or words

• Demonstrate, in context, an understanding of unit consistency, iteration, and the concept of matching when measuring an attribute

• Identify which unit of measurement will need the most iterations to measure the same length

Based on teacher observation, this is what second graders knew and were able to do:

- Understood that Mom's hand would go further in 6 iterations than Jean's hand
- Knew that in order to measure the length, they would need to count
- Indicate what they were counting and how many they had counted using drawings, number sentences, and words
- Understood the opposite effect of size of unit to number of units required to measure a particular attribute

Areas of difficulty for second graders:

- Determining whether to count spaces or lines in the hand span
- Using consistent units
- Understanding the process of iteration, and how to count the lines when they need to overlap
- Redrawing the picture instead of using the picture given
- Drawing representations of the attribute being measured that include the relevant distance being measured
- When the units didn't match the attribute in the drawing, students did not persist in recalculating or redrawing so they would match

Strategies used by successful students:

- Include the hands given in the drawing in their total count of units
- Keep track of unit iteration using jumps, brackets, drawings, and/or labels

- Identified the span of the unit (by lines or by spaces) and then carefully counted out the units, keeping track of each iteration
- Use the given drawing or accurately recreate their own drawing, including all relevant characteristics of the attribute being measured



MARS	Task	4 Raw	Scores

Task 4 Scores	Student Count	% at or below	% at or above
0	413	6.8%	100.0%
1	1202	26.5%	93.2%
2	1555	51.9%	73.5%
3	597	61.7%	48.1%
4	453	69.1%	38.3%
5	394	75.6%	30.9%
6	272	80.0%	24.4%
7	330	85.5%	20.0%
8	888	100.0%	14.5%

There is a maximum of 8 points for this task.

The cut score for a level 3 response, meeting standards, is 4 points.

Over 38% of the students scored at or above the cut score for this task. More than 93% of the students were able to identify which unit would measure more of the attribute, given the same number of iterations, and more than half could also identify which unit would require more iterations to measure the whole length. Almost 15% of the students met all the demands of this challenging task.

High Ho	High Horse						
Points	Understandings	Misunderstandings					
0-2	More than 93% of the students were able to identify which unit would measure more of the attribute, given the same number of iterations. Almost 35% of the students scoring at this level were also able to correctly estimate that the smaller unit would require more iterations to measure the same length.	Students did not identify the relevant characteristics of either of the units of measure in the task. Students did not use any labels or markings to keep track of their counting.					
3	Students could keep track of either the lines or the spaces that make up one hand span, and they used this information to count how many iterations would be needed to measure the full length.	Students worked inconsistently at this level. Their drawings and labels may be inaccurate or difficult to read, resulting in incomplete counting on Parts 2 and 3. Nearly three-quarters of these students drew or recreated the drawing. Most drew the horse with some detail, but did not accurately recreate the scale attached to the attribute being measured.					
4	Almost two-thirds of these students accurately counted the number of iterations for Part 2 and 3, using the drawing. More than a quarter of the students could provide an accurate and complete explanation, using an accurate drawing and counting, for either Part 2 or Part 3. Students who used labeling or counting marks to keep track of their iterations, rather than drawing hand shapes, were more likely to count accurately and match the length.	Most students did not label their drawings with consistent numbers or labels to explain their thinking. Students may have used the provided drawing to do the counting, but showed no evidence of their work. Almost half the students offered either an incorrect drawing or an unrelated number fact as evidence for their accurate counting. Students may not have noticed or may not have persisted in recalculating when the drawings they made didn't match the distance being measured.					
5-6	Students used a variety of strategies, including counting by 2s or 3s, explicitly identifying whether they were counting lines or spaces, and using the drawing provided to keep track of counting. These students used marks and labels to model the hand spans.	Just over 17% of these students were using an accurate counting strategy, but then forgot to include the hands provided in the drawing in their total counts. They may have counted inaccurately, or used inconsistent units when marking the drawings.					
8	Almost 15% of the students met all the demands of this challenging task. Students identified the number of spaces or lines needed to span one unit, then accurately and completely iterated that unit across the attribute of the length. Students understood the effect of unit size on total iterations.						

Implications for Instruction

Students in second grade are learning to use both non-standard and standard units of measurement. The best way to make sense of measurement is in context. According to John Van de Walle (*Teaching Student-Centered Mathematics, K-3, 2006*), students need a variety of experiences in order to understand, among other things, how to select a unit; and how to execute a measurement using that unit.

Whether the student is working with standard or non-standard units, it is important that measuring tasks be open-ended enough that students are required to develop their own approach. The temptation ma be to carefully explain to students how to use a variety of units to measure, and then spend time actually practicing the measuring.

Van de Walle cautions that this approach will "shift students' attention to the procedure (following your instruction) and away from developing an understanding of measurement using units." (Van de Walle, 2006)

A description of Activity 8.4: How Long Is the Teacher? (p. 230) follows: Explain that you have just received an important request from the principal. She needs to know exactly how tall each teacher is. The students are to decide how to measure the teachers and write a note to the principal explaining how tall their teacher is and how they decided. Next, explain that it may be easier if you lay down and students measure how long you are instead of how tall. Do this at several stations around the room. Have students make marks at your feet and head and draw a straight line between your "head" and "foot".

> Explain that the principal says you can use any ONE of these things to measure with. (Provide several choices. For each choice of unit, supply enough units to more than cover your length. Be aware of the numbers that will result so be sure to sue units that are at least six inches long. Paper bags, straws, play dollar bills, photocopies of a large footprint, etc.)Put students in pairs and allow them to select one unit with which to measure.

Essential questions to guide the discussion after the activity:

- How did different children measure?
- Did students who measured with the same unit get the same answers? Why not? (This could be caused by 'matching' problems...students may have lined up the measuring unit with gaps or overlaps. It may also be caused by an inconsistency in units.)
- How could the principal make a line that is just as long as the teacher?

- Provide similar activities (measuring other things) or do this activity again. This time, give students only two or three of each measuring unit. How can the students use these few objects to measure a longer length? (iteration)
 - Do they just use all the units? (consistency of unit issue)
 - Do they use their fingers to indicate where to lay down the next unit? What kind of markings or labeling do they use to keep track of how many units they've iterated? (Place a cube? Make a mark? Etc.)

If you do activities such as "How Long Is the Teacher?", students will have many opportunities to make sense of measurement in context. There may be misconceptions that need to be addressed. In order to surface some of the issues around consistency and matching, have several groups of students measure the same length using the same units. If (when) they get different answers, the essential questions listed above will provide an important discussion around these big ideas.

Reengagement Lesson

Understanding how and why measurement works is an important part of each student's mathematical growth. Van de Walle's "How long is the Teacher" and other activities from Chapter 8 of *Teaching Student-Centered Mathematics*, provide important opportunities to negotiate unit and iteration.

Planning reengagement lessons, using student work from High Horse, is also important, as these lessons will help students identify and begin to use successful strategies and recording options for measurement activities. In this reengagement activity, student work is used to help mitigate mistakes students were making around how to count the iterations of Jean's hand span.



Make a poster that has "Part 3" and the task at the top. Write in this student's response:

"I looked at the lines and counted by twos because Jean's hand fits in 2 rectangles."

Students use a picture of the Horse from the task to identify where these "2 rectangles" are in Jean's hand span. Can students find the "2 rectangles" in all three of the hand spans given as part of the original picture?

Give students a chance to articulate their own understanding and ask questions. Can they explain in their own words why counting '2 rectangles' works? Can they explain why this would or wouldn't work for Mom's side of the horse?



Introduce this picture of the horse task, or recreate it on a large poster.

Call their attention to the marks on Jean's side of the horse. What do they notice? How many are there? What size are the marks? Are they all the same length, or are some longer? Are some shorter?

How is this recording connected to the statement made by the previous student in their explanation?

Use the Essential Questions to guide discussion and discovery.

- How does one student's recording strategy match the explanation of the other student?
- Did these two students get the same answers? What is the evidence that they were measuring with the same unit?
- How does the student know that each line is the right length?
- On a blank Horse paper, ask students to work together to use this recording or marking strategy to determine how many of mom's hands we'll need to measure the horse.
 - Can we use the same "two-ness" for Mom's side?
 - How can we know that our marks are the right size?
 - What does each mark represent?

As you go through student work for this task in your own classroom or school, ask yourself if this type of comparative analysis might be a way to help students compare and contrast strategies, and help them to identify connections across several strategies. This deepening understanding is what will make it possible to pick up the strategy and use it again, as their own, in different contexts.

Performance Assessment Task			
High Horse			
Grade 2			
The task challenges a student to demonstrate understanding of measurement. Students must			
measure an object using two different units and compare their results. A student must understand			

measure an object using two different units and compare their results. A student must understand the smaller the unit is in size, the larger the number of units it takes to measure an object. The student must maintain the same scale size of a unit in measuring an object. In measuring, there cannot be gaps between units and units cannot overlap.

Common Core State Standards Math - Content Standards

Measurement and Data

Represent and interpret data

2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

Common Core State Standards Math - Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
2	2009	8	4	38%