Problem of the Month: *Game Show*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month, *Game Show,* students use mathematical concepts of game theory, probability, and expected value. The mathematical topics that underlie this POM are knowledge of game strategies, reasoning, graph theory, sample spaces, fairness, probability ratios, experimental and theoretical probability, counting principles/strategies, and expected value.

In the first level of the POM, students play a simple game where the game is structured such that the same player wins every time. Students determine whether the game is fair for all players and who is favored. They explain why the outcome is always in favor of one player. In Level B, the students play and analyze a NIM type game, then determine a strategy where they can always win. They explain their solutions and why it will always work. In Level C, students are presented with a more complex NIM type game that requires strategies to change as the opponent makes his or her moves. Students develop and explain a strategy for winning the game. In Level D, the students are asked to analyze the famous Monty Hall game that involves selecting which door to pick to come up with the grand prize. Students justify their strategy and explain their chances of winning. In the final level students are asked to determine the best strategy for playing a game involving a spinner and payoff using expected value. Students must defend their strategy to maximize their winnings.

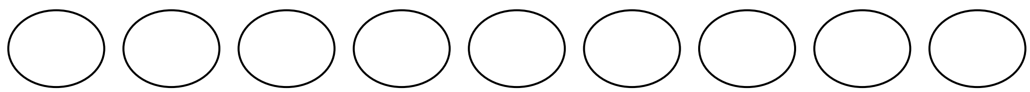
Mathematical Concepts
In the Problem of the Month *Game Show,* students use mathematical concepts of game theory, probability, and expected value. The mathematical topics that underlie this POM are knowledge of game strategies, reasoning, graph theory, sample spaces, fairness, probability ratios, experimental and theoretical probability, counting principles/strategies, and expected value.

Problem of the Month

Game Show

Level A

On television there is a new Game Show called *Take Two*. The game is played with two players. There are nine coins lined up in a row.



Each player takes turns. On a turn a player must take two coins away.   So the game starts with the first player taking away two coins. Then it is the other player's turn. The second player takes two coins away. The game continues with alternate turns. The player who can't pick up exactly two coins loses. If there is only one coin left when it is a player's turn, then the player whose turn it is loses. The other player wins.

Which player do you want to be - the first player or the second player?

Explain your choice?

Is this a fair game? Why or why not?

Level B

Many viewers complained about the game show called *Take Two*, so the producers changed the show's name and rules. The new show is called *Pick Up Coins*.

They also changed the number of coins from 9 to 22 and changed the rules so on a turn a player could choose to pick up either one coin or two coins.

So the new rules are: Start with 22 coins. The players take turns. On a turn a player must take either one or two coins away. Then it is the other player's turn. The second player takes either one or two coins away. The game continues with alternating turns. The player who picks up the last coin loses. The other player wins.

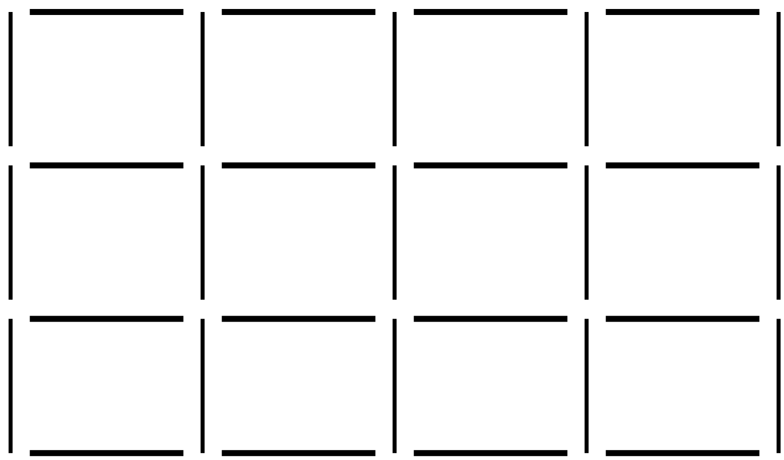
Determine a strategy for playing this game.

Explain a strategy that may ensure you win the game.

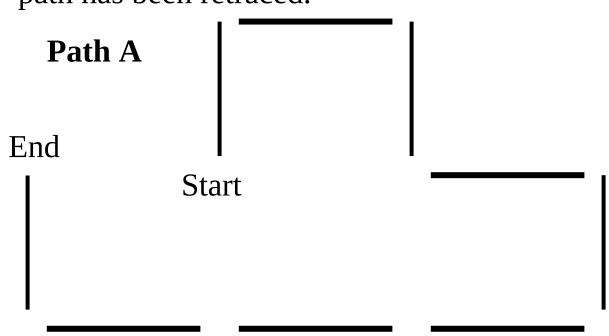
Is this a fair game? Explain your reasoning.

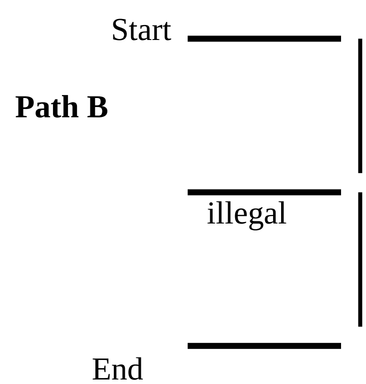
Level C

The game show *Pick Up Coins* is getting top ratings. So the rival network has begun a new show called *Pick Up Sticks*. The game starts with an arrangement of 31 sticks in a 3 by 4 rectangular-shaped grid as shown below:



The game is played by two players who alternate taking turns. A turn consists of picking up sticks along a path. During any one turn, a player may start wherever she or he likes, picking up as many sticks as he or she wishes as long as the sticks follow a path. A path is a sequence of adjacent sticks, starting at one end of the stick, leading to a stick at the opposite end and so on. For example, Path A is legal and Path B is illegal because the path has been retraced.



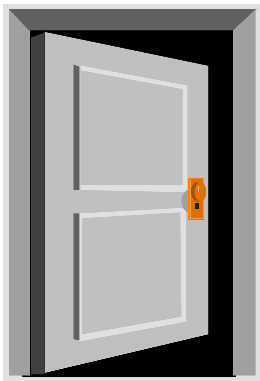


The winner is the player who picks up the **last stick!**

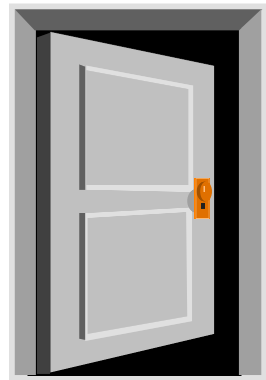
Determine a strategy for playing this game. Should you start first or second? Explain a method for ensuring you win.

Level D

It's time to play *Let's Make It Real* hosted by Mighty Fall.


Door 1


Door 2


Door 3

As you know there are three doors. Behind one door is a fabulous prize. Behind the other two doors are bogus prizes. When you pick the first door, another door will open and show you a bogus prize, leaving the door you picked and the third door closed. You once again pick one of the two closed doors. You will either get the fabulous prize or the bogus prize.

Determine the best strategy for winning the fabulous prize.

Explain using mathematics why your strategy is the best.

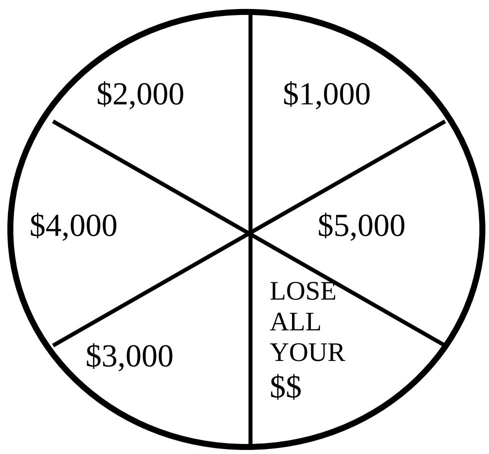
Level E

“Hi, I’m Pat Saywhat and this is Banana. Welcome to

WHEEL OF TORTURE!

“Today we have several contestants who will try their luck and risk their earnings to become today’s Big Wheel winner (crowd roars).

“The rules of the game are quite simple. We have several contestants backstage in soundproof booths. One by one we will invite a contestant to come out and try her/his luck at the Big Wheel. As you can see our Big Wheel has six equal sectors (pie-shaped regions).   In each of the sectors is a dollar amount, except for one that is labeled, LOSE ALL YOUR $$.



“A contestant may spin the wheel as many times as s/he would like until either s/he decides to stop and keep the money, or the big wheel lands on LOSE ALL YOUR $$. If that happens the contestant loses the game, but we have some very nice consolation prizes. The outcome of each player’s turn is unknown to the other contestants, so each player must use the best strategy, since s/he is not aware of what has transpired prior to her/his turn. After the last contestant has completed her/his turn at the Big Wheel, the player with the most money wins the game and keeps all the money (crowd roars). Banana, I sure hope these contestants have come up with a sound mathematical strategy!”

Use your knowledge of mathematics to determine the optimum strategy for playing the game.   Describe your strategy. Please instruct a player of the steps to follow in playing the game and at what point to stop and keep the cash. Explain the reasoning behind your game strategy. What mathematics did you use to arrive at your game strategy? Why do you believe that it is the best possible strategy?

Problem of the Month

Game Show

Primary Version Level A

Materials: The nine chips or tokens for each group

Discussion on the rug: Teacher starts a discussion about fair games. **"What makes a game fair? Tell us about a game that is unfair."** Teacher demonstrates how to play the game with two players. **"Each player takes two coins on each turn. If you can't pick up two coins, then the game is over. The last player to pick up two coins is the winner."**

In small groups: Each group plays the game several times. Have the pairs switch who starts first.

After the games, the teacher polls the class and records the outcomes for the entire class.
"Which player has the best chance of winning?"
"Does the same player always win?"
Explain why you think it happens."

Teacher asks the following questions:
"Is this a fair game?"
"Why is this game fair or not fair?"
"How could you change this game to make it better?"

At the end of the investigation have students either discuss or dictate a response to the summary questions above.

Problem of the Month
Game Show
Task Description – Level A
This task challenges students to determine whether or not a game is fair for all players and who is favored; the game has been structured so that the same player wins each time. Students must explain why the outcome is always in favor of one player.
Common Core State Standards Math - Content Standards
<u>Measurement and Data</u> **Represent and interpret data.** 1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Game Show

Task Description – Level B

This task challenges students to play and analyze a game similar to NIM. Students are then asked to determine a strategy where they can always win. Students must explain their solution and why it will always work.

Common Core State Standards Math - Content Standards

Measurement and Data
Represent and interpret data.
1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Operations and Algebraic Thinking
Work with equal groups of objects to gain foundations for multiplication.
2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month
Game Show
Task Description – Level C
This task challenges students to develop and explain a strategy for winning a more complex type of NIM game that requires strategies to change as the opponents make their moves. Students must develop and explain a strategy for winning the game.
Common Core State Standards Math - Content Standards
<u>Measurement and Data</u> **Represent and interpret data.** 1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **<u>High School – Statistics and Probability - Making Inferences and Justifying Conclusions</u>** **Understand and evaluate random processes underlying statistical experiments** S-IC.2. Decide if a specified model is consistent with results from a given data-generating process
Common Core State Standards Math – Standards of Mathematical Practice
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problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP. 7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(\mathbf{x} – \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x–1)(x+1),(x–1)(x2+x+1),and(x–1)(x3 +x2+x+1)might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month:
Game Show
Task Description – Level D
This task challenges students to analyze the famous Monty Hall game where one selects a door to get to the grand prize. Students are asked to justify their strategy and explain their chances of winning.
Common Core State Standards Math - Content Standards
<u>Statistics and Probability</u> **Investigate chance processes and develop, use, and evaluate probability models.** 7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. 7.SP.8.c Design and use a simulation to generate frequencies for compound events
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month
Game Show
Task Description – Level E
This task challenges students to determine the best strategy for playing a game involving a spinner and payoff using expected value. Students must defend their strategy to maximize their winnings.
Common Core State Standards Math - Content Standards
<u>High School - Probability and Statistics - Using Probability to Make Decisions</u> **Calculate expected values and use them to solve problems.** S-MD.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S-MD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. S-MD.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. S-MD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?
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Problem of the Month

Game Show

Task Description – Primary Level

This task challenges students to explore the idea of the fairness of a game. The teacher leads a class discussion about their ideas of what constitutes an unfair game. The teacher instructs the students on a game using nine chips or tokens, demonstrates the game, and then has small groups of students play the game together. After the small groups play the game, the teacher records the outcomes or data for the entire class. Discussion questions are asked of the students such as: Which player has the best chance of winning? Does the same player always win? Is this a fair game? Why is this game fair or not fair?

Common Core State Standards Math - Content Standards

Measurement and Data

Represent and interpret data.

1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Common Core State Standards Math – Standards of Mathematical Practice

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