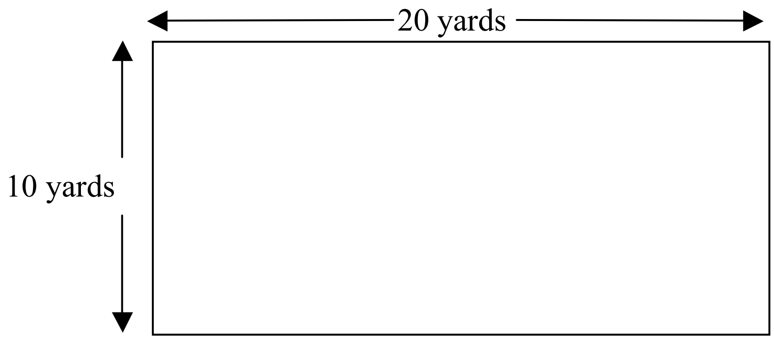
Fair Play

This problem gives you the chance to:
• find areas and perimeters of rectangles

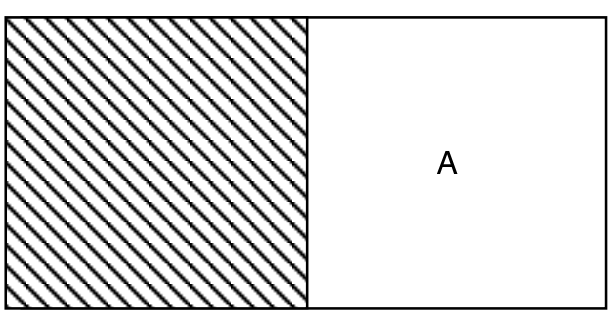
The Grade 4 students have a play area.
These are its measurements.

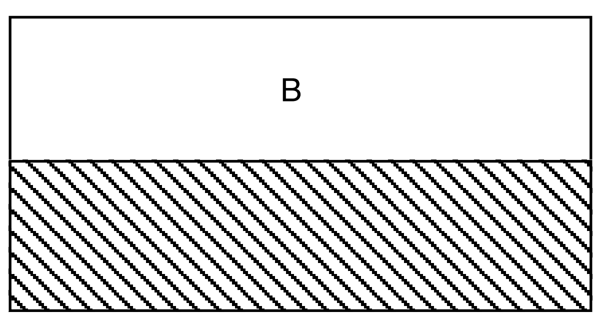


1. What is the area of the play area? ________________ square yards
   Show how you figured this out

2. The students would like a fence to be put around the area to stop balls going too far.
   What will the total length of the fence be?
   Show how you figured this out. ____________ yards

3. The girls say that the boys take up too much space with their ball games.
   They want the area to be split into two equal parts.
   Here are two possible ways of dividing the area.



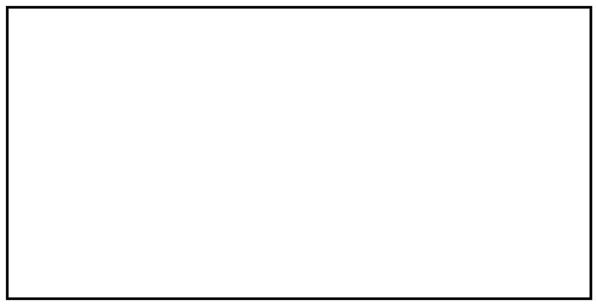


What are the perimeters of these areas?

A = ______________yards

B = ______________yards

4. Draw a straight line that divides the play area into two equal parts in a **different** way.



7

Fair Play		Rubric
The core elements of performance required by this task are: • to find areas and perimeters of rectangles. Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **200** square yards Shows work such as: 20 x 10 =200	1 1	2
2. Gives correct answer: **60** yards Shows work such as: 10 +20 + 20 +10 = 60	1 1	2
3. Gives correct answer: A = **40** yards Gives correct answer: B = **50** yards	1 1	2
4. Makes an area such as: a right triangle.	1	1
Total Points		**7**

Fair Play

Work the task and look at the rubric. What does a student need to understand about area and perimeter to be successful on this task? What are the big mathematical ideas?

Look at student work for part 1, finding the area. How many of your students put:

200	60	20	30	Other

What were students doing who got an answer of 60? 20? 30? How are these misconceptions different? What follow up activities or discussions might you plan to help each of these types of errors?

Look at student work on part 2, finding the perimeter. How many of your students put:

60	20	30	70	40	10	Other

What might students have been thinking who got answers of 20, 10 or 30? What might have confused them about perimeter?

Now look at student work for part 3, finding the perimeter of half the playground. How many of your students put:

40,50	60,60	30,30	40,30	100,100	10,20	Other

How might students have found some of these other answers? What were they looking at?
How do you help students become diagram literate? What opportunities do students in your class have to read and interpret diagrams? Did you see evidence of students marking dimensions on the diagrams to help them solve in part 3?

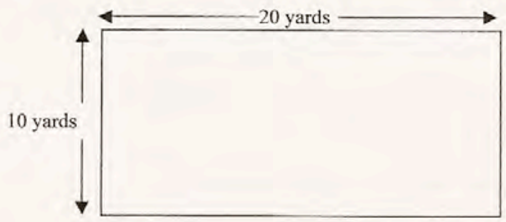
Now look at student work on part 4, dividing the playground in half a different way.
How many of your students could make a correct drawing?
How many of your students made figure A or B?
How many of your students did not draw a straight line to make the division?
What other types of errors did you see?

Looking at Student Work on Fair Play

Student A is able to calculate area and perimeter easily. Notice that the student marks measurements on the diagram as a tool.

Student A

The Grade 4 students have a play area.
These are its measurements.



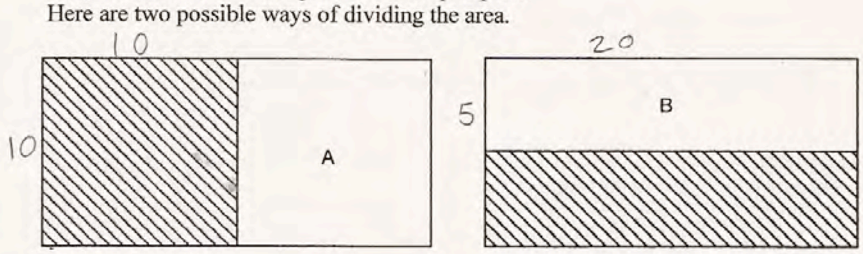
1. What is the area of the play area? 200 square yards
   Show how you figured this out

   10×20=200

2. The students would like a fence to be put around the area to stop balls going too far.
   What will the total length of the fence be? 60 yards
   Show how you figured this out.

   20+20=40
   10+10=+20
   60

3. The girls say that the boys take up too much space with their ball games.
   They want the area to be split into two equal parts.
   Here are two possible ways of dividing the area.

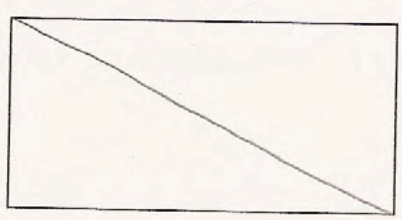


What are the perimeters of these areas?

A = 40 yards

B = 50 yards

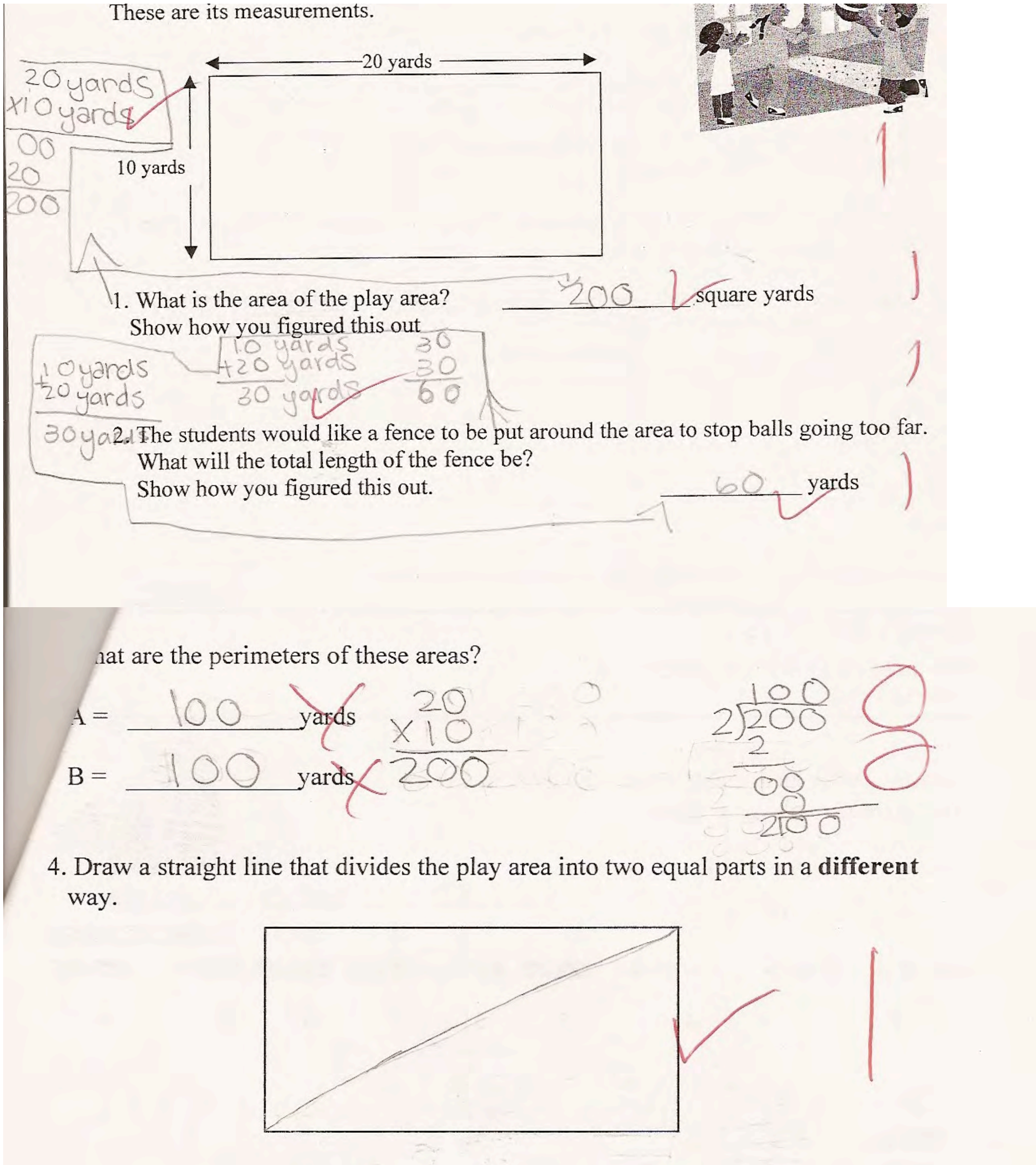
4. Draw a straight line that divides the play area into two equal parts in a **different** way.



?

Student B is also able to make the calculations for area and perimeter in part 1 and 2. The use of labels makes the reasoning chain easy to follow. However in part 3, finding the perimeter of half the area, the student finds half the area instead of the newly formed perimeter.

Student B


These are its measurements.

20 yards

10 yards

20 yards
x10 yards
00
20
200

1. What is the area of the play area? 200 square yards
Show how you figured this out

10 yards
+20 yards
30 yards

10 yards
+20 yards
30 yards

30
30
60

2. The students would like a fence to be put around the area to stop balls going too far.
What will the total length of the fence be?
Show how you figured this out. 60 yards

...hat are the perimeters of these areas?

A = 100 yards

B = 100 yards

20
x10
200

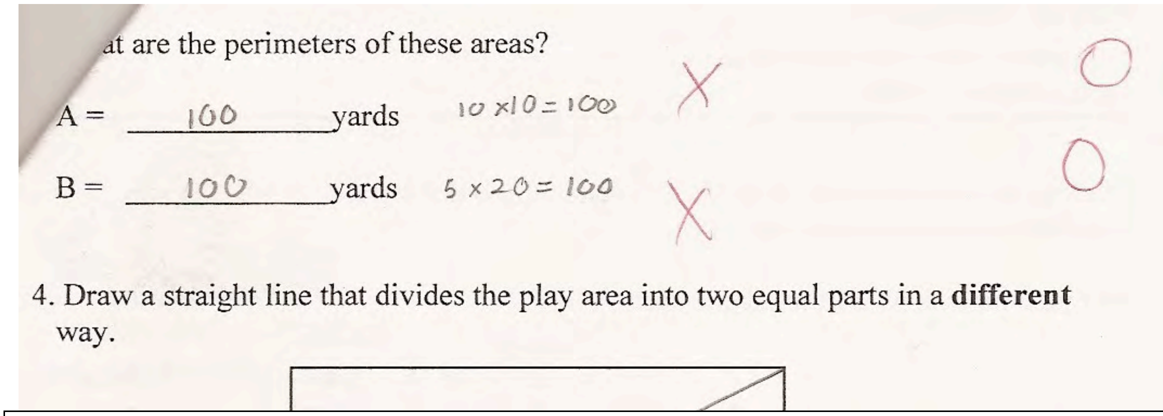
100
2)200
2
00
0
2100

4. Draw a straight line that divides the play area into two equal parts in a **different** way.

Grade Four

Student C is also able to calculate area and perimeter in part 1 and 2.Student C uses the new dimensions of the shape, but also calculates half the area. *Why do you think it is difficult for students to find the perimeter in the new shape?*

Student C



...at are the perimeters of these areas?

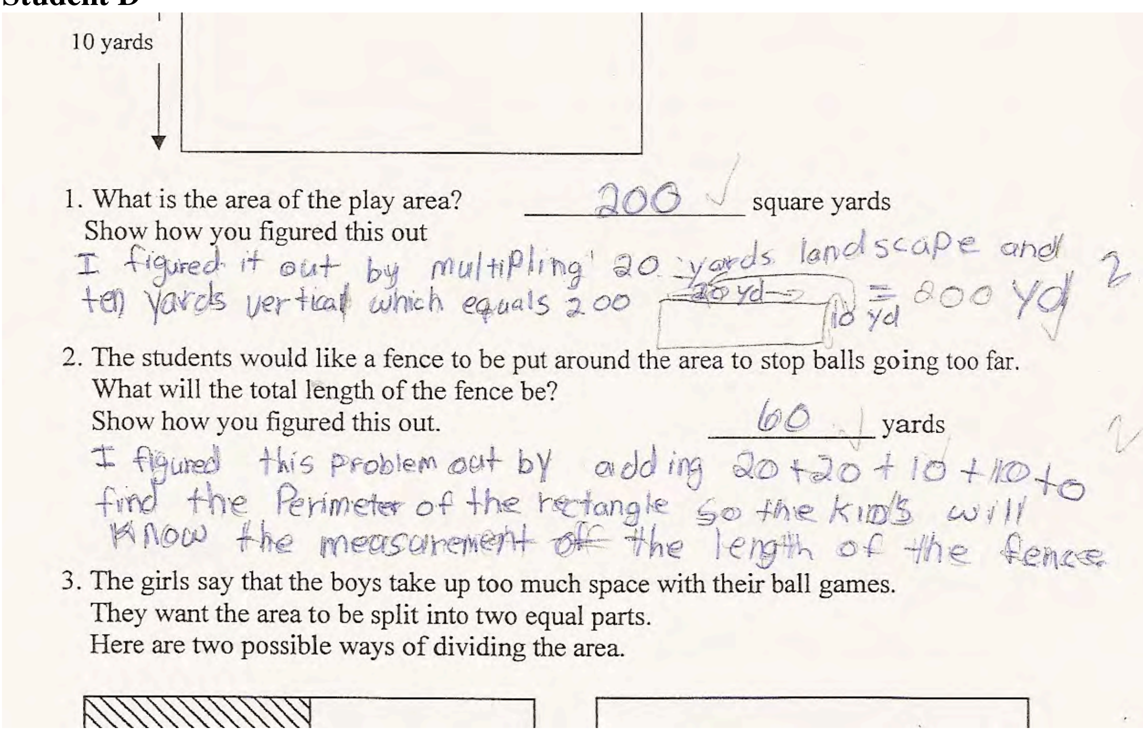
A = 100 yards 10 x 10 = 100 X

B = 100 yards 5 x 20 = 100 X

4. Draw a straight line that divides the play area into two equal parts in a **different** way.

Student D is also very clear about how to find area and perimeter, giving lengthy explanations. In part 3 the student makes the assumption the if the area is halved the perimeter is halved. *Why doesn't this work? What experiences does this student need?*

Student D



10 yards

1. What is the area of the play area? 200 square yards
Show how you figured this out
I figured it out by multipling 20 yards landscape and ten yards vertical which equals 200 20 yd 10 yd = 200 yd

2. The students would like a fence to be put around the area to stop balls going too far.
What will the total length of the fence be?
Show how you figured this out. 60 yards
I figured this problem out by adding 20 + 20 + 10 + 10 to find the Perimeter of the rectangle so the kids will know the measurement ~~of~~ the length of the fence.

3. The girls say that the boys take up too much space with their ball games.
They want the area to be split into two equal parts.
Here are two possible ways of dividing the area.

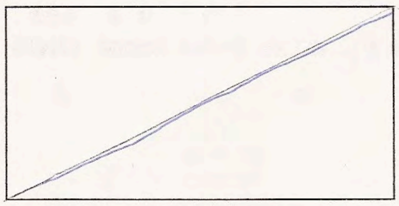
Student D, continued

What are the perimeters of these areas?

A = 30 yards

B = 30 yards

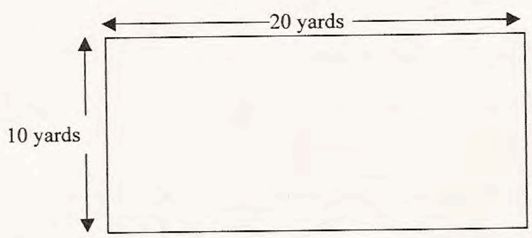
4. Draw a straight line that divides the play area into two equal parts in a **different** way.



Student E is able to find the area and the perimeter. In part 3 the student comes up with unusual numbers to calculate the new perimeter.

Student E

These are its measurements.



1. What is the area of the play area? 200 square yards
   Show how you figured this out

10 × 20 = 200

20 + 20 + 20 + 20 + 20 + 20 + 20 + 20 + 20 + 20 = 200

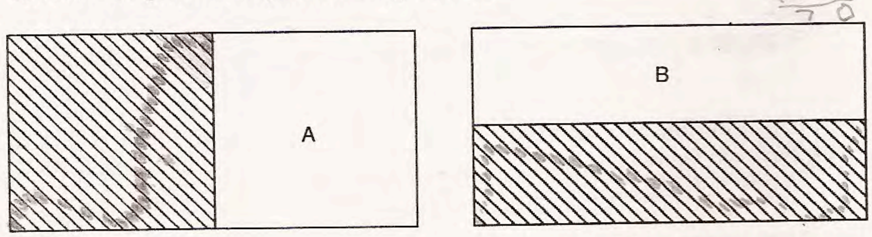
2. The students would like a fence to be put around the area to stop balls going too far.
   What will the total length of the fence be? 60 yards
   Show how you figured this out.

20 + 20 = 40
10 + 10 = 20
40 + 20 = 60

3. The girls say that the boys take up too much space with their ball games.
   They want the area to be split into two equal parts.
   Here are two possible ways of dividing the area.

35 + 35 = 70

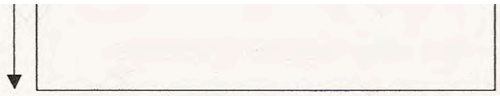
28 + 28 = 56



Grade Four

Student F calculates the area in part 1. In part 2 the student confuses perimeter with one of the dimensions. Then in part 3 the student seems to have calculated the real perimeter and then the perimeter of B. *What question would you pose for this student?*

Student F



1. What is the area of the play area? 200 ✓ square yards 1
   Show how you figured this out 1

   20
   x 10
   200 ✓

2. The students would like a fence to be put around the area to stop balls going too far.
   What will the total length of the fence be?
   Show how you figured this out. 10 X yards 0

0

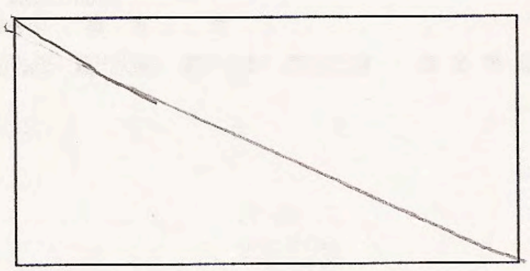
3. The girls say that the boys take up too much space with their ball games.
   They want the area to be split into two equal parts.

What are the perimeters of these areas?

A = 60 X yards

B = 50 ✓ yards

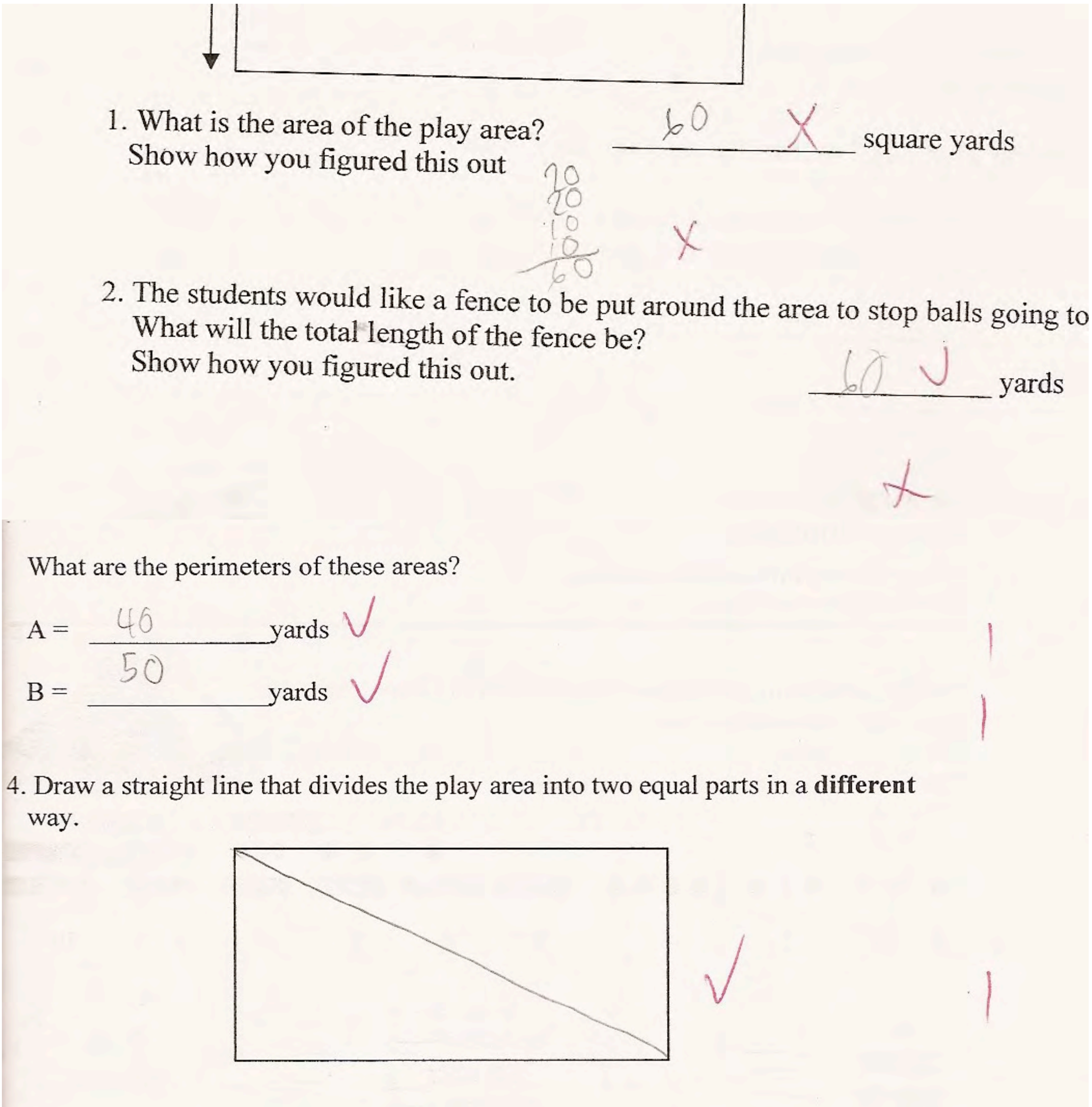
4. Draw a straight line that divides the play area into two equal parts in a **different** way.



✓

Student G only knows how to calculate perimeter. *What might be your next steps with this student?*

Student G



1. What is the area of the play area? 60 square yards
Show how you figured this out

20
20
10
10
60

2. The students would like a fence to be put around the area to stop balls going to
What will the total length of the fence be?
Show how you figured this out. 60 yards

What are the perimeters of these areas?

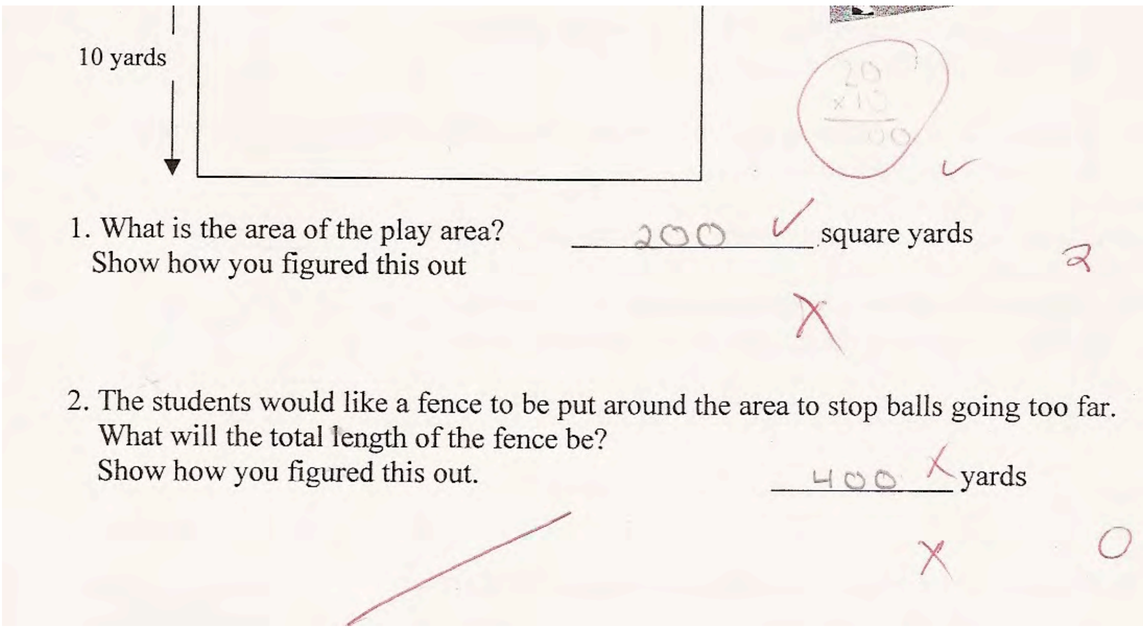
A = 40 yards

B = 50 yards

4. Draw a straight line that divides the play area into two equal parts in a **different** way.

Student H calculates the area. Then it is unclear what the logic is for the calculation in part 2. In part 3 the student shows an understanding of procedure for finding perimeter, but doesn't understand how to interpret the new diagrams for the playground and uses the wrong dimensions. In part 4 the student changes the dimensions to try to get half the area instead of drawing a line. Notice that the figure is no longer a rectangle. *How do we help students develop diagram literacy? How do we help students understand that just because a drawing looks like a rectangle does not mean that it is a rectangle? What does it mean "not drawn to scale"? Do students ever get opportunities to try to investigate these ideas?*

Student H



10 yards

20
×10
200

1. What is the area of the play area? 200 square yards
   Show how you figured this out

2

2. The students would like a fence to be put around the area to stop balls going too far.
   What will the total length of the fence be?
   Show how you figured this out. 400 yards

0

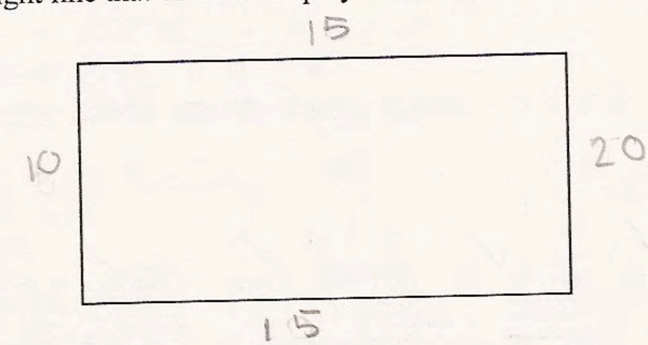
Student H, continued

…t are the perimeters of these areas?

A = 60 yards × 0

B = 60 yards × 0

4. Draw a straight line that divides the play area into two equal parts in a **different** way.



× 0

1. 20
×10
200

2. 200
+200
400

3. A
20
20
10
+10
60

B
20
20
10
+10
60

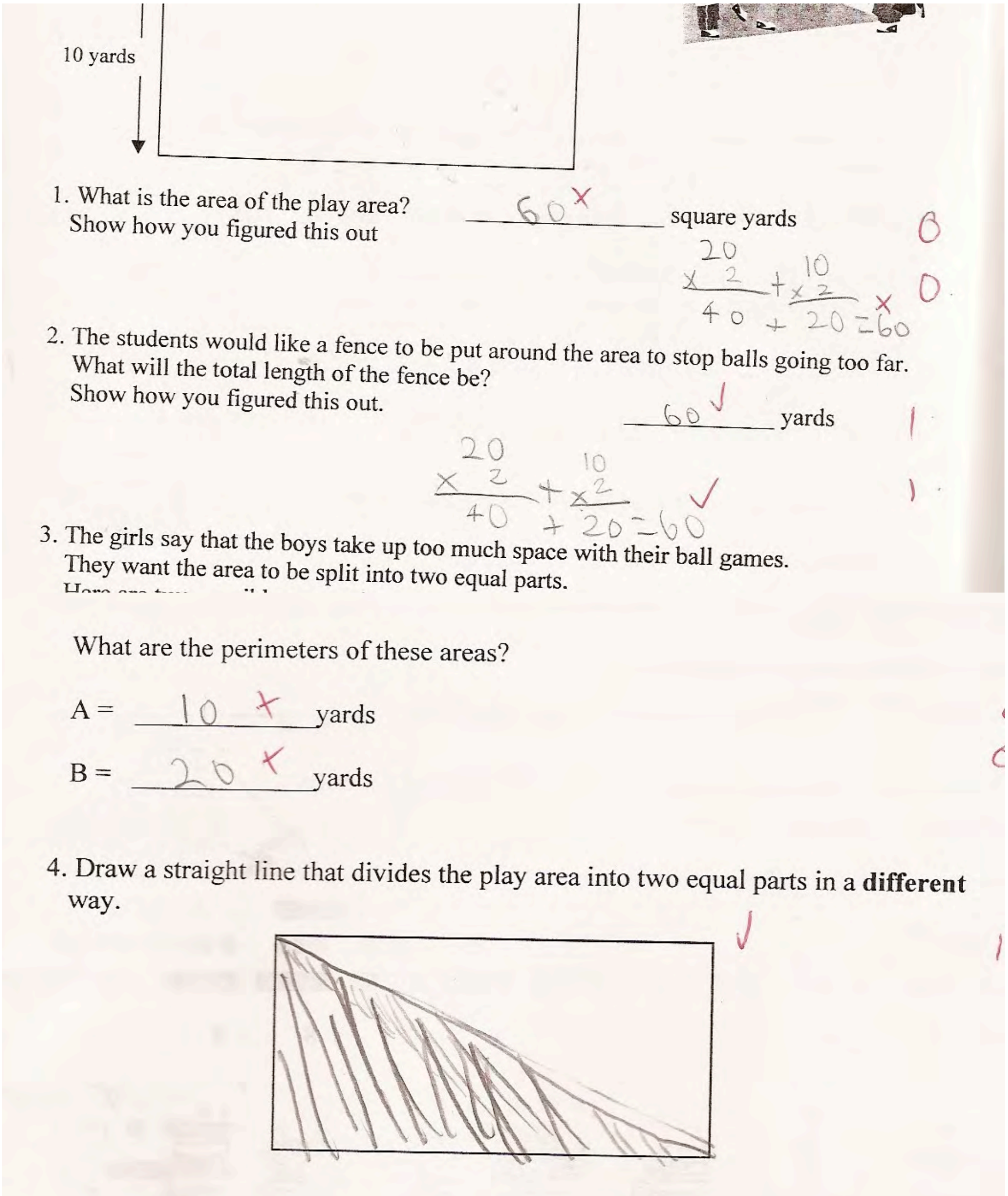
4. 15
+15
30
10
+20
60

2/2

Grade Four

Student I does not distinguish a difference between area and perimeter in part 1 and 2. In part 3 the student appears to give the width of the new play areas rather than the new perimeter.

Student I


10 yards

1. What is the area of the play area? 60 square yards 0
Show how you figured this out
20 × 2 = 40, 10 × 2 = 20, 40 + 20 = 60 0.

2. The students would like a fence to be put around the area to stop balls going too far.
What will the total length of the fence be?
Show how you figured this out. 60 yards 1
20 × 2 = 40, 10 × 2 = 20, 40 + 20 = 60 1.

3. The girls say that the boys take up too much space with their ball games.
They want the area to be split into two equal parts.

What are the perimeters of these areas?

A = 10 yards

B = 20 yards

4. Draw a straight line that divides the play area into two equal parts in a **different** way.
1

Student J draws an unusual playground for dividing the area in half.

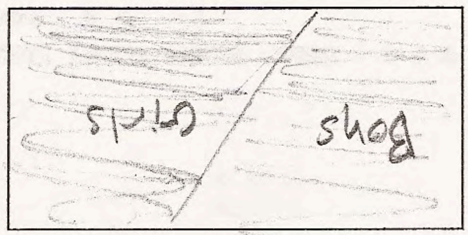
Student J

What are the perimeters of these areas?

A = 30 yards x 0

B = 30 yards x 0

4. Draw a straight line that divides the play area into two equal parts in a **different** way.



1

Grade Four

4th Grade Task 2 Fair Play

Student Task	To find the area and perimeter of a rectangle. Reason about how dividing the area in half effects perimeter. Find new ways of dividing the shape in half by understanding symmetry.
Core Idea 4 Geometry and Measurement	**Use characteristics, properties, and relationships of two-dimensional geometric shapes and apply appropriate techniques to determine measurements Examine, compare, and analyze attributes of geometric figures.**

Mathematics of the task:

- Finding area and perimeter of a rectangle using given or derived measurements
- Reasoning about how dividing a shape changes the dimensions
- Reading and interpreting a diagram
- Decomposing a shape into halves

Based on teacher observation, this is what fourth graders know and are able to do:

- Divide the rectangle into two equal parts
- Find the perimeter of the original rectangle

Areas of difficulty for fourth graders:

- Distinguishing between area and perimeter
- Finding area
- Finding the measurements of the new play area (using the diagram)
- Confusing dimensions and perimeter
- While students might know perimeter for one part of the problem, their thinking was fragile and was not consistent throughout the task

Strategies used by successful students:

- Defining the term (area or perimeter) before calculating
- Marking dimensions on the new diagrams

MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 4

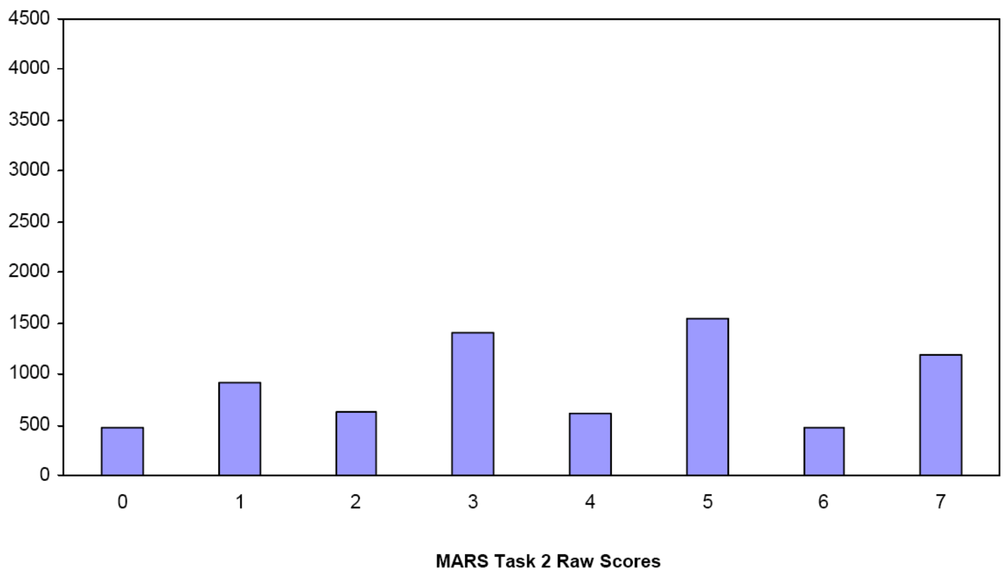
Task 2 - Fair Play

Mean: 3.82 StdDev: 2.16

Table 21: Frequency Distribution of MARS Test Task 2, Grade 4

Task 2 Scores	Student Count	% at or below	% at or above
0	478	6.6%	100.0%
1	920	19.2%	93.4%
2	636	28.0%	80.8%
3	1414	47.5%	72.0%
4	616	55.9%	52.5%
5	1538	77.1%	44.1%
6	476	83.7%	22.9%
7	1186	100.0%	16.3%

Figure 30: Bar Graph of MARS Test Task 2 Raw Scores, Grade 4



The maximum score available for this task is 7 points.
The minimum score needed for a level 3 response, meeting standards, is 4 points.

Most students, 94%, could divide a rectangle in half. Many students, 72%, could also find the perimeter of the original rectangle and show how they figured it out. More than half the students,52%, could show their work for finding area, but may have struggled with the calculation. 16% of the students could meet all the demands of the task, including finding area and perimeter of the original shape, perimeters for the new shapes, and divide a rectangle into halves. Almost 7% of the students scored no points on this task. 89% of the students with this score attempted the task.

Fair Play

Points	Understandings	Misunderstandings
0	89% of the students with this score attempted the task.	9% of the students repeated one of the rectangles from page 1 of the task when attempted to divide the play area in half.
1	Students could divide the area of a rectangle into halves.	Students struggled with perimeter. 8% of the students had the correct answer in part 2, but showed no work. 11% thought the perimeter was 20 (confusing dimension with perimeter). 5% thought the perimeter was 30 (only adding the dimensions shown in the diagram).
3	Students could calculate perimeter for the original rectangle and divide a rectangle into two equal parts.	Students struggled with finding the area. 29% of the students confused area with perimeter(60). 6% thought the area was 30. 5% thought the area was 20.
4	Students could calculate perimeter for the original rectangle and divide a rectangle into two equal parts. They could also either find the area, but not show work or show the procedure for area but made mistakes in the arithmetic.	Students struggled with finding the perimeter of the new play areas. Students confused dimensions with perimeter (10, 20). Some students only added the new sides (10 or 40). 14% thought that if the area was halved, the perimeter would be halved (30,30). Some did not see the significance of the details in the new play area (60,60).
7	Students could read and interpret a diagram to derive dimensions for the new play area and calculate the new and original perimeters. Students could also calculate area and divide the area in half visually.	

Implications for Instruction

Students need to use a diagram to reason about the size of all of its sides. Students should recognize that opposite sides of a rectangle are equal. Students should be able to reason about the new dimensions after a line is drawn to cut the shape in half. Diagram literacy is an important skill and students need frequent opportunities to use diagrams to solve problems, including being able to write down dimensions.
Students should also understand how to visually divide the shape into equal areas or equal-size parts. These parts do not need to be symmetrical or congruent.
Students should be familiar with using formulas to find area and perimeter of simple shapes. Students need opportunities to work with more complex shapes and subdivide them into familiar parts for finding areas and perimeter.

Ideas for Action Research – Re-engagement Lesson

Re-engagement – Confronting misconceptions, providing feedback on thinking, going deeper into the mathematics. *(See overview at beginning of toolkit).*

1. Start with a simple problem to bring all the students along. This allows students to clarify and articulate the mathematical ideas.
2. Make sense of another person's strategy. Try on a strategy. Compare strategies.
3. Have students analyze misconceptions and discuss why they don't make sense. In the process students can let go of misconceptions and clarify their thinking about the big ideas.
4. Find out how a strategy could be modified to get the right answer. Find the seeds of mathematical thinking in student work.

Students in this lesson had difficulty using diagrams as a tool. To start a re-engagement lesson it is often important to have students articulate fundamental ideas before moving to the more demanding parts of the task.

For this task, wanting the emphasis to be on diagram literacy, I might start with the original rectangle and ask students what the dimensions are for the other sides. I would give them a chance for individual think time and then pair/share, before starting the discussion. The purpose is to maximize the amount of conversation and give all students the opportunity to verbalize their ideas. Next I might ask some questions to get students to discuss the difference between area and perimeter. (For examples see the introduction to the lesson on Pizza Crust, www.InsideMathematics.org)

After the introduction section is complete ask students to help solve part 1 area, and part 2 perimeter for the original figure. Depending on the level of the class, you might want to give them on misconception on perimeter by asking, "*When I looked at student papers I saw two different answers for perimeter. Do you think this is a problem that could have 2 solutions? Francis said the perimeter was 10 + 20. What is Francis thinking?"*
After students have an opportunity to discuss Francis strategy, I might say, "*Phil said the answer is 10 + 20 + 10 + 20 = , but Art said the answer is 30 x 2 = … What are they thinking? Where do the numbers come from? Who do you think is correct?"*

I might introduce the next section of the task by posing the question: *"Lisa says that part 3 can't be solved because there are no dimensions on the diagram. Can you help Lisa solve the problem?"* Again to maximize participation and learning, I would allow first individual think time, then pair/share, before opening up the question to class discussion. *How do you think students might respond?*

Now look through your student work or work in the toolkits with colleagues. *How might you finish the lesson? Are there any snippets of student work that you could use to pose a question? What are the big mathematical ideas that you want students to make sense of? What kind of question can you ask at end of the lesson to help students explain what they learned?*

Performance Assessment Task
Fair Play Grade 4
The task challenges a student to demonstrate understanding of the concept of measurement. A student must understand area and perimeter of rectangles and the difference between the two measurements. A student must make sense that when area remains the same, the perimeter can change. A student must be able to determine a line of symmetry that divides a rectangle into two equal areas.

Common Core State Standards Math - Content Standards

<u>Measurement and Data</u>
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

<u>Geometry</u>
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Common Core State Standards Math – Standards of Mathematical Practice

MP.6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP.7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
4	2009	7	4	54 %