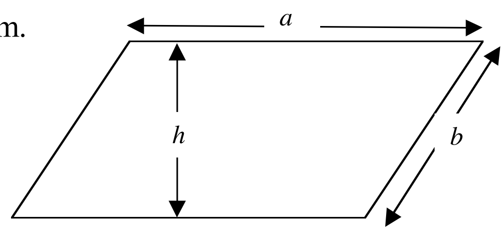
Expressions

This problem gives you the chance to:
- work with algebraic expressions for areas and perimeters of parallelograms and trapezoids

1. Here is a parallelogram.



The area of a parallelogram is the product of its base times the perpendicular height.

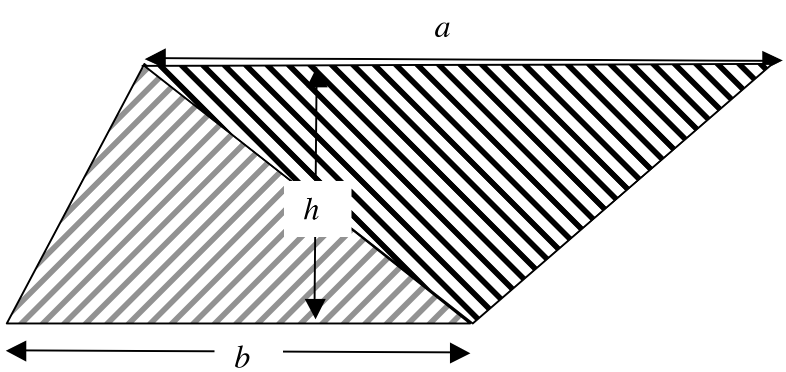
a. Which of these are correct expressions for the area of this parallelogram?
Draw a circle around any that are correct.

ab $\quad \frac{1}{2}ab$ $\quad ah$ $\quad \frac{1}{2}ah$ $\quad 2a+2b$ $\quad 2(a+b)$ $\quad abh$

b. Which of these are correct expressions for the perimeter of the parallelogram?
Draw a circle around any that are correct.

ab $\quad \frac{1}{2}ab$ $\quad ah$ $\quad \frac{1}{2}ah$ $\quad 2a+2b$ $\quad 2(a+b)$ $\quad abh$

2. Here is a trapezoid. It is made up of two triangles, each with height h.



Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$.

6

Expressions — Rubric

		points	section points
	The core elements of performance required by this task are: • work with algebraic expressions for areas and perimeters of parallelograms and trapezoids Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1.a	Gives correct answer: ***ah*** circled and no others circled	1	
b.	Gives correct answers: ***2a + 2b*** and ***2(a + b)*** Deduct 1 point for 1 extra and 2 points for more than 1 extra.	2 x 1	3
2.	Provides a convincing development of the required expression such as: Shows the areas of the two triangles are $\frac{1}{2}ah$ and $\frac{1}{2}bh$ Adds these two expressions to get $\frac{1}{2}(a+b)h$	2 x 1 1	3
	Total Points		**6**

Expressions

Work the task and look at the rubric. What are the algebra tools a student needs to know to do this task? What would you like to see if for complete explanation or proof in part 3?

____

____

____

____

Look at student work for part 1a, finding the area of a parallelogram. Remember the formula is given in words. Now look at student work. How many of your students put:

ah	Omitted ah	ab	abh	1/2ab	1/2ah	2a +2b	2(a+b)

What surprises you about these results? How often do students get to work with algebra in the context of geometry? How might this context help students to see algebra as a sense-making tool?

Now look at student work for part 1b, finding the perimeter of a parallelogram. How many of your students put:

Both formulas	Omit 2a+2b	Omit 2(a+b)	ab	1/2ab	ah	1/2ah	abh

Aside from the geometry, how many students didn't see the equivalency between 2a+2b and 2(a+b)? Why do you think this was difficult for students? Why do you think students had difficulty expressing perimeter in a geometric setting?

Now look at work for part 2. How many students could complete the entire argument? In their work, how might you encourage them to improve their answers, make them clearer and highlight the mathematics and logic of each step?

Now look at types of errors. How many students:

- Made no attempt on this part of the task____________________
- Put either 1/2 a or 1/2 b as the area of a small triangle__________
- Tried to use numbers instead of variables to solve the problem__________
- Found the area of the small triangles but didn't combine them to try and complete the argument or proof________________
- Other errors ___________________

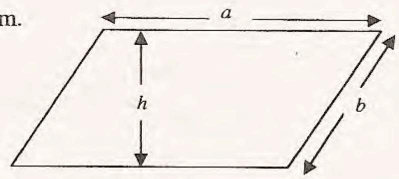
How often do students in your class get the opportunity to make and test conjectures about geometric or other contexts using algebra? What are some of your favorite problems?

Looking at Student Work on Expressions

Student A is able to identify the expressions for finding area and perimeter of a parallelogram. The student is able to see that 2a+2b is equivalent to 2(a+b). In part 2, Student A labels the two triangles in order to define which areas are being found by each area expression. Then the student uses words and symbols to discuss combining the two separate areas into a single expression.

Student A

1. Here is a parallelogram.



The area of a parallelogram is the product of its base times the perpendicular height.

a. Which of these are correct expressions for the area of this parallelogram?
Draw a circle around any that are correct.

ab $\quad \frac{1}{2}ab$ $\quad ah$ $\quad \frac{1}{2}ah$ $\quad 2a+2b$ $\quad 2(a+b)$ $\quad abh$

1

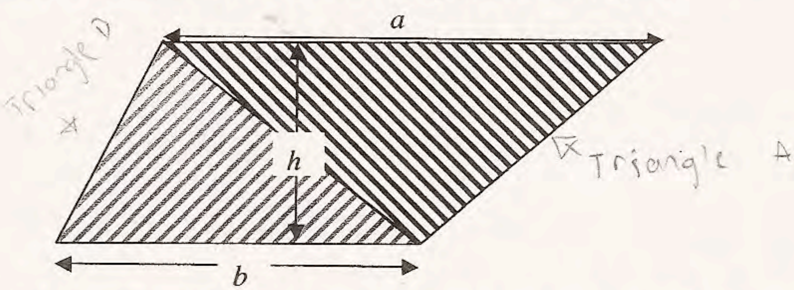
b. Which of these are correct expressions for the perimeter of the parallelogram?
Draw a circle around any that are correct.

ab $\quad \frac{1}{2}ab$ $\quad ah$ $\quad \frac{1}{2}ah$ $\quad 2a+2b$ $\quad 2(a+b)$ $\quad abh$

1

1

2. Here is a trapezoid. It is made up of two triangles, each with height h.



Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$.

$\frac{1}{2}(b)h + \frac{1}{2}(a)h =$

2

1

$B = \frac{1}{2}(b)h$

$A = \frac{1}{2}(a)h$

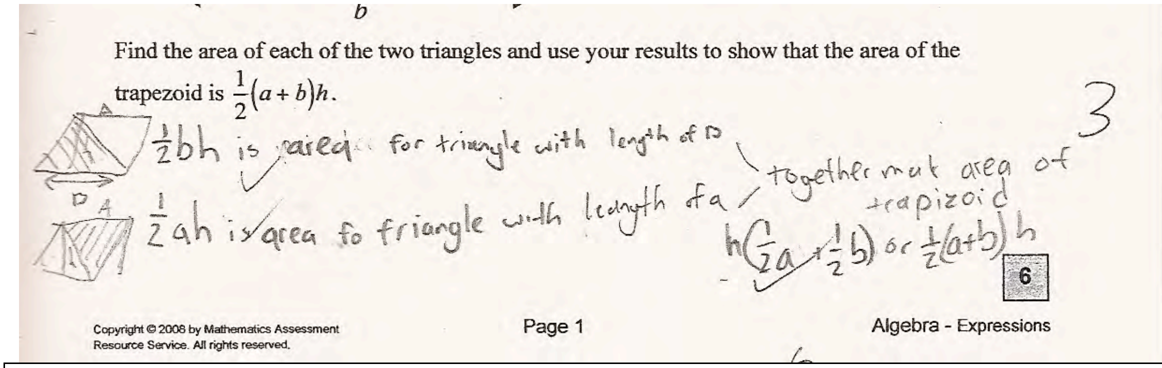
Ther fore by combining these two, you'll get $\boxed{\frac{1}{2}(a+b)h}$

6

6

Student B uses diagrams to think about the expressions for area of each triangle. Then the student talks about combining the two expressions. Student B factors out first the height and then the 1/2 to make the combined expression equivalent to the original formula. Understanding how to close an argument and show the steps back to the original statement is an important piece of logical reasoning.

Student B



Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$.

$\frac{1}{2}bh$ is area for triangle with length of b

$\frac{1}{2}ah$ is area fo triangle with length of a

together make area of trapizoid

$h(\frac{1}{2}a + \frac{1}{2}b)$ or $\frac{1}{2}(a+b)h$

3

6

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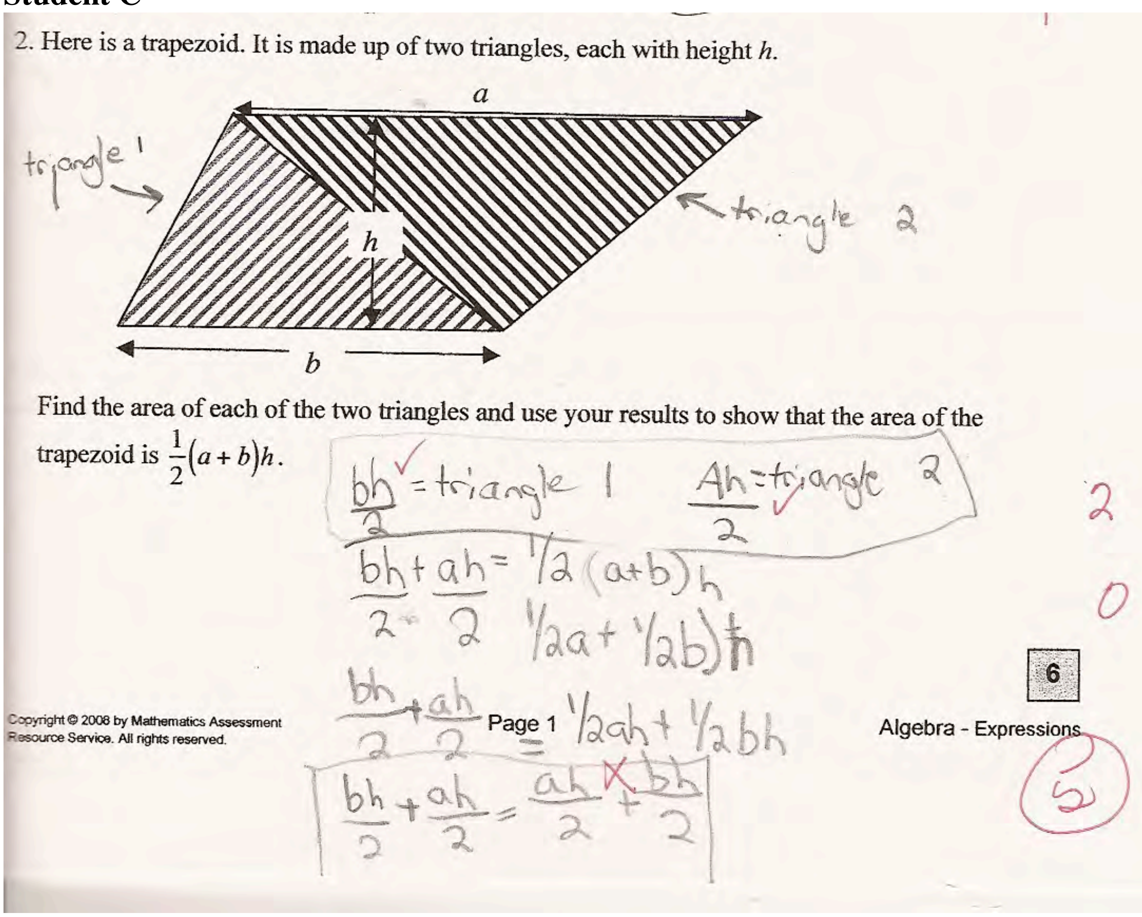
Student C has all the information needed to make the conclusion, but either doesn't understand how to factor out the expression or recognize a need to make the final statement the same as the original.

Student C



2. Here is a trapezoid. It is made up of two triangles, each with height h.

Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$.

$\frac{bh}{2}$ = triangle 1 $\frac{Ah}{2}$ = triangle 2

$\frac{bh}{2} + \frac{ah}{2} = \frac{1}{2}(a+b)h$

$\frac{1}{2}a + \frac{1}{2}b)h$

$\frac{bh}{2} + \frac{ah}{2} = \frac{1}{2}ah + \frac{1}{2}bh$

$\frac{bh}{2} + \frac{ah}{2} = \frac{ah}{2} + \frac{bh}{2}$

2

0

6

2

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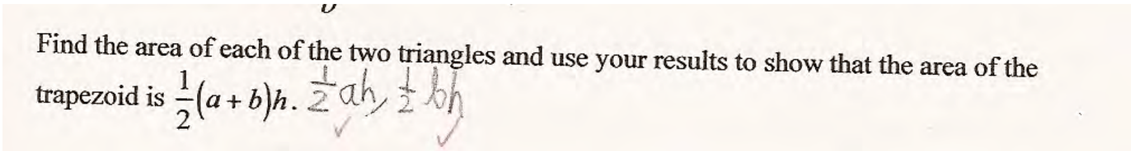
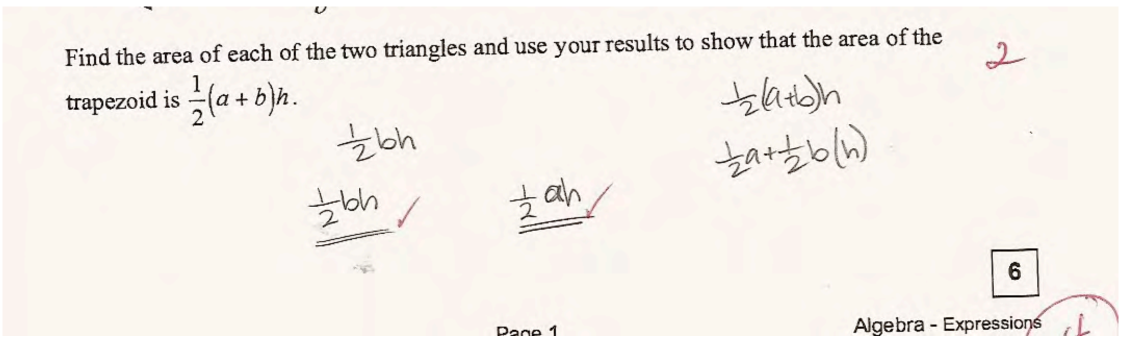
Student D finds the area of each separate triangle, but does not combine the terms or make any attempt to show how that relates to the original formula.

Student D


Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$. $\frac{1}{2}ah$, $\frac{1}{2}bh$

Student E finds the area of the separate triangles and then seems to try to work backward from the original formula to get the two expressions. The student appears to be attempting distributive property on the right, but does not carry it through correctly.

Student E


Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$.

2

$\frac{1}{2}(a+b)h$

$\frac{1}{2}a+\frac{1}{2}b(h)$

$\frac{1}{2}bh$

$\frac{1}{2}bh$ $\frac{1}{2}ah$

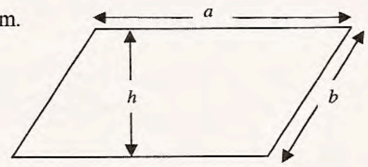
6

Page 1 Algebra - Expressions

Student F might be debating between two different area formulas, using one strategy for triangle a and a different strategy for triangle b. The students seems to settle on ah + bh. The student then factors this expression but does not know how to get the 1/2 into the problem. *If you could interview this student, what question might you want to ask? If you could pose some other problems, what might help you understand where the thinking or skills break down?* Notice that student F puts multiple choices for area in part 1a and doesn't recognize equivalent expressions for perimeter in part 1b.

Student F

1. Here is a parallelogram.



The area of a parallelogram is the product of its base times the perpendicular height.

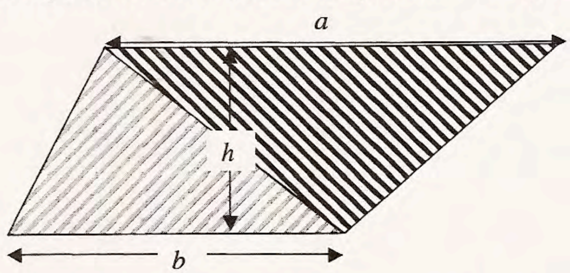
a. Which of these are correct expressions for the area of this parallelogram?
Draw a circle around any that are correct.

ab $\quad \frac{1}{2}ab$ $\quad ah$ $\quad \frac{1}{2}ah$ $\quad 2a+2b$ $\quad 2(a+b)$ $\quad abh$ $\quad$ 1 $\quad$ 0

b. Which of these are correct expressions for the perimeter of the parallelogram?
Draw a circle around any that are correct.

ab $\quad \frac{1}{2}ab$ $\quad ah$ $\quad \frac{1}{2}ah$ $\quad 2a+2b$ $\quad 2(a+b)$ $\quad abh$ $\quad$ 1

2. Here is a trapezoid. It is made up of two triangles, each with height h.



Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$.

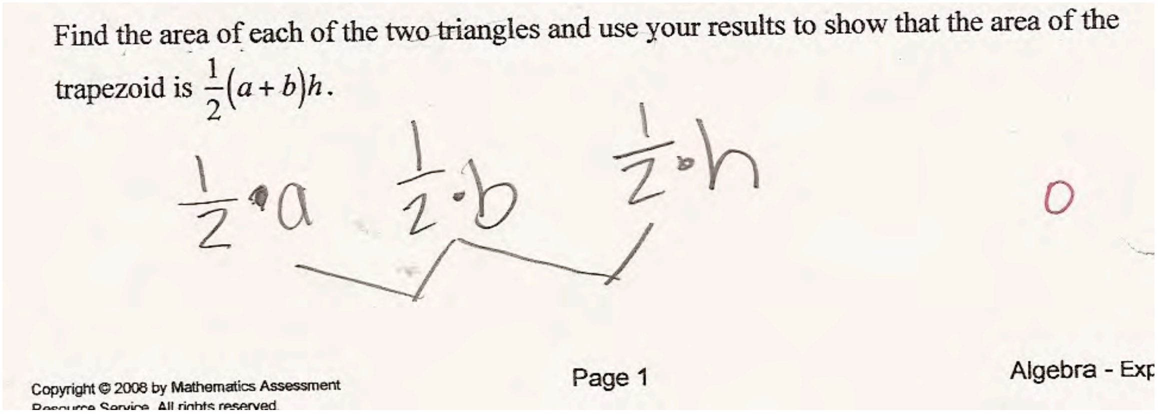
ah $\quad \frac{1}{2}(a+b)h$ $\quad$ 1 $\quad$ x

$\frac{1}{2}bh$ $\quad ah+bh = (a+b)h$

$\boxed{\frac{1}{2}(a+b)h}$ $\quad$ 0 $\quad$ 6

Many students struggled with using the distributive property on the original formula. Student G distributes the 1/2 on variables inside and outside the parentheses. Student H distributes the 1/2 over both variables and then separately distributes the h.

Student G


Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$.

$\frac{1}{2} \cdot a \quad \frac{1}{2} \cdot b \quad \frac{1}{2} \cdot h$

0

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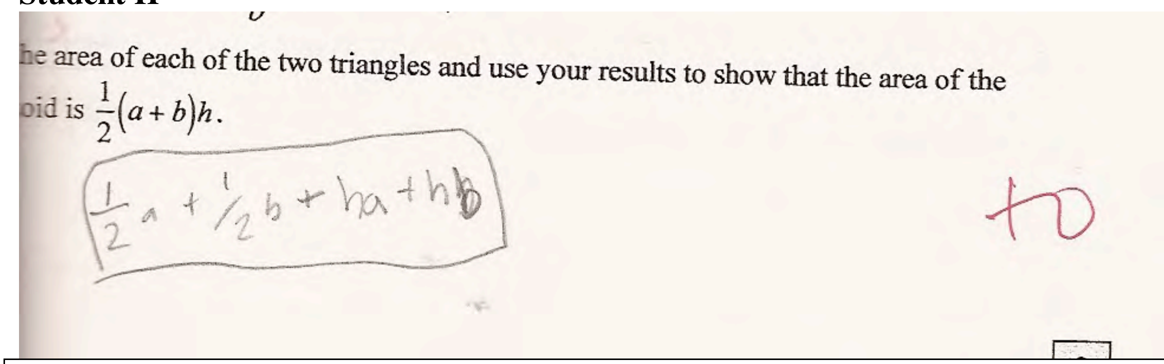
Student H

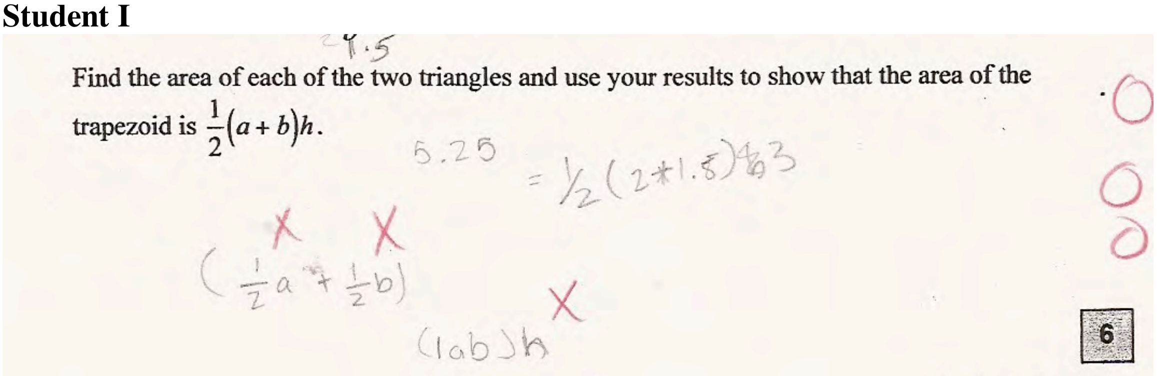

he area of each of the two triangles and use your results to show that the area of the oid is $\frac{1}{2}(a+b)h$.

$\frac{1}{2}a + \frac{1}{2}b + ha + hb$

+0

Student I struggles with the concept of like and unlike terms. The student also tries to replace the variables with numbers to check the solution.

Student I


Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$.

5.25 $= \frac{1}{2}(2+1.5)3$

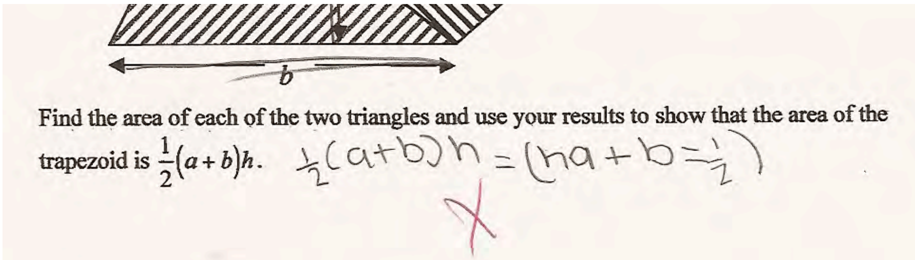
$(\frac{1}{2}a + \frac{1}{2}b)$ x x

$(1ab)h$ x

0 0 0 6

Student J tries to use distributive property inappropriately and also teases apart the expression in an attempt to set up an equation. On the right, the student also seems to use parentheses more in a linguistic sense to separate things rather than as a mathematical notation.

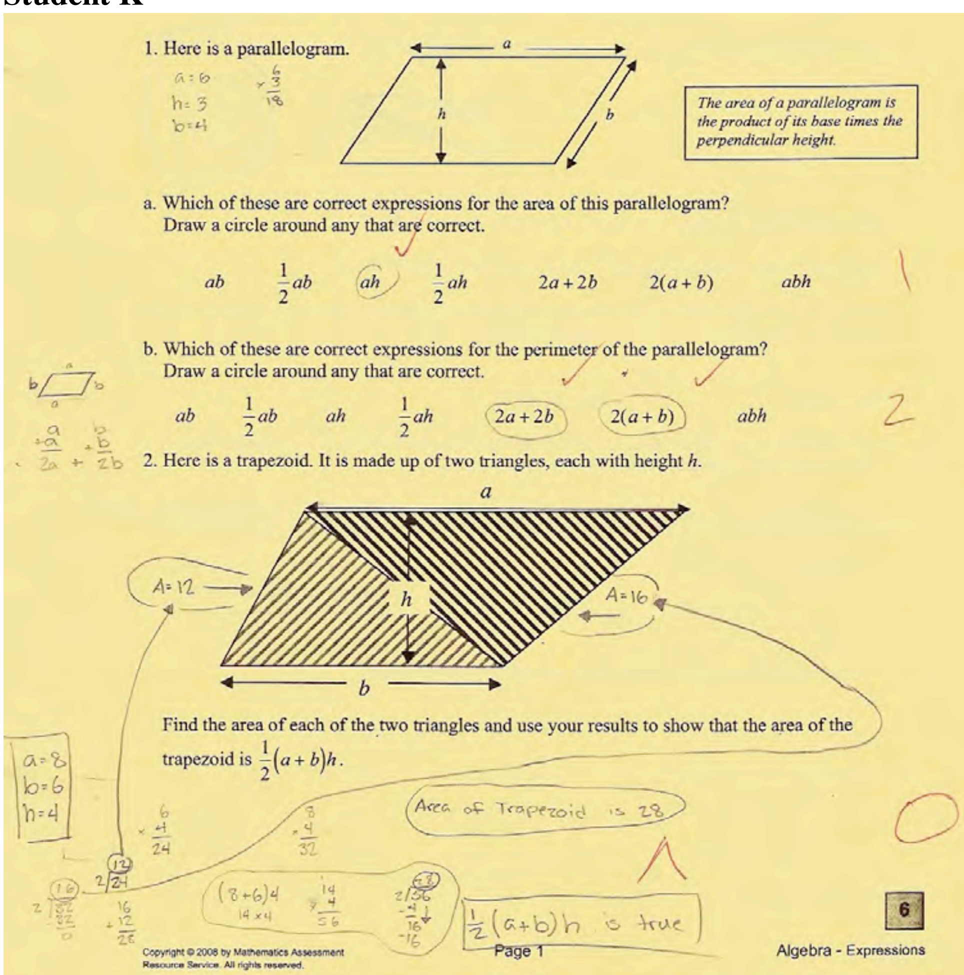
Student J



Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$. $\frac{1}{2}(a+b)h = (ha + b = \frac{1}{2})$

Student K can't think with variables. To identify the expressions in 1a and b, the student needs to put in numbers to think through the process and then substitute back in the variables. In part 2 the student checks out one case of using numbers to check that the formula is true, but doesn't have the sense of the importance of variables to build a generalizable proof for all cases. *How do we help students learn to think with variables? How do we help develop in students the idea of algebra as a tool for generalization instead of set of manipulations?*

Student K



1. Here is a parallelogram.

The area of a parallelogram is the product of its base times the perpendicular height.

a. Which of these are correct expressions for the area of this parallelogram? Draw a circle around any that are correct.

ab $\quad \frac{1}{2}ab$ $\quad ah$ $\quad \frac{1}{2}ah$ $\quad 2a+2b$ $\quad 2(a+b)$ $\quad abh$

b. Which of these are correct expressions for the perimeter of the parallelogram? Draw a circle around any that are correct.

ab $\quad \frac{1}{2}ab$ $\quad ah$ $\quad \frac{1}{2}ah$ $\quad 2a+2b$ $\quad 2(a+b)$ $\quad abh$

2. Here is a trapezoid. It is made up of two triangles, each with height h.

Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b)h$.

A=12 A=16

a=8 b=6 h=4

Area of Trapezoid is 28

$\frac{1}{2}(a+b)h$ is true

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Page 1

Algebra - Expressions

Algebra Task 1 Expressions

Student Task	Work with algebraic expressions for areas and perimeters of parallelograms and trapezoids.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Use symbolic algebra to represent and explain mathematical relationships.

Mathematics of this task:

- Using variables to find area and perimeter of a parallelogram
- Recognizing equivalent expressions by factoring or using distributive property
- Using algebra to make and prove a generalization
- Explaining the steps of factoring or using distributive property to make two expressions equivalent

Based on teacher observations*, this is what algebra students know and are able to do:*

- Students were able to recognize expressions for finding perimeter of a parallelogram
- Many students could recognize equivalent expressions for perimeter

Areas of difficulty for algebra students:

- Finding the area of a parallelogram/ translating from words to variables (the formula for area was given in a verbal form)
- Thinking with variables instead of numbers
- Decomposing the trapezoid into two triangles
- Using factoring and/or distributive.0 property to make equivalent expressions
- Using algebra to make a generalization

MARS Test Task 1 Frequency Distribution and Bar Graph, Course 1

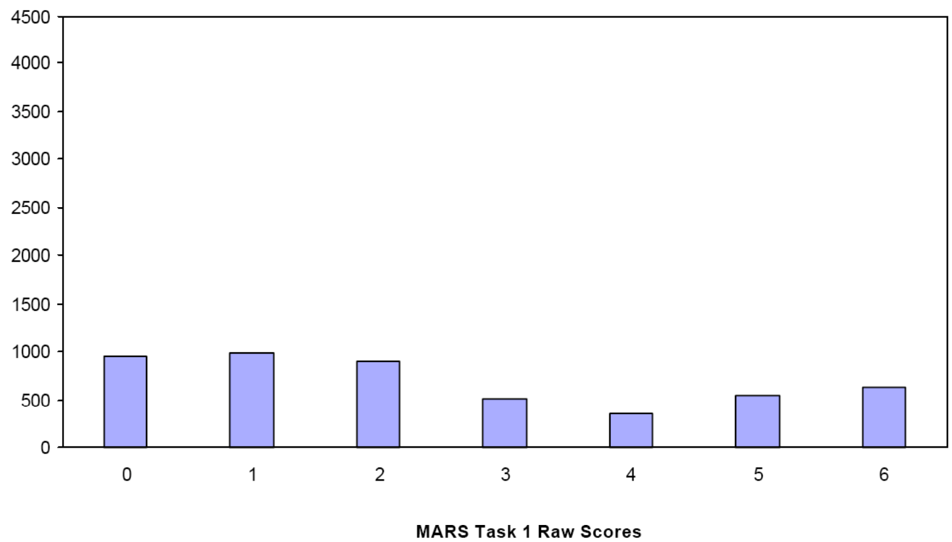
Task 1 - Expressions

Mean: 2.51 StdDev: 2.05

Table 45: Frequency Distribution of MARS Test Task 1, Course 1

Task 1 Scores	Student Count	% at or below	% at or above
0	955	19.6%	100.0%
1	993	39.9%	80.4%
2	897	58.3%	60.1%
3	508	68.7%	41.7%
4	354	75.9%	31.3%
5	539	87.0%	24.1%
6	636	100.0%	13.0%

Figure 54: Bar Graph of MARS Test Task 1 Raw Scores, Course 1



The maximum score available for this task is 6 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Many students, 80%, could find one expression for the perimeter of a parallelogram. More than half the students could either find one expression for area and perimeter of a parallelogram or find two expressions for a parallelogram. Some students, about 31%, could find two expressions for perimeter of a parallelogram, and find the area of the two small triangles. 13% of the students could meet all the demands of the task including finding an expression for the area of a parallelogram and using algebra to show how to combine and factor the expressions for the area of two triangles into the formula for the area of a trapezoid.

Expressions

Points	Understandings	Misunderstandings
0	82% of the students with this score attempted the task.	Students chose too many expressions for perimeter and didn't see equivalent expressions. 26% of the students omitted 2a+2b for perimeter. 31% omitted 2(a+b) for perimeter. About 5% of the students chose each of the following options:ah, 1/ah, ahb, ab, and 1/2 ab.
2	Students with this score could either find both expressions for perimeter for a parallelogram or find one expression for area and one expression for perimeter.	Students had difficulty identifying the formula for area, even though the verbal rule was given. While many students picked ah, they often picked other choices as well. 35% of the students did not pick ah as one of the choices. Almost 14% of the students picked ab for area, confusing side length with height. 14% picked 1/2ah and 12% picked 1/2ab for the area formula, confusing area of a triangle with area for a parallelogram. About 5% picked each of the perimeter formulas for area.
3	Students could identify expressions for area and perimeter.	Students had difficulty trying to make an algebraic generalization about area of a trapezoid. Some students did not decompose the shape into two triangles. 10% of the students substituted numbers for variables. 8% left the height out of the formula for the small triangles (1/2 a or 1/2 b). 10% found the area of the small triangles, but did no further work to show how to combine the areas and create an equivalent expression to the given formula.
4	Students with this score could find the expressions for perimeter and the area of the small triangles in part 2 or find one perimeter expression and explain all of part 2.	Students applied algebra inappropriately: combining unlike terms, not being able to factor expressions, using distributive property incorrectly.
5		Students couldn't or didn't combine the areas of the small triangles to make a complete argument in part 2.
6	Students could use a geometric context to write expressions with variables for area and perimeter of a parallelogram and a trapezoid.	

Implications for Instruction

Students at this grade level have been working with area and perimeter of parallelograms since 4th grade and area of triangles since 5th grade. At this grade level students need to be able to move from the specific solution to generalizing about why the formula works and where it comes from.

Students in algebra have learned a variety of tools, such as using a variable to stand for a side of any length, combining like terms, or factoring polynomials. The purpose of these procedures or tools is to be able to prove numerical relationships or why something works or discover under what circumstances it won't work.

Students need to be exposed to a variety of contexts for applying their skills to make and test conjectures, to prove mathematical relationships. Procedures that can't be transferred to new situations are not useful and will probably have a short retention rate.

Some of the most interesting data from algebra shows that students on cumulative tests do best on the most recently taught topic, rather than building from foundational knowledge at the beginning of a course. Students need more opportunities to connect their thinking to its use in context.

Ideas for Action Research – Building Logical Arguments

Students, even at very young ages, are capable of learning the reasoning chains to make logical proofs. They just need to be pushed with good questioning strategies. In the book, Thinking Mathematically, Integrating Arithmetic and Algebra in Elementary School, the authors explore classroom experiences with children in 2nd, 3rd, and 4th grades using variables and learning the logic of proof. In the chapter on justification and proof, second-grader Susie is able to use number properties to justify why $-5 - (-5) + 5 = 5$. She is first able to write that $a + b - b = a$. She justifies it by saying that any number minus itself equals 0, so $b - b = 0$. Then any number + 0 equals itself. Finally she is able to say that if both of these statements are true the original will also be true. "It is productive to ask children whether their conjectures are always true and how they know that they are always true for all numbers…..We consistently have been surprised at what children are capable of when given the opportunity." This book comes with a video of students making and testing conjectures. Some of the videos might be useful in the classroom to give students models of how to use and discuss mathematics.

Fostering Algebraic Thinking offers some grade level appropriate activities to help students build their capacity for generalization and making proofs. Consider the problem of finding combinations of consecutive numbers to make the different answers of 1 to 35. After students have explored this problem they might be asked to describe patterns that they found with the consecutive numbers. Students might provide a range of solutions from all the numbers made of 3 consecutive numbers can be divided by three to a number N is a consecutive sum of *m* numbers if *m* divides evenly into N and *m* is an odd number. This resource offers students many intriguing problems to work on their abilities to make logical arguments and develop proofs using variables. It also offers suggestions on types of questions that help students build habits of mind that lead them to make better justifications.

A related MARS task is the Sum of Two Squares, Course 2- 2005, available on the Noyce website. It starts with the premise from Lewis Carroll, that $2(x^2 + y^2)$ is always the sum of two squares where x and y are a pair of non-zero integers. Students are given opportunities to investigate the conjecture with numbers, then describe the relationship in words and finally challenged to prove why this is always true using algebra. The discussion on this task would be a good place for practicing the types of questioning strategies suggested in the two references above.

Performance Assessment Task
Expressions Grade 9
The task challenges a student to demonstrate understanding of the concepts of algebraic properties and representations. A student must be able to represent and analyze mathematical situations and structures using algebraic symbols. In this case, a student must make sense of areas and perimeters of parallelograms and trapezoids using algebraic expressions. A student must understand how to use symbolic algebra to represent and explain mathematical relationships. A student must determine how to explain the steps to make two expressions equivalent.
Common Core State Standards Math - Content Standards
<u>**High School – Algebra – Creating Equations**</u> **Create equations that describe numbers or relationships.** A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
Common Core State Standards Math – Standards of Mathematical Practice
MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.6 Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2008	6	4	31 %