# Early Mathematics a resource for teaching young children



a publication of

The Charles A. Dana Center at The University of Texas at Austin

# **Early Mathematics**

# A Resource for Teaching Young Children

Kindergarten

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This edition was developed in Microsoft Word.

October 2012 release.

As always, we welcome your comments and suggestions for improvements. Please contact us at dana-txshop@utlists.utexas.edu or at the mailing address above.

# About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center strengthens our nation's education systems to provide a reliable path to upward mobility for all students. Our work focuses on mathematics and science education, with an emphasis on strategies for improving student engagement, motivation, and persistence. We are dedicated to nurturing students' intellectual passions and ensuring that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy.

We advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards. We help our partners adapt promising research to meet their local needs.

We develop innovative curricula, tools, protocols, instructional supports, and professional development systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance to school and district leadership teams, advise community colleges and states, and collaborate with national partners on work such as our Urban District Leadership Networks, Academic Youth Development project, and Advanced Mathematical Decision Making course.

We have significant experience and expertise in the following:

- Standards development and implementation, systemic reform, and district capacity building
- Education leadership, instructional coaching, and teaching
- K-14 course design and development, learning networks, and programs for bridging critical transitions
- Research, content development, and publishing

The Center was founded in 1991 at The University of Texas at Austin. Our staff of nearly 80 researchers and education professionals has worked with dozens of school systems in nearly 20 states and with 90 percent of Texas's more than 1,000 school districts. We are committed to ensuring that the accident of where a child attends school does not limit the academic opportunities he or she can pursue. For more information about our programs and resources, see our homepage at www.utdanacenter.org.

### **About the Common Core State Standards for Mathematics**

This resource is aligned to the Common Core State Standards for Mathematics.

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# About the development of this resource

This new revised and expanded edition of *Early mathematics: A resource for teaching young children* consists of materials for 20 sessions for each of four grades—prekindergarten, kindergarten, grade 1, and grade 2—for a total of 80 sessions. We were able to develop these materials because of a generous December 2010 grant from the **Novce Foundation**.

First edition (2011)

Twenty of these sessions (10 for prekindergarten and 10 for second grade) were initially developed in spring and summer 2011 by early mathematics education experts Brian Mowry (prekindergarten) and Carolyn Moore (second grade), and reviewed in summer and early fall 2011 by ACE: A Community for Education leaders Chetan Makan and Mary Ellen Isaacs, both of whom are experts in designing and implementing early childhood tutoring programs that can be implemented at scale. The materials were also reviewed by Patti Bridwell, who has expertise in professional supports for teachers and tutors.

This first edition was released in fall 2011 as a proof-of-concept resource titled *Early mathematics: Resources for tutoring young children*. These initial 20 sessions were field-tested in fall 2011 by tutors from the Dana Center's ACE: A Community for Education (www.utdanacenter.org/ace) program in Austin, Texas (prekindergarten sessions), and by tutors from Experience Corps (www.experiencecorps.org) in Philadelphia, Pennsylvania (second-grade sessions).

Second edition (2012)

A key finding from the fall 2011 proof-of-concept field testing was that the material as written was probably too complex for paraprofessionals (e.g., tutors) to deliver, but that it could be very effective if delivered by classroom teachers. Based on this feedback, we have substantially revised the initial 20 sessions for this new edition, changing the intended users of this resource from paraprofessionals to classroom teachers.

All 80 sessions are built on recommendations in the 2009 National Research Council report *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Committee on Early Childhood Mathematics; Christopher T. Cross, Taniesha A. Woods, and Heidi Schweingruber, editors). Center for Education, Division of Behavioral and Social Sciences and Education. Washington, D.C: The National Academies Press.

In particular, these session materials speak to the recommendation that:

Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with *more mathematics instruction time devoted to number* than to other topics.

Accordingly, our materials focus primarily on number.

# **About the Noyce Foundation**

The Noyce Foundation<sup>1</sup> aims to help young people become curious, thoughtful, and engaged learners. The Noyce Foundation focuses on a few key areas:

- improving the teaching of math and science in public schools;
- developing leadership to support student achievement;
- supporting education policy and research; and
- expanding opportunities for students to experience hands-on science in out-of-school settings.

The Noyce family created the Noyce Foundation in 1990 to honor the memory and legacy of Dr. Robert N. Noyce, cofounder of Intel and inventor of the integrated circuit—which fueled the personal computer revolution and gave Silicon Valley its name.

Although he was an individual of daunting talents and intellect who was honored by two presidents as well as by his academic and industry peers around the world, Bob Noyce also remained a humble and approachable man who believed fervently in democracy. In everything the Noyce Foundation undertakes, it remains committed to promoting the qualities that Bob Noyce embodied: optimism, creativity, risk taking, and determination.

In recognition of Bob's concern about the narrowing pipeline of students interested in—and committed to—science-related careers, the Noyce Foundation has focused on mathematics, science, and associated work in research and policy. Much of the Foundation's focus has been on improving instruction in mathematics, science, and early literacy in public schools.

As schools began to intensify their focus on math and literacy in response to No Child Left Behind—leaving science behind in the process—the Noyce Foundation emphasized support for out-of-school science programs that show promise of sustaining and engaging student interest through middle school, a time when students tend to make critical decisions about what subjects they want to pursue in the future. The Noyce Foundation informal science initiative includes support for leadership development in science centers.

For more information about the Noyce Foundation, visit its website at **www.noycefdn.org.** For more information about the Silicon Valley Mathematics Initiative, see **www.svmimac.org.** 

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<sup>&</sup>lt;sup>1</sup> This description of the Noyce Foundation's mission and history was adapted from content retrieved from its homepage (www.noycefdn.org) and its About Us page (www.noycefdn.org/aboutus.php) on October 9, 2012.

# **Acknowledgments**

# With special thanks . . .

The Dana Center thanks the Noyce Foundation for its generous support of this project. The Noyce family created the Noyce Foundation in 1990 to honor the memory and legacy of Dr. Robert N. Noyce, cofounder of Intel and inventor of the integrated circuit—which fueled the personal computer revolution and gave Silicon Valley its name. For more information about the Noyce Foundation, visit its website at **www.noycefdn.org.** 

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# Introduction

# **Background**

Early Mathematics—A Resource for Teaching Young Children provides a series of instructional tasks, aligned with the Common Core State Standards for Mathematics, that teachers can use to instruct children in prekindergarten, kindergarten, grade 1, and grade 2. The complete resource includes content for 20 sessions for each of these four grade levels.

The tasks were developed for whole-class instruction with some small-group work, but they are also easily adaptable for tutoring sessions. The estimated timeframe for each session is as follows:

Grade level	Estimated time per session
Prekindergarten	30 minutes
Kindergarten	45 minutes
Grade 1	45 minutes
Grade 2	45 minutes

Most sessions have a literature focus to draw children into the content and/or to keep them connected to a context.

These session materials do not provide everything a child needs to know about a given topic, such as *number*. Rather, each session provides a series of instructional tasks to help you teach selected content and practices described in the Common Core State Standards for Mathematics. You should feel free to modify the sessions as appropriate to meet the individual needs of children in your classroom.

# Alignment

We have embedded key Common Core State Standards for Mathematical Practice in each session to help bring out crucial ideas. In most sessions, though, additional Standards for Mathematical Practice beyond those selected may also be relevant.

We chose the content for these sessions based on what content we believe will have the most significant effect on student learning. The language below is drawn from the National Council of Teachers of Mathematics 2006 publication, *Curriculum Focal Points for Prekindergarten Through Grade Eight Mathematics: A Quest for Coherence* (prekindergarten) and the Common Core State Standards for Mathematics (kindergarten onward).

# Prekindergarten

 developing an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison.

# Kindergarten

representing, relating, and operating on whole numbers, initially with sets of objects;

### Grade 1

- (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;
- (2) developing understanding of whole number relationships and place value, including grouping in tens and ones;
- (3) developing understanding of linear measurement and measuring lengths as iterating length units.

### Grade 2

- (1) extending understanding of base-ten notation;
- (2) building fluency with addition and subtraction.

# **Structure**

Each session is divided into three instructional formats—Activate, Engage, and Develop.

The **activate** portion introduces the content in the session and objectives that will be developed in the forthcoming session. In prekindergarten and kindergarten, this section can often occur as a part of the morning circle routine (e.g., calendar, morning message), or it can serve as a transition activity that incorporates songs, movement, and other instructional activities developed to capture the interest and attention of younger students with emerging attention spans.

Then children will **engage** in the content through an activity centered on the content and practices in the standard(s) being addressed. In prekindergarten and kindergarten, this time is spent mostly in whole (or large) group so that the teacher can model the mathematical thinking that children will apply in the Develop section. For younger children, keep in mind that wholegroup sessions are designed to last no longer than 20 minutes.

Each session ends with **develop**, which provides children an opportunity to share and analyze their understandings and/or methods. In prekindergarten and kindergarten, the activities in this section can take place during centers, small group, or math station time. Throughout, the role of the teacher will primarily be to ask probing questions to help children make sense of the content in the session.

# **Early Mathematics**

# A Resource for Teaching Young Children

# Kindergarten

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# **Session 1: Mouse Count**

# **At A Glance**

This session begins as the teacher introduces children to the formal language—how many, the number word list (*one, two, three,* etc.)—associated with counting. The teacher then reads the book *Mouse Count* to model behaviors that children will demonstrate as they encounter similar counting situations (e.g., reciting the counting word list, counting on, and counting backward) (K.CC.2). Next, the teacher models how to count by taking inventory of the number of children who came to school. The session concludes as the teacher observes pairs of children counting collections of materials from the classroom. The focus is on using counting to describe how many objects are in a set (K.CC.5).

# Common Core State Standards

# **Counting and Cardinality**

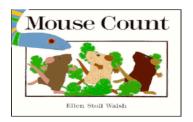
Know number names and the count sequence.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

# Count to tell the number of objects.

- K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
- MP 5 Use appropriate tools strategically.

- Mouse Count by Ellen Stoll Walsh
- Empty jar
- Sock snake puppet
- 10 counters or toy mice
- Sets of classroom counters for children to inventory
- Sticky notes or index cards
- Writing utensils



# Activate

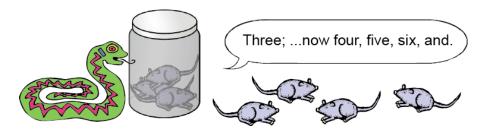
- Introduce *Mouse Count* by Ellen Stoll Walsh. Discuss the number of mice displayed on the book's cover.
  - Ask, "How many mice do you see? How did you figure out the number? What words would the snake use if he were to count the mice?"
- Explain that there are words people say when they count to help them know which items they have counted and how many.
  - "When you count, touch the first object you see and say the word one. Then, you say the word two for the next object you touch, followed by three for the object that comes after."

"One, two, three, four, five, six, seven, eight, nine, and ten."

12345678910

# **Engage**

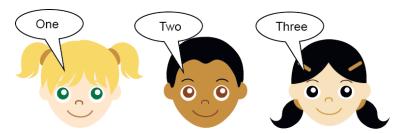
- If children are not familiar with *Mouse Count*, read the book in its entirety for the first reading, with only minor pauses to check for children's comprehension.
  - For example, "How many mice has the snake collected so far? How many more do you think he will find?"



- If this is a book children have read several other times, consider acting out the story using props such as a sock to represent the snake, a set of counters/toy mice, and a jar for collecting the materials.
- As you read the story, highlight/act out the following counting episodes:
  - 1. The snake stops to search for more mice and then begins to count on from the number of mice he had collected so far (e.g., he stops at three and then counts on, "four, five, six, seven").
  - 2. Upon their successful escape, the mice begin to "uncount" themselves, a process called *counting backward* (e.g., "ten, nine, eight, seven, six, five, four, three, two, one").

# Develop

- After reading the story, explain that you will conduct a child count to determine how many children there are altogether in the class.
- Go around the circle, tapping the head of each child and saying the corresponding counting word. Once you reach the last child, emphasize how the last counting word you said tells how many children there are altogether.



- If time permits, model how to count backward, starting with the last child you counted during the initial round.
- Gather suggestions from children about other items they have counted at home, at school, at the store, on the playground, etc.
- Pair children with partners and assign each pair a bin of classroom counters or art supplies (e.g., crayons, scissors, markers) to count and inventory.
- Provide each pair sticky notes or index cards to use to record their count. Encourage
  children to refer to a classroom number line or hundreds chart as they record their assigned
  collection.
- As children are working, take anecdotal/observational notes in relation to their counting skills and familiarity with numbers. Use this information to guide further instruction and grouping decisions for the remainder of the unit.
  - Do children know the rote-counting word sequence? For example, do they start at 1 and continue the standard sequence? If so, up to what number?
  - Do children demonstrate one-to-one correspondence? For example, is their counting deliberate or does their finger hover or swipe across several objects at once while they recite the counting sequence?
  - Do children have a system of keeping track? That is, do they double-count objects, counting an object more than once?
  - When asked how many, do children count again or do they know that the last number they said as they were counting tells how many items there are altogether?
  - o Are children able to identify numerals or use numerals to label the sets they count?

# **Session 2: Keeping Track and Counting to 10**

# **At A Glance**

The teacher uses chain links to represent and compare the number of classmates who came to school to the number of children on the class roster. Children revisit this activity (or variations of it) as routine practice for reciting the counting word sequence (K.CC.4a) as well as representing and comparing numbers beyond 10 (K.CC.6). Children then practice a strategy for keeping track whereby they lift a finger for each cube the teacher drops into an empty jar. The focus for this activity is on using a discrete model to represent a quantity presented as an accumulation of successive sounds versus a visible whole. At the close of the session, children play a counting game called "Rescue 10 Mice" in which they roll a dot cube and collect a corresponding number of cubes. Children arrange the cubes they collect in a tens-frame 2-by-5 array. The objective is to count and keep track of a set using a model that shows how numbers 0-9 are related to 10.

# **Common Core State Standards**

# **Counting and Cardinality**

# Count to tell the number of objects.

K.CC.4a Understand the relationship between numbers and quantities' connect counting to cardinality. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

# Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.<sup>1</sup>

<sup>1</sup>Include groups with up to ten objects.

# MP 4 Model with mathematics.

- Alphabetized class roster replicated on chart paper
- Chain links
- Cubes
- Empty jar
- Blackline master, "Tens-Frame"
- Blackline master, "Rescue 10 Mice" game board
- Blackline master, "1–3
   Dot Cube"
  - 1 Janice Akerman
  - 2 Raul Balboa
  - 3 Lizette Cardona
  - 4 Jermaine Frost
  - 5 Paul Hendricks
  - 6 Rachel Jenks
  - 7 Thomas Lowry 8 Lucinda Meador
  - 9 Brian Murray
  - 10 Dominique Newman
  - 11 Cindy Nielson
  - 12 Barak Onsar
  - 13 Hope Overton
  - 14 Nathan Piedmont
  - 15 Hailey Price
  - 16 Beth Rodriguez
  - 17 Kelly Smith
  - 18 Jamal Thorton
  - 19 Enrique Vasquez
  - 20 Deidra Williams

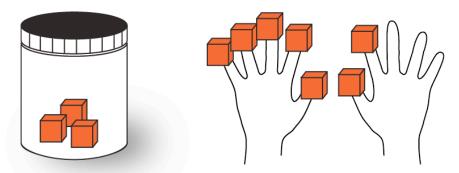
# **Activate**

- Show an official class roster with children's names arranged top down numerically in alphabetical order. If possible, make a poster-size replication of the roster on chart paper so that each child can easily see his/her name in relation to the number it corresponds to on the list.
- Tell children to stand and give you a link as you call off their names in alphabetical order.
- As each child hands over a link, count and check off his/her name on the list to highlight the one-to-one relationship—one link for each child. Connect the links as you count, making one continuous chain.
- Once you reach the last child, compare the number of children standing with the number of names on the list.
  - Ask, "Is everyone present at school today? How do you know?"
- Count the number of children whose names on the roster have not been checked off to help the class see any equivalency or quantitative differences (e.g., two children are absent).
- Explain that you will use the chain to keep track of how many children are present each day at school.

# Engage

- Display the jar you used to reenact the *Mouse Count* story presented in Session 1.
- Present a pretend scenario to the class that builds upon the counting chain routine you introduced in the Activate section.
- Tell children that their job is to help find a place to hide the mice that escaped from the jar.
  - "How many mice would you need to collect? How could you be sure that you had rescued all the mice that had escaped?"
- Explain to children that you are going to drop 1 cube inside the empty jar for each mouse the snake had captured so that they can keep track of how many mice they need to rescue.
- Hide the jar behind a screen or inside a box and instruct children to raise 1 finger and count out loud for each cube they hear you drop inside the container.
- After you drop the last cube inside the jar, emphasize that the final counting word you said tells how many mice are inside the jar.
- Then, count the number of fingers children had raised after the final count, once again emphasizing the last counting word—ten.

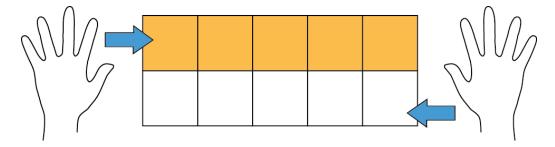
• Match each cube from the jar to all the fingers on one child's hands to confirm the equivalency between the two sets (i.e., 10 fingers and 10 cubes).



- Lead children to make sense of the connection between the chain-link model you created to represent the number of children in the class and the finger tally they made to help them remember how many mice needed rescue.
- "We can use words, numbers, pictures, and objects to help us remember how many people, animals, or other items there are in a collection."

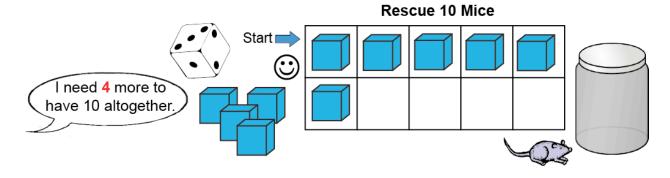
# Develop

- Introduce the tens-frame as another model for representing how many.
- Compare and connect the 2-by-5 arrangement of squares of the tens-frame array to the 5-and-5 configuration of fingers on two hands.
  - 1. Count the number of squares on the first row of the tens-frame.
  - 2. Then, count the number of fingers on one hand.
  - 3. Emphasize the numerical equivalency.
  - 4. Repeat the same procedure for the bottom row and the opposite hand.

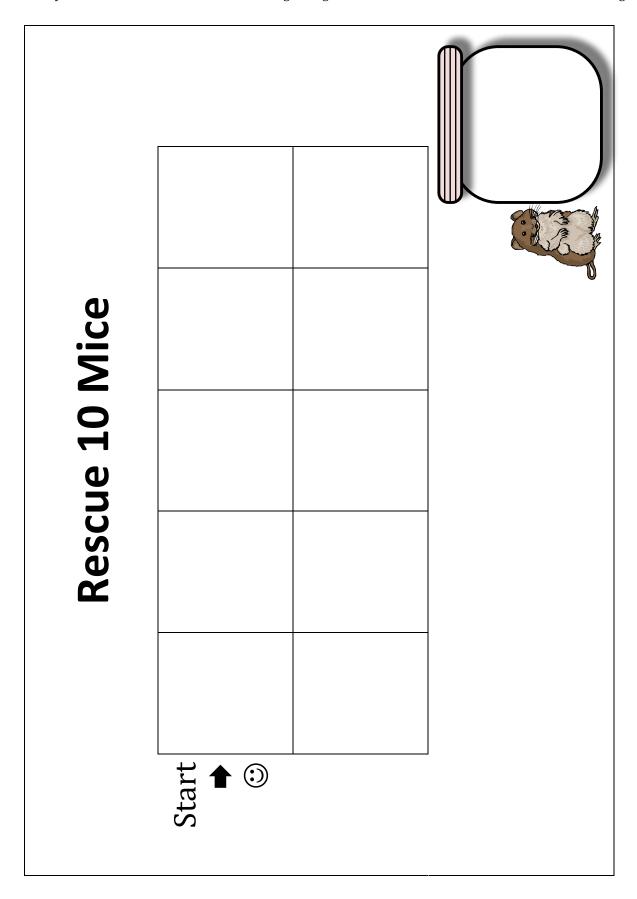


- Model how to fill a tens-frame using the cubes you counted to represent the mice in the pretend scenario.
  - o Start at the top left square and move to the right as you place 1 cube per square.
  - Sweep back down to the left on the bottom row and go across in the same direction left to right—continuing to count the cubes up to 10.

- Introduce and model how to play the game, "Rescue 10 Mice."
  - 1. Roll a 1-3 dot cube.
  - 2. Start at the top left square of the tens-frame game board and place as many cubes (mice), moving left to right, in the top row.
  - 3. Continue rolling and counting until all 10 squares are filled.
- Allow children to play the game with a partner during center/small-group time.
- As children play, prompt them to think about the number of cubes they have collected in relation to the total they need to collect altogether—10.
  - "According to the dot cube, how many cubes do you need to collect for this roll? How many have you collected so far? How many more do you need to have exactly 10?"



# **Tens-Frame**



# 1-3 Dot Cube

Cut apart the along the edge of the outline. Then, fold on the dotted lines and tape together to make a cube.





















# Session 3: Ordering Numbers Horizontally 1-10

# **At A Glance**

Children revisit the chain-link attendance routine introduced during Session 2. Children then double-check the attendance count they conducted using the class sign-in sheet by counting off of the alphabetized class roster. The objective of this activity is to help them make sense of two critical principles of number theory: 1) the order in which items in a set are counted does not change the numerical value of the collection (K.CC.4b); and 2) such a change only affects the ordinal values (e.g., first, second) that describe the order in which each object was enumerated (K.CC.4c). Finally, children compare the numerical position of an assortment of colored mouse cards arranged up against a row of numbered cups. During center time, children play a game in which they identify a missing numeral in a horizontal lineup. The overall objective of both activities is for children to use ordinal vocabulary to describe the position and location of numerals on a number line.

# **Common Core State Standards**

# **Counting and Cardinality**

# Count to tell the number of objects.

- K.CC.4b Understand the relationship between numbers and quantities; connect counting to cardinality. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- K.CC.4c Understand the relationship between numbers and quantities; connect counting to cardinality. Understand that each successive number name refers to a quantity that is one larger.
- MP 4 Reason abstractly and quantitatively.

- Chart paper for morning sign-in
- Alphabetized class roster created during Session 2
- Chain links
- Paper cups numbered 1–10 with their bottoms up
- Blackline master,
   "Mouse Color Cards"
- Sentence strip with green sticker adhered to the bottom far-left side
- Number Line Card blackline master

Class Sign-In Sheet  1 Josh 2 Amy 3 4 5 6 7 8		
2 Amy 3 4 5 6 7 8 9		_
3 4 5 6 7 8 9	1	Josh
4 5 6 7 8 9	2	Amy
5 6 7 8 9	3	
6 7 8 9	4	
7 8 9	5	
8	6	
9	7	
	8	
	9	
10	10	
11	11	
12	12	

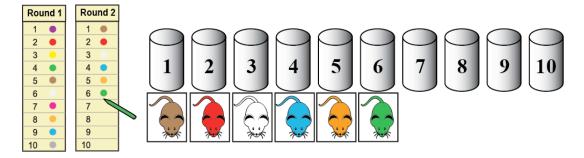
# Activate

- As children enter the classroom, have them sign in on a numbered class list.
- During math time, display the sign-in sheet next to the alphabetized class roster you created during Session 2.
- Take attendance by calling on each child to stand, take a chain link, and count off in the order his/her name appears on the numbered sign-in sheet.
- Emphasize the last counting word children say when you reach the end of the list to highlight the total number of classmates who are present that day.
- Double-check and make a visual representation of today's attendance total by having each child connect his/her link together to form one continuous chain. For this count, however, have children stand and connect their links in the order that their names appear on the alphabetized class roster.
- Compare the number of links in the chain to the number of children on the class roster.
  - Ask, "How many children are present/absent today? How do you know? Are there as many links in the chain as there are names on the roster? Why?"

# Engage

- Lead children to see how the number of a quantity remains the same regardless of the order in which you count the items in the set.
  - "Whose name did I call first when I took attendance from the sign-in sheet? Whose name did I call first when I went by the class roster? Did the order we counted change the total number of children here today?"
- Call on volunteers to help you arrange a set of cups numbered 1–10 bottoms up in numerical order (left to right) on the carpet.
- Assist children in positioning/ordering the numerals.
  - "Which numeral comes first/next/last?"
- Shuffle the colored mouse cards and arrange them in a deck face down on the carpet.
- Call on another group of volunteers to come up one at a time and take a card off the deck, placing the mouse they select against a numbered cup that matches the order it was chosen (e.g., the first card selected should be displayed against Cup No. 1).
- Start at the top of a sheet of chart paper and make a numbered list 1–10 running vertically down the left edge.
- Record the results of the mouse card draw in the order the cards were arranged against the numbered cups.

- Discuss the ordinal value of each card.
  - "Which color comes first/second/third? How do the numbers on the cups help you know the position of the cards?
- Collect the mouse cards and reshuffle. Go a second round, calling on another group of volunteers to select a card off the top of the pile and then place it against a numbered cup that corresponds to the order it was chosen.
- Record the results of this second draw on the opposite side of the chart paper.

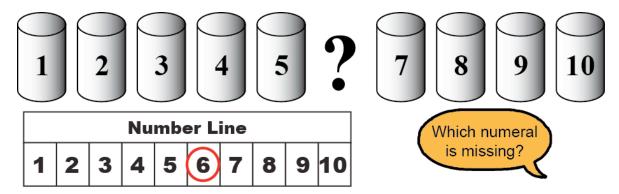


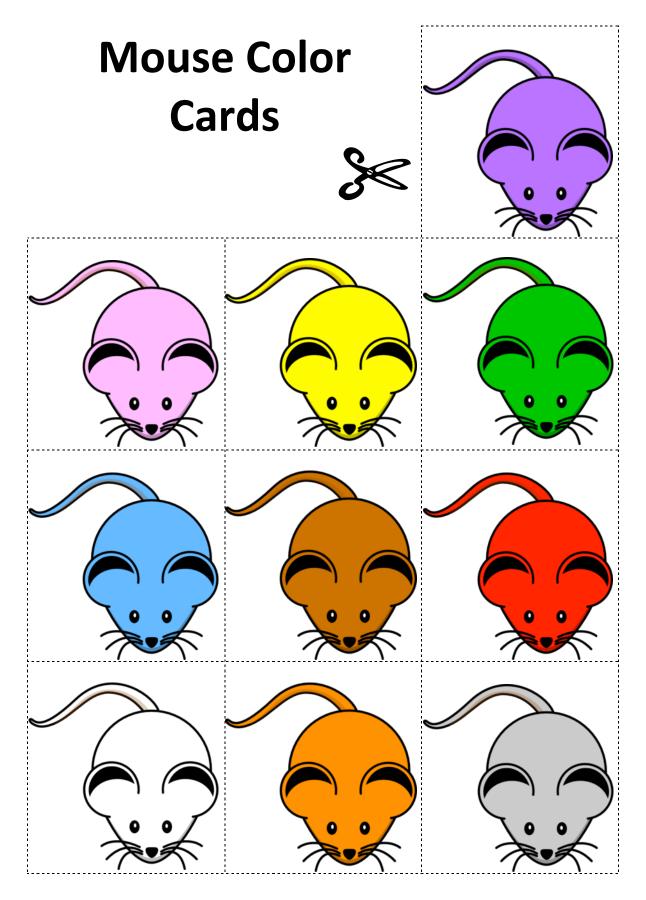
- Compare the positional location of the colored cards from the two rounds.
  - "How has the order changed? What is the same/different?"
- Explain that numerals are sometimes used to describe the order in which something or someone is positioned in a line.

# Develop

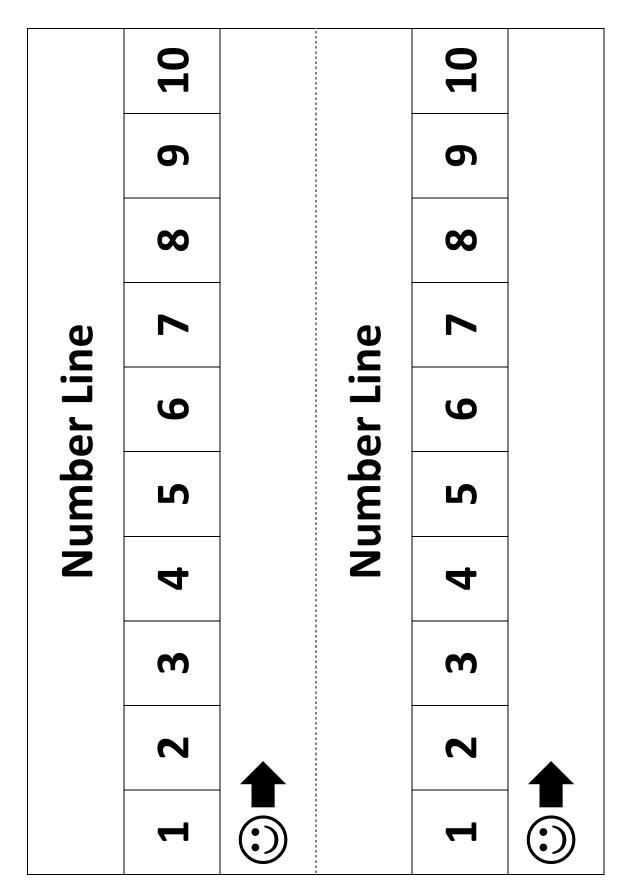
- Introduce a new game, "Missing Number," that children will play during center/small-group time.
- Display a sentence strip with a happy face/dot sticker adhered to its bottom far-left side. Call on a volunteer to help you demonstrate the game:
  - 1. One partner starts at the sticker and arranges the numbered cups in numerical order 1–10 on top of the sentence strip.
  - 2. The other child uses a number line to check the accuracy of her/his partner's numerical arrangement.
  - 3. The partner with the number line closes his/her eyes as the child who was responsible for ordering the cups removes one cup from the arrangement.
  - 4. The checking partner opens her/his eyes and refers back to the number line to determine which numeral is missing from the cup lineup.
  - 5. After confirming the identity of the missing cup, the players switch roles—arranger/hider and checker.

- During center/small-group time, observe and ask questions to help children reflect on and describe the location of numerals on the number line.
  - "Which numeral is missing? Between which two numerals is it located? Which numeral comes after/before it?"





The Charles A. Dana Center at the University of Texas at Austin



# Session 4: Ordering Numbers Vertically 1-10

# **At A Glance**

Children revisit a variation of the Missing Numeral activity introduced in Session 3 during which they described the ordinal position (first, second, etc.) and location (before, after, in between) of numerals situated on a horizontal number line (K.CC.2). The teacher then presents a new scenario in which children locate the numerical position of each floor in a 10-story building drawn on chart paper. The focus is on describing the vertical, bottom-to-top sequence of numerals as ordered on a formal y-axis. This activity, including the "Up and Down the Elevator" game introduced during center/small-group time, provides children with a visual representation for linking a numeral's height as it is situated on a number line (higher or lower) to its cardinal value (greater than/less than) relative to other numbers (K.CC.7).

# **Common Core State Standards**

**Counting and Cardinality** 

Know number names and the count sequence.

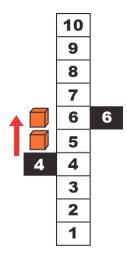
K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

### Compare numbers.

K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

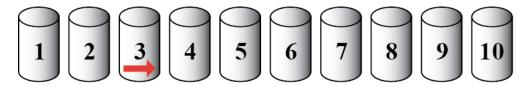
MP 2 Reason abstractly and quantitatively.

- Paper cups numbered
   1–10 with their bottoms
   up
- Linking cubes
- Chart paper
- Crayons/markers
- Sentence strip game board with numerals 1–10 written vertically from bottom to top
- Blackline master, "1–10
   Numeral Cards"



# Activate

- Call on volunteers to help you order the numbered cups you introduced in Session 3.
  - Ask, "Which numeral comes first/second? Which numeral comes after 5?"
- Inform children that you will pretend to be the snake from *Mouse Count* and that you are going to hide a pretend mouse under one of the cups in the arrangement.
- Instruct children to close their eyes as you hide a cube. Tell them to open their eyes and listen carefully as you provide clues that describe under which cup the cube is hiding.
  - "The mouse is under a cup that comes after the number 3."

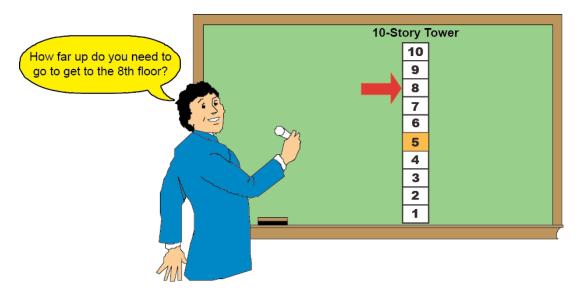


- Call on a volunteer to locate where the cube might be based on the first clue. If the child's guess is incorrect, provide an additional clue.
  - "That was a good guess. However, the mouse is under a cup that comes <u>after</u> the number 3 and is right next to the number 5."
- If time permits, go another round, hiding a cube and providing a variety of clues that help children to focus on the ordinal positions of the horizontal, left-to-right arrangement of numerals on the number line.
  - Additional clues: "The cube is hiding under the sixth cup, between 7 and 9, etc."

# Engage

- Draw a 10-story building on a sheet of chart paper. Divide/mark off each floor with a
  horizontal line so that children see a rectangular array of 10 squares running vertically
  from top to bottom.
- Explain that you want to label each square with a numeral to help people who are getting on and off the elevator know on which floor they are located.
- Prompt the class to think about how to transfer the order of numerals from a left-to-right 1–10 progression to a vertical ascension going from bottom to top on the building.
  - "Which numeral do we start with on the bottom? Which numeral goes above the 1?
     Which numeral will be at the very top?"
- Inform children that you want them to help you to paint each floor in the 10-story building a different color.
- Distribute 10 different color crayons to volunteers.

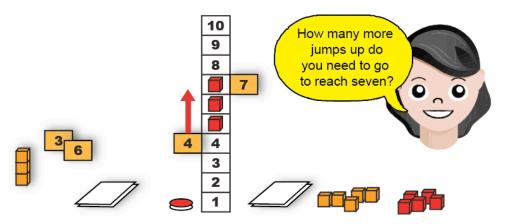
- Prompt children to use ordinal language and positional vocabulary to figure out and describe which floor they are assigned and how many spaces they have to go up on the elevator to reach it.
  - For example, "The fifth floor will be painted red. Who has a crayon to help me color that square? How many floors up will you go from the first floor?"
- Continue giving similar prompts and directions as you call on the other volunteers to paint their assigned floors. Do not go in numerical order so that children have to pay attention to and think about the ordinal language you are using to describe locations.



# Develop

- Introduce a game similar to the exercise you modeled.
- Display a 10-story building (vertical number line) sentence strip game board on the carpet. Designate the bottom floor with a stick figure.
- Have available a container of red and yellow cubes and a coin or double-sided red/yellow counter.
- Call on a volunteer to help you demonstrate the game, "Up and Down the Elevator."
  - 1. Assign each player a location (left or right) in relation to the building game board.
  - 2. Both players shuffle and stack a deck of numeral cards (1–10) face down on their assigned side of the building.
  - 3. Players take a card off the top of the deck and match it to its respective location on the vertical number line on the sentence strip game board.
  - 4. Players flip a coin (or double-sided counter) to determine if they are playing higher (heads/red) or lower (tails/yellow) to win the match of cards.
    - o If the flip indicates lower (tails/yellow), the player with the lowest card on the number line takes the pair of cards.

- o If the flip indicates higher (heads/ red), the player with the highest card on the number line takes the cards.
- If the cards are identical, the draw is a tie, and each partner discards his/her respective card.
- 5. Both players take as many cubes (red if the game is being played for higher cards and yellow if the game is being played for lower cads) as the number of spaces between their respective numbers on the number line.
- 6. The game continues until all cards from the deck are exhausted.
- 7. The player with the most cards wins.



- Prompt children to compare the results and differences.
  - "Did we (the players) make more jumps up/down the number line? How do you know?"
- As children play the game, ask questions to help them compare and think about the difference between their two cards.
  - o "How far up/down the elevator do you need to go to reach [child's name]'s card?"

10	1–10 Numeral Cards  Make a set for each player.	
9	4	3
8	5	2
7	6	1

# **Session 5: Concrete and Pictorial Representations**

# **At A Glance**

Children revisit the attendance chain routine with an emphasis on understanding that since each chain represents one child from the class, the number of children present at school should correspond to the number of links in the chain (K.CC.4b). The teacher then introduces a pretend scenario based on a similar objective as the one from the attendance routine whereby the class helps take inventory of the number of markers inside a box to verify if the snake from *Mouse Count* has been sneaking items out of the classroom. Before dismissing to centers, children make an object and pictorial/symbolic representation (K.CC.4a) of a set of 6–10 items displayed in a plastic bag. The session concludes as children share strategies related to what they have learned about counting, precision, and keeping track.

# **Common Core State Standards**

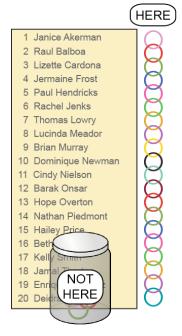
# **Counting and Cardinality**

# Count to tell the number of objects.

- K.CC.4a Understand the relationship between numbers and quantities; connect counting to cardinality. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
- K.CC.4b Understand the relationship between numbers and quantities; connect counting to cardinality. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

# MP 6 Attend to precision.

- Alphabetized class roster displayed on chart paper
- Chain links
- Empty jar
- Box of 8 crayons or markers
- Chart paper
- Baggies of 6–10 objects per pair of children
- Paper and sticky notes

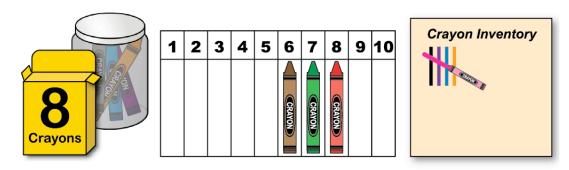


# Activate

- Display the class roster you replicated on chart paper during Session 2.
- Instruct each child to stand and count off as you take attendance going down the list.
- Take one chain link for each child that stands and says his/her assigned number. After each count, connect the links to form one continuous chain.
- Drop a link inside an empty jar for each child who is not present.
- After you reach the last child on the roster, count each link in the chain. Emphasize how the last counting word you say for the final link in the chain is the same number children said as they were counting off for roll call.
  - Ask, "How many children? How many chains? That means there are as many children as there are chains."
- Explain that you will use the chain throughout the day to make sure that each classmate is accounted for as they return to the classroom from lunch and recess.
- Throughout the day, as children return and enter the room, remove the links from the chain and distribute one per child. Count off as each child takes a link.
  - "I will know that everyone is safe inside the classroom if there is a link for each child."

# Engage

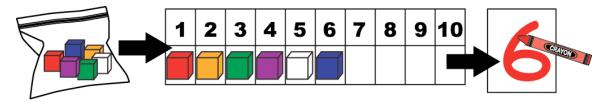
- Display a box of 8 crayons/markers. Explain that the snake from *Mouse Count* (refer to him as "Sneaky Snake") has been taking crayons out of the classroom.
- Lead children to think about what the numeral on the box represents.
  - "What does this number say? If you did not know how to read the numeral, how could you determine how many crayons there are inside the box?"
- Explain that numerals are useful because they show how many items there are without having to count each object one by one.
- Inform children that you want to take inventory of the number of crayons inside the new box you brought to school because the snake has been removing items from the classroom.
- Show children a 1–10 number line. Explain that when you empty the box of crayons and place one crayon underneath each numeral, starting from the left at 1 and moving to the right, you will land on the numeral 8, which matches the numeral displayed on the box.
- Demonstrate how to arrange the crayons along the number line.
- After confirming that there are 8 crayons, write the numeral on a sheet of chart paper. Below the numeral, make a tally mark representation of each crayon. Encourage children to count with you as you make a corresponding mark on the chart paper for each crayon in the box.



- Place the set of crayons inside a see-through jar and explain to children that you will leave
  the lid slightly ajar to confirm your speculation that the snake is taking items overnight
  when everyone is gone.
- Inform the class that you will check the number of crayons inside the jar against the tally mark representation you just made on the chart when children return to school the next day.

# Develop

- Distribute baggies containing 6 to 10 counters to pairs of children. Adjust the quantity based on each child's fluency with numbers and accuracy with counting. (If necessary, refer to the observational notes you took from Session 1 during which you documented the children's facility with counting.)
- Direct the pairs to figure out how many counters there are in their assigned bag and then represent that amount pictorially and with a written numeral on a sticky note (or a sheet of paper.)
- Allow children to use a number line (1–10) to determine how many counters are inside their baggies. Remind them to start at 1 (indicated by a happy face) and then move forward going left to right as they place one object underneath a corresponding numeral. The last number they reach on the number line shows how many counters are inside the bag.



- Dismiss children to math stations or centers as they finish. During center/small-group time, allow them to select among the games that have been introduced thus far—Rescue 10 Mice, Missing Number, or Up and Down the Elevator—or continue counting and taking inventory of items in the classroom.
- At the close of math time or the end of the day, allow children to share the counting strategies they learned as they played the games or engaged in the various number activities introduced up to this point.

# Session 6: Mental Representations for Images of 5

# **At A Glance**

Before this session, the teacher removes and hides 2 crayons from the inventory jar introduced in Session 5. At the start of math time, the teacher guides children to figure out how many crayons are missing from the jar by matching the 6 remaining crayons to the 8 hash marks represented on the inventory chart display. The focus is on using one-to-one matching strategies to compare the difference between two sets (K.CC.6). Next, the class discusses different strategies for determining how many objects are in a set for large (e.g., 8) and small (e.g., 2) quantities as well as amounts in between (e.g., 5). Finally, at center time, children practice creating and visualizing mental representations for color tile arrangements of 5, which can be described and expressed in terms of part-to-whole relationships verbally and with numerical equations (e.g., 4 + 1) (K.OA.3).

# **Common Core State Standards**

# **Counting and Cardinality**

# Count to tell the number of objects.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

# Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. <sup>1</sup> Include groups with up to ten objects.

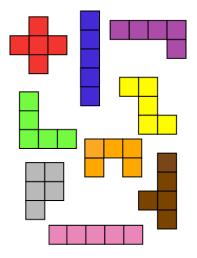
# **Operations and Algebraic Thinking**

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

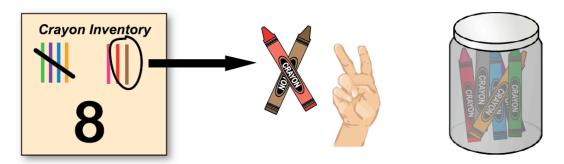
K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

# MP 2 Reason abstractly and quantitatively.

- Inventory jar with the 8 crayons introduced in Session 5
- Crayon Inventory Chart from Session 5
- Dot cube (1–6)
- Paper plates (1 per child)
- Color tiles
- Crayons/markers
- Blackline master, "My 5 Picture" recording sheet
- 5-Tile Arrangement Cards (A–P) blackline master



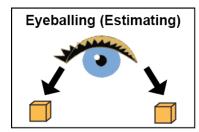
- Before class, remove 2 crayons from the 8 inside the crayon inventory jar that you introduced in Session 5. Hide the 2 crayons somewhere in the classroom.
- Display the crayon inventory jar.
- Refer to the pictorial representation of the crayon inventory as you match each crayon currently inside the jar to a corresponding hash/tally mark on the chart.
- Circle the 2 hash marks that did not have a match and compare the difference/discrepancy between the two amounts.
  - Ask, "Are there the same number of crayons inside the jar as there were the day we made our inventory? How do you know? How many are missing?"
- Call on a volunteer to use her/his fingers or take a set of cubes that matches the number of crayons missing from the jar.
- Direct the volunteer to go to the location where Sneaky Snake decided to hide the crayons.
- Confirm the volunteer's estimate by matching each missing crayon he/she found to the fingers/cubes used to represent the difference.
  - "Yes, there were 2 crayons missing/2 fewer crayons than yesterday."

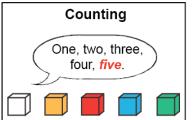


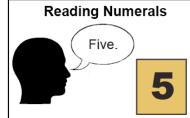
#### Engage

- Congratulate the volunteer for her/his detective and number sense skills.
- Place the two missing crayons back inside the jar. Challenge children to use their fingers to show how many crayons are in the jar now.
- Discuss why so many classmates might have different finger estimates.
  - "Can you see how many are inside without counting? What could we do to make sure how many are in the jar? What does the inventory chart we made tell us about the number of crayons in the jar? What would happen if we counted each crayon?"
- Introduce different strategies for figuring out how many.
  - "Sometimes when there are very few items to look at, we can just eyeball/estimate an amount without counting one by one."

- "Sometimes when the amount is too large to eyeball/estimate, we need to count to confirm our answer."
- "You can also read numerals on a label to know exactly how many are inside."



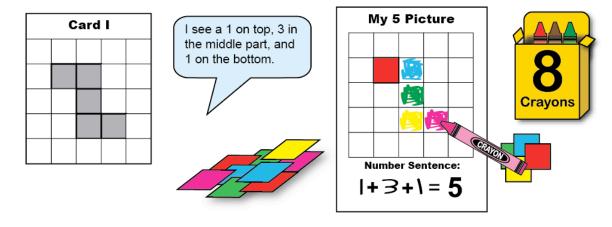




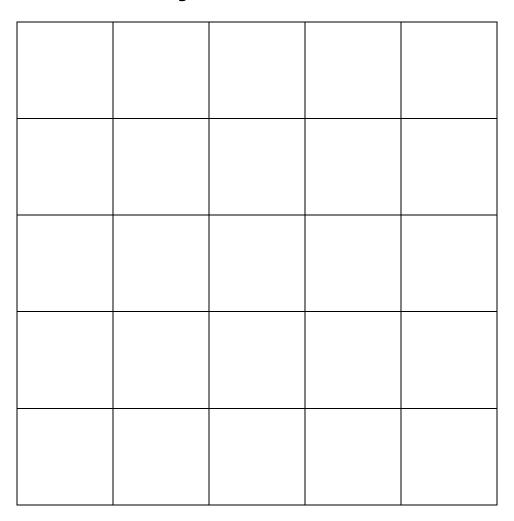
- Inform children that they can select any of the strategies you discussed to figure out the number of pips/dots there are on one side of a dot cube.
- Allow children to get a handful of tiles and a paper plate.
- Flash the side of the cube showing 5 dots for 5 seconds and then cover it up.
- Have children arrange a set of color tiles on their paper plate to match the dot image they saw on the cube.
- When children finish arranging the tiles, uncover the 5-dot image on the cube. Allow children to confirm or correct their arrangement so that it looks just like the configuration of dots they see on the cube.
- Encourage children to share which strategy they used to figure out how to recreate the 5-dot image.
  - "How did you know how many tiles to get? How did you remember the dot image in your head?"
- Record the number sentences children verbalize to describe how they remembered the dot image (e.g. "I saw 2 on top, 1 in the middle, and 2 on the bottom." [2 + 1 + 2]; "I saw 3 going down the middle and then 1 in the bottom corner and 1 in the top corner." [3 + 1 + 1].)
- Count the dots on the cube to confirm that there are 5. Then, have children count the tiles on their plate to make sure they have the same amount.

- Display a set of color tiles, the My 5 Picture recording sheet, and the set of lettered 5-tile arrangement cards (A–P).
- Select a volunteer to work with you as you model the center time/small-group activity.
- One player is the card flasher, and the other player is the tile arranger.
  - 1. The tile arranger counts out a set of 5 tiles—as many fingers on one hand.
  - 2. The card flasher selects a 5-tile arrangement card and counts out loud to 5 as he/she shows it to his/her partner.

- 3. After counting to 5, the card flasher turns over the card and waits as his/her partner arranges the tiles on the recording sheet so that they match the image shown on the card.
- 4. When the tile arranger finishes recreating the 5-tile image, the card flasher turns over the card and helps his/her partner make any necessary corrections or adjustments on the recording sheet.
- 5. Once the partners are sure both images match, the tile arranger records the tile arrangement on his/her recording sheet.
- 6. The players then switch roles.
- As children are playing the game, encourage them to use number sentences to describe the part-part-whole arrangement of the color-tile image (e.g., "I see 4 and then 1 on top.").
- Instruct children to write a corresponding equation on their recording sheet (e.g., 4 + 1 = 5).



## **My 5 Picture**



### **Number Sentence:**

= 5

Card A				Card B				
C	ard	С		Card D				

Card E				Card F					
	C	ard	G		Card H				

	Card J						
	Card K		Card L				

C	Card N						
C	Card O		Card P				

#### Session 7: Part-Part-Whole Relationships with 6

#### **At A Glance**

Children engage in a variation of the attendance chain routine whereby they sort themselves into two age categories—5 and 6 years old—and then represent and compare the data with objects (links), pictures (a bar-type graph), and words (*greater than/less than/as many as*). The focus is on comparing two sets using counting and one-to-one matching strategies (K.CC.6). During math time, children think about the attendance/age data they represented in terms of part-part-whole relationship—how many classmates out of the total present are 5 years old and 6 years old. At center time, children practice describing part-part-whole relationships by representing how many pompom mice they throw inside/outside a ring. The objective is building their fluency with recognizing combinations of 6 (K.OA.3)—a forerunner for developing fluency with fact families.

#### **Common Core State Standards**

#### **Counting and Cardinality**

#### Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.<sup>1</sup>

<sup>1</sup>Include groups with up to ten objects.

#### **Operations and Algebraic Thinking**

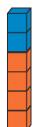
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

MP 4 Model with mathematics.

- Chart paper for attendance sign-in
- Red and blue dot stickers
- Chain links
- Two-column graph for representing the class age data
- Cubes
- Markers
- Pom-poms
- Hula hoop
- Blackline master,
   "Collect 6 Mice Inside/
   Outside Recording Sheet





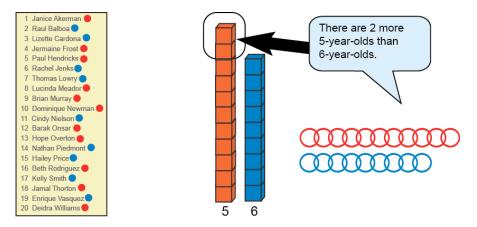


- Have children sign in on the attendance chart in the order they enter the room in the morning. Instruct them to indicate how old they are by placing a colored dot sticker (red for 5 years old and blue for 6 years old) next to their name.
- Display the attendance sign-in chart during group time and discuss the spread (appearance) of the age data.
  - Ask, "What does the sign-in sheet tell us about the ages of your classmates? How many of your classmates are 5 years old? How many have already turned 6? How can we check to be sure how many children in our class are 5 or 6 years old?"
- Take attendance by having children stand and take a link that corresponds to the dot sticker next to their name on the sign-in sheet.
- Tell children with red links (those who are 5) to form one line and the other classmates with blue links (those who are 6) to form another line opposite and parallel to the red line.
- Take a link from every child, counting one by one and forming a chain as you go down each line to determine the number of children in the two age categories.
- Discuss and compare the two groups.
  - "Are there more/fewer children in the class who are 5 years old/6 years old? How do you know?"

#### Engage

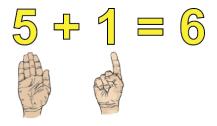
- At math time, revisit the attendance/age data spread sheet and chain-link representation.
- Transfer the records from either one of those data sources to a sheet of chart paper divided in two columns labeled by the respective ages represented in the classroom.
- Start at the bottom of the chart and move up as you make a discrete bar-type graph with each child clearly differentiated as a discrete unit (e.g., one square per child).
- Emphasize the height difference between the two bars as the number that tells/shows how many more children are represented in the taller bar.
  - "How many more children would need to be 5 years old/need to turn 6 years old for there to be the same number in each age group/category?"

- Use formal language to describe the age difference in the classroom.
  - o For example, "There are two more children in the age 5 group than the age 6 group."

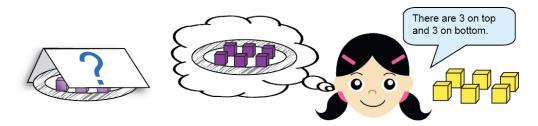


- Join the two chains (the blue and red) to emphasize the total number of children represented on the age survey.
- Count the two chains combined and emphasize the total (e.g., 9 + 12 = 21). Then, discuss and describe the relationship of the parts to the whole (e.g., 9 out of the 21 children in the class are 6 years old; 12 out of the 21 are 5 years old).

- Instruct children to use their fingers to show how old they are.
- Discuss how the 5-year-olds would need to arrange the fingers on both hands to show how old they are when they turn 6.
  - "How many fingers would you have on one hand? How many would you need to show on the other hand? How many fingers would you be showing altogether?"



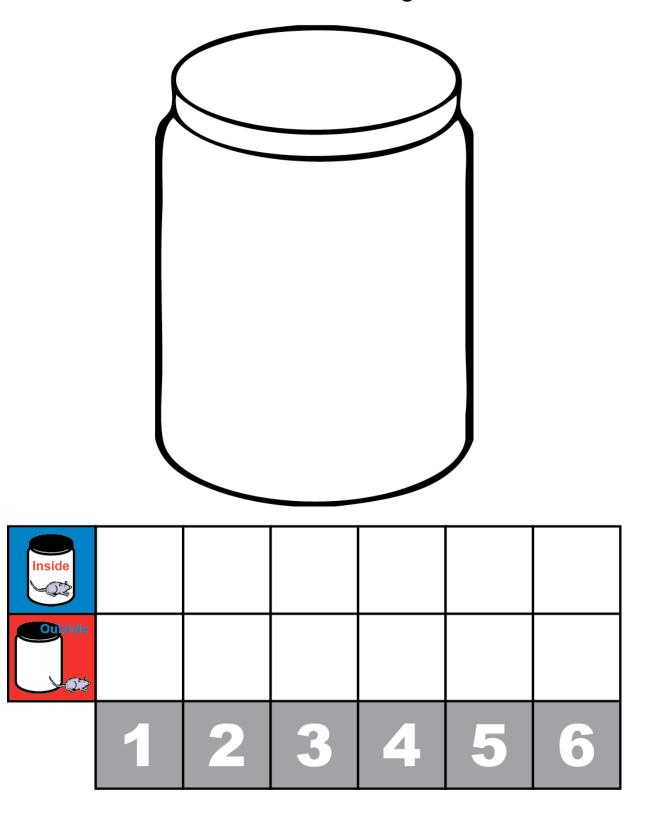
- Use 6 cubes to represent candles for celebrating children's sixth birthday. Arrange the candles in a 2-by-3 array on a paper plate.
- Show the candle arrangement for 5 seconds and then cover it up. Allow children to take a handful of cubes and recreate the image they saw.



- Uncover the arrangement so that children can confirm or correct their configurations.
- Allow children to share how they remember seeing the image in their heads (e.g. "I saw 3 on top and 3 on the bottom."; "I saw 2, 2, and 2 going up and down.").
- Record number sentences on chart paper to numerically represent children's part-partwhole descriptions.
- Inform children that when they are at centers, they will play a game called "Collect 6 Mice."
- Demonstrate how to play the game.
  - 1. Stand at the edge of a strip of tape and try to throw 6 pom-pom mice one at a time inside a hula hoop or a large picture of an empty jar illustrated on chart paper.
  - 2. Use the Collect 6 Mice Inside/Outside Recording Sheet to represent the results of the throw (e.g., 4 inside and 2 outside).
- As children play the game, make a record of all the combinations of 6 they make on a sheet of chart paper (e.g., 4 + 2, 3 + 3, 1 + 5, 6 + 0).
- Prompt children to describe the results of their game in terms of parts to a whole (e.g., "5 of the 6 I threw are inside the jar, and 1 of the 6 fell outside.").



# Collect 6 Mice Inside/Outside Recording Sheet



#### Session 8: Counting Forward and Backward (1-10)

#### **At A Glance**

Children revisit the attendance chain routine, counting and comparing the number of children present at school. The teacher extends the activity by demonstrating how children will uncount themselves as they depart for home, similar to the mouse escape scenario in Mouse Count. The focus is on helping children develop a beginning understanding of subtraction (counting backward) as the inverse operation of addition (counting forward). Children then practice counting forward and backward from 10 as they "finger walk" along their arm. The objective is to help them associate the directionality of the 1–10 sequence of numbers on a number line. At center time, children play a game whereby they count and compare the number of moves they make up and down an elevator inside a 10-story building. The focus is on counting on/back from any number 1–10 (K.CC. 2) and using objects to represent, compare, and keep track of that additive/subtractive count (K.OA.2).

#### **Common Core State Standards**

**Counting and Cardinality** 

Know number names and the count sequence.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

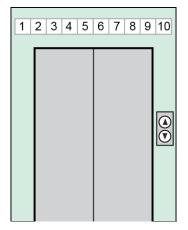
**Operations and Algebraic Thinking** 

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MP 6 Attend to precision.

- Mouse Count by Ellen Stoll Walsh
- Attendance sign-in sheet (chart paper)
- Chain links
- Chart paper
- Markers
- Cubes
- Blackline master for "Going Up, Going Down" game board and 1–10 numeral cards



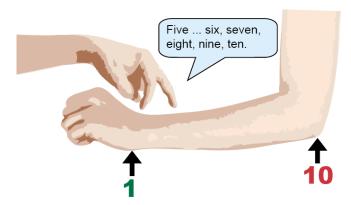
- Have children sign in on a sheet of chart paper as they enter the room in the morning.
- At morning greeting/circle time, post the chart and take attendance in the order that the children signed in.
- As you go down the list, count out loud and construct a chain, connecting a link for each child whose name appears on the list.
- Go back and count the number of links in the chain and then the names on the list, emphasizing how the last counting word you said for both counts is the same.
  - Say, "There are as many links as there are children who signed in on the list."
- Explain to children that as they depart for home in the afternoon, you will uncount them by removing one link from the attendance chain for each child you dismiss.
- Turn to the page in *Mouse Count* where the mice are escaping from the jar.
  - "What number did the mice start with as they uncounted themselves from the jar?
     What was the last number they said?"
- Demonstrate how you will uncount them at dismissal time by starting at the bottom of the attendance chain and removing one chain at a time as you count backward.



#### Engage

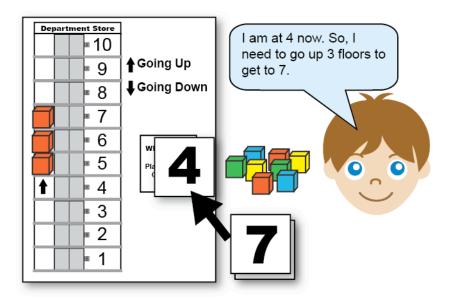
- Display a horizontal 1–10 number line on a sheet of chart paper.
- Tell children to hold their right arm (from wrist to elbow) horizontally in front of their face, similar to the orientation of the number line.
- Inform children that they will model walking along their arm by alternating their pointing and middle fingers back and forth as they recite the forward counting sequence to 10, starting with the number 1 at the wrist and stopping at 10 once they reach the elbow.

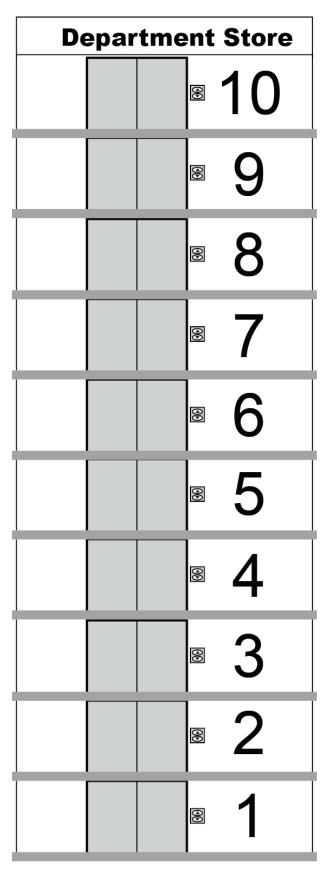
- After modeling how to say one counting word for each step forward along the arm, allow children to practice the counting forward exercise. Point and refer to the number line you made on the chart as they recite and practice.
- Now, direct children to count in reverse as they finger walk backward along their arm, starting at 10 on the elbow and ending at 1 upon reaching the wrist.
- Next, direct children to find a point on their arm that is halfway between the wrist and elbow.
- Refer to the number line on the chart as you prompt children to think about where the number 5 would be located in between the wrist and elbow on their arm.
- Have children finger walk on their arm as they count forward from the number 5.
- If time permits, allow them to use their arms to practice counting forward and backward from any number 1–10 in reference to where that numeral would be located between their wrist and elbow.



- Inform the class that you want to build a 10-story tower with cubes.
  - "Where would the bottom floor be located? What number name would we use to label the bottom floor (e.g., first or one)?
- Refer to the horizontal number line you created on chart paper to help children think about the position of numerals in a vertical translation.
  - "If you were in an elevator, which number would you press to get to the top floor?
     Which number would you press if you wanted to reach the fifth floor?"
- As you build the 10-story tower, direct children to say the corresponding counting word in the 1–10 sequence for each cube you stack.
- Select a volunteer to help you introduce and demonstrate the game, "Going Up, Going Down." One player is the card shuffler and the other the elevator operator.
  - 1. The card shuffler begins the game by shuffling a deck of numeral cards 1–10 and placing them face down on the carpet.

- 2. Then, the shuffler takes the first card off the deck and places it on the game board.
- 3. The other player (the elevator operator) starts at the bottom floor (1) and counts up to that number, placing one cube per floor to keep track of how far he/she goes up.
- 4. The shuffler counts and makes a tower with the number of cubes the elevator operator used to keep track of the first move.
- 5. Next, the shuffler removes another card off the top of the deck and places it on top of the previous card played.
- 6. The elevator operator must either count up or down from the floor he/she is currently on and then count out and arrange another set of cubes on the game board to show how many floors up or down he/she has to move.
- 7. The game continues until the shuffler turns over the 10 card, which gets the elevator operator to the top of the building.
- 8. At the end of Round 1, both players work together combining and counting the number of cubes used to show how many moves up and down the elevator operator had to make throughout the entire trip.
- 9. For Round 2, the players switch roles.
- 10. At the end of the game, the players compare cubes to determine which round required the most moves up and down the elevator.
- Work with and observe small groups of children as they play the game during center time.
- Ask questions to help children think about the number of cubes they need to arrange on the game board to show how many spaces up or down they need to move.
  - For example, "If you are on the 4th floor now, what do you need to do to reach the 7th floor? How many spaces up/down do you need to go?"





**†** Going Up **↓** Going Down

What floor?

Place number card here

1–10 Num "Going Up, G	1	
4	3	2
5	6	7
10	9	8

#### Session 9: Illustrating Counting Books (0-10)

#### **At A Glance**

Children briefly perform a balloon blastoff countdown in which they recite the backward sequence of numbers they learned during Session 8. The teacher then introduces a series of cube towers 1–10, which children order from least to greatest in a left-to-right progression, similar to the sequence of pages in a counting book. The focus is on understanding that each successive number represents a quantity that is one larger (K.CC.4c). Before releasing children to center/small groups, the teacher introduces a counting book that children illustrate with a partner to practice representing quantities to 10 pictorially using sets of objects. While the pairs complete the counting books in small groups, other children count out and use numerals to represent sets of objects presented pictorially in a tens-frame illustrated on counting jar job cards (0–10) (K.CC.3).

#### **Common Core State Standards**

#### **Counting and Cardinality**

#### Know number names and the count sequence.

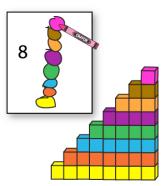
K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a numeral 0-20 (with 0 representing a count of no objects).

#### Count to tell the number of objects.

K.CC.4c Understand the relationship between numbers and quantities; connect counting to cardinality. Understand that each successive number name refers to a quantity that is one larger.

#### MP 6 Attend to precision.

- Balloon
- Mouse Count by Ellen Stoll Walsh
- Cube towers representing quantities 1–10
- 10 blank sheets of paper stapled together in a book
- Markers
- Linking cubes
- Me and My Friend's Jar Count Book (0–10) blackline master
- Counting Jar Job Cards (0–10) blackline master



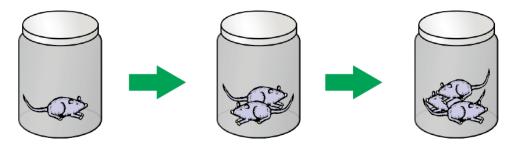
- Show children a balloon and inform them that you will use it to demonstrate a game called "Rocket Countdown."
- Encourage children to count out loud to keep track of how many blows it takes you to fill the balloon. After each blow, pause to catch your breath.
  - Ask, "How many blows have I made so far? How many more blows do you think it will take to fill the balloon completely?"

**Note:** Give each blow the same amount of air so that children can make an accurate estimate for how many breaths are necessary fill the balloon to its capacity.

- Once the balloon is filled, keep your fingers pressed together on the opening to prevent air from escaping.
- Begin a countdown whereby you count backward from 10. Have children count out loud in unison as you model the backward counting process.
  - Recite the chant, "Ten, nine, eight, seven, six, five, four, three, two, one, and ... blastoff!"
- Release your fingers from the balloon and let it shoot up and away.

#### Engage

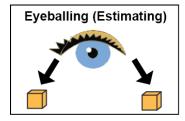
- Refer to the pages in *Mouse Count* where the snake begins dropping 1 mouse at a time into the jar before he stops and realizes that he only has 3 inside.
- Turn to the page where there is only 1 mouse inside the jar.
  - "How many mice did the snake start off with at the very beginning?"
- Turn to the page with the illustration of the jar filled with 2 mice.
  - "How many are in the jar now?"
- Turn to the subsequent illustration showing 3 mice inside and discuss and compare the "one-more" accumulation of mice.
  - "How is the number of mice in the jar changing from page to page?"

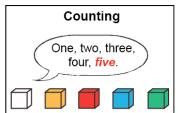


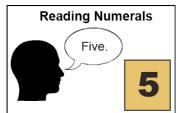
• Show children a cube and explain that it will grow as you add one more cube to the top until it is 10 cubes tall.

- Show children a book with a series of 10 blank pages stapled together. Encourage them to
  provide suggestions on how you could keep track and illustrate the cube tower's one-more
  growing pattern within the pages of the blank book.
- Display a group of 10 cube towers—each representing one unit of growth (1–10).
- Call on volunteers to help you order the towers left to right from least to greatest to represent the sequence of illustrations that will go on each page in the book.
- Challenge them to describe and compare the one-more pattern represented in the horizontal, left-to-right lineup of the 1–10 cube towers.
  - "How do the towers change as we move from left to right?"
- Draw a picture of each tower on every subsequent page of the blank book.

- Display pages of the Me and My Friend's Jar Count Book, which children will work with a partner to illustrate during small-group/center time.
- Call on a volunteer to locate the page that has the numeral 2.
  - "How many mice would I draw on this page? How do you know?"
- Prompt the child to use his/her fingers to show the number of mice that would go inside the jar. Then, ask the child to make two circles inside the jar to represent the quantity specified by the numeral.
- After the child draws the pictorial representation, call on another volunteer to shade in the bottom 2 left-hand boxes inside in the tens-frame.
- Encourage children to review and determine which of the three counting strategies introduced during Session 6 might be most useful for completing the counting book.
- Show the page that has the jar with the numeral 10.
  - "Which counting strategy could you use to illustrate the number of mice that would go inside the jar on this page?"
- Remind children that eyeballing is a good strategy for figuring out small amounts up to 5 and counting one by one is a useful strategy for making larger sets of about 5 or more.

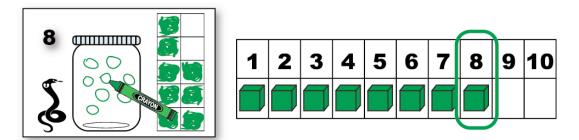




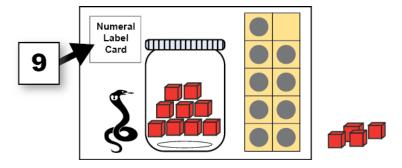


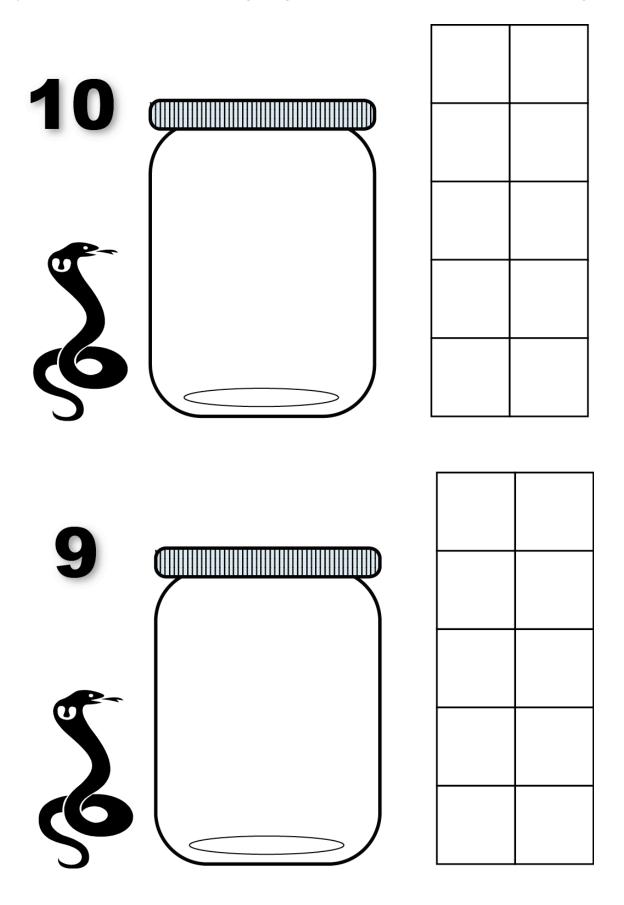
 Introduce the number line as a tool/strategy for counting and creating quantities to match a numeral.

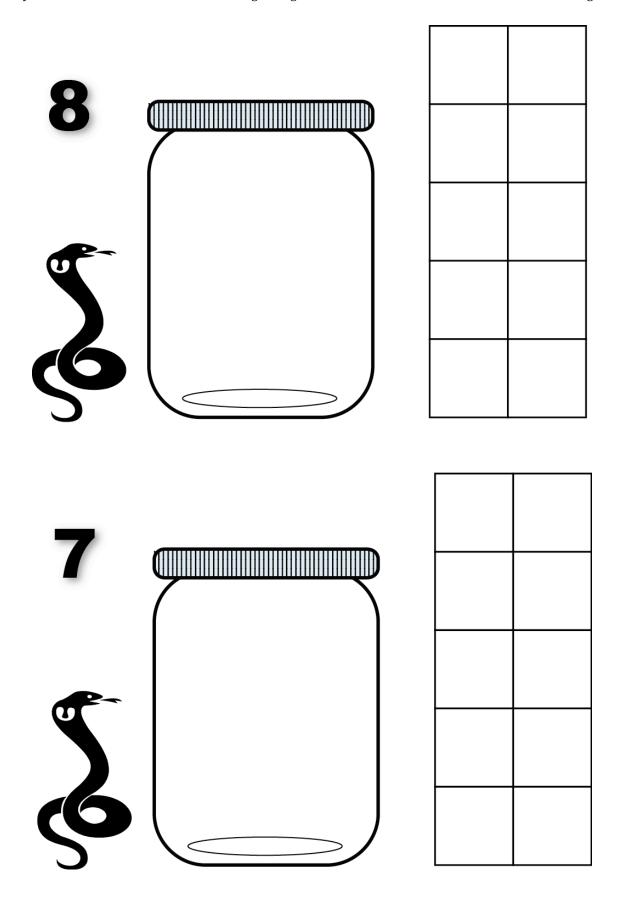
- "If you do not know how to read the numerals on each page, you can refer to a number line and then count out an equal set of objects below each numeral on the number line starting at 1 and stopping at the numeral you are trying to figure out."
- Explain that 0 represents a set with no objects.
- Supervise and observe small groups of pairs as they fill in and illustrate the counting books during center time.



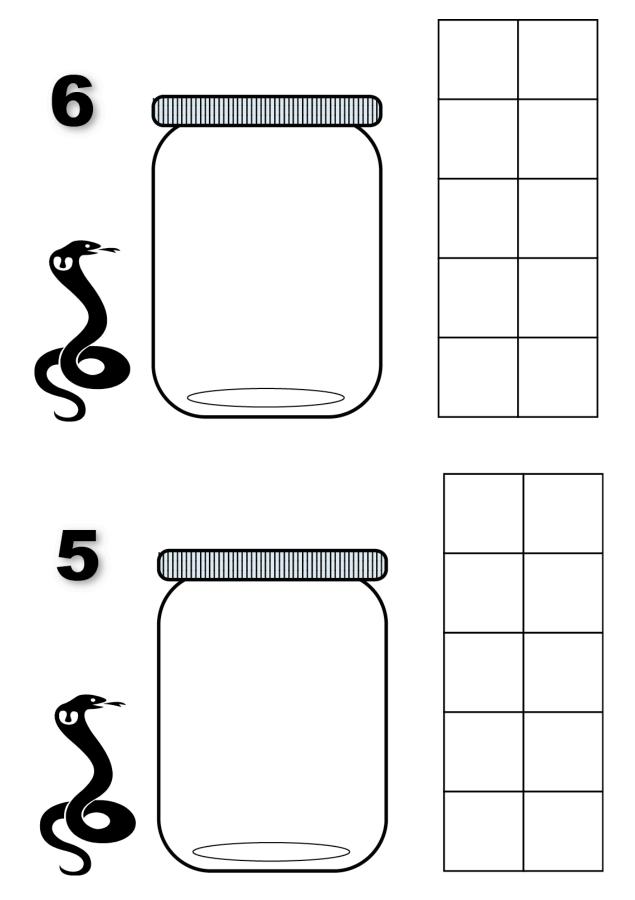
- As one child counts and makes a pictorial representation for each jar per page, his/her partner shades in an equal number of boxes on the empty tens-frame to the side.
- Make sets of the Counting Jar Job Cards (0–10) for children to work on at the math center.
  They count as many cubes onto the job card as the number of dots displayed on the tensframe representation located to the right. Then, children label that quantity with a
  matching numeral card.
- Other center time choices should include the games/activities introduced up to this point in the unit—Going Up, Going Down; Collect 6 Mice; My 5 Picture; and Rescue 10 Mice.



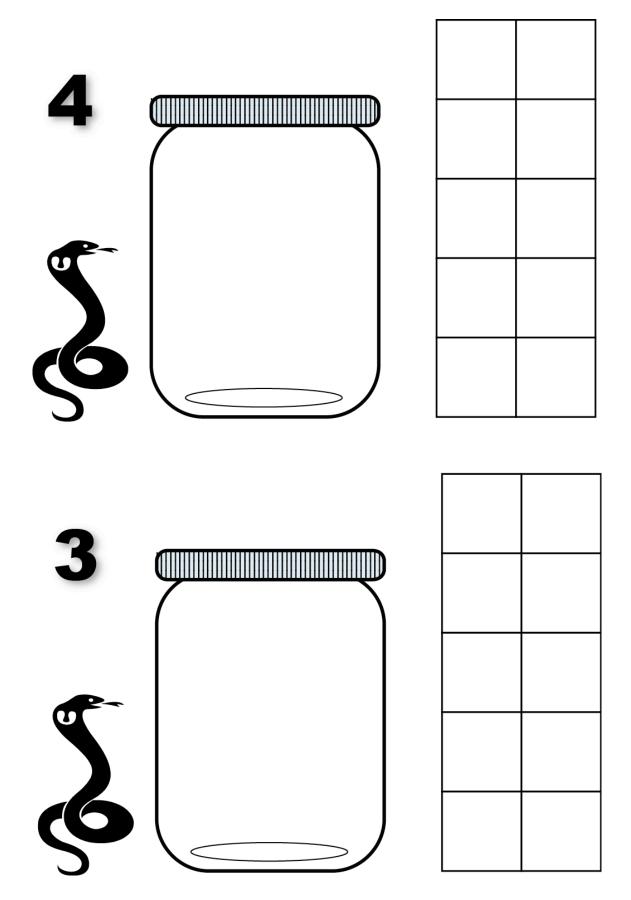




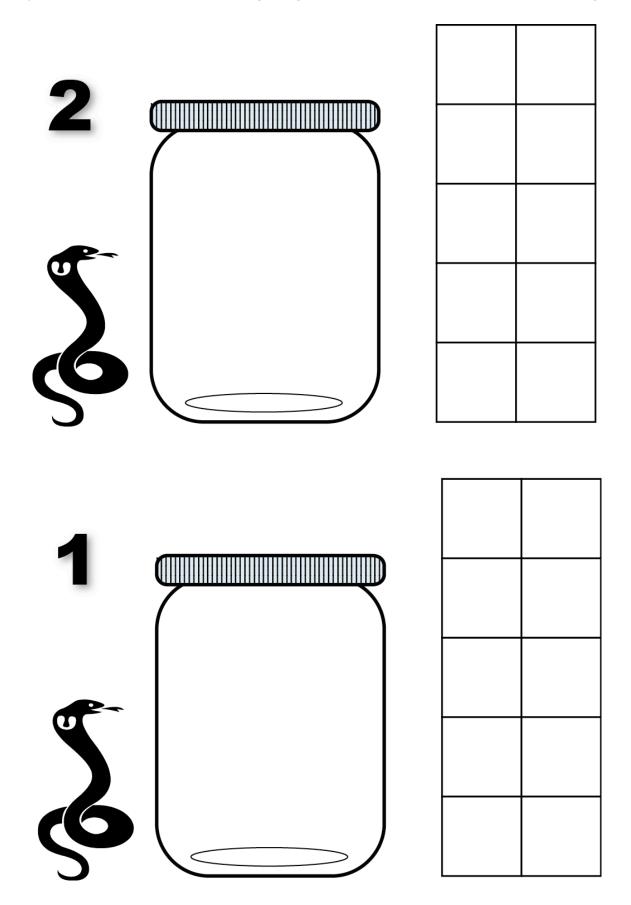
The Charles A. Dana Center at the University of Texas at Austin



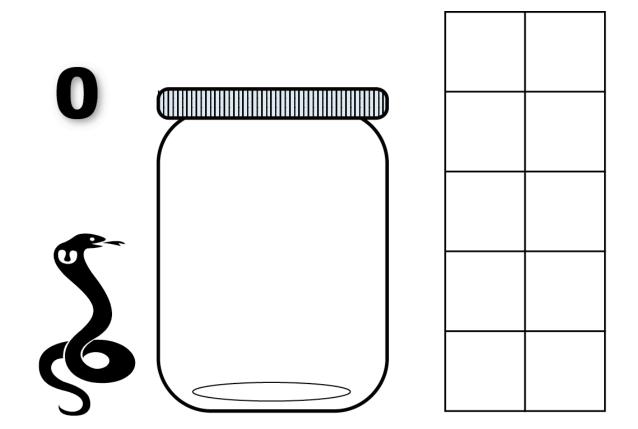
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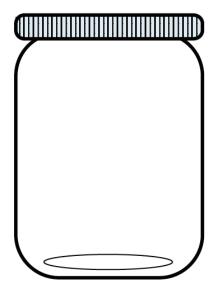
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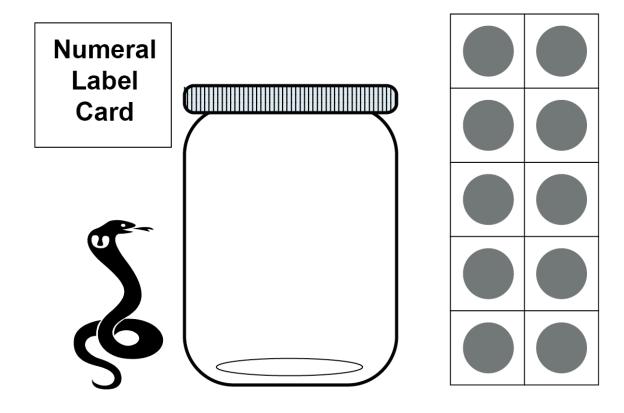


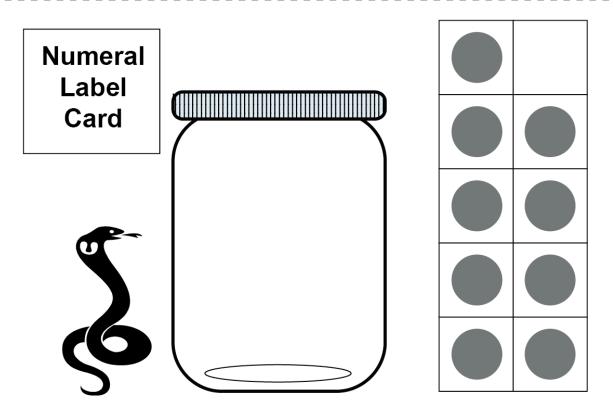
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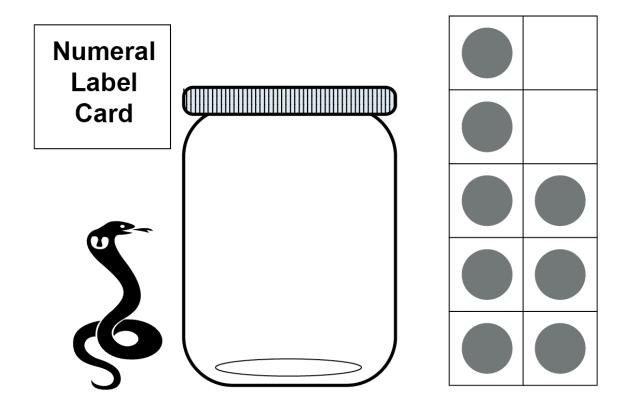


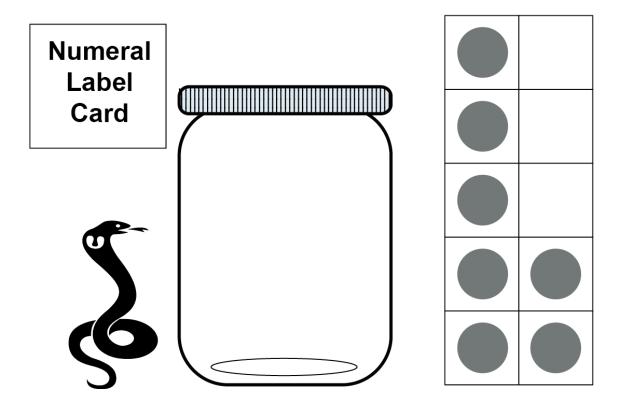
# \_\_\_\_and \_\_\_\_'s Jar Count Book

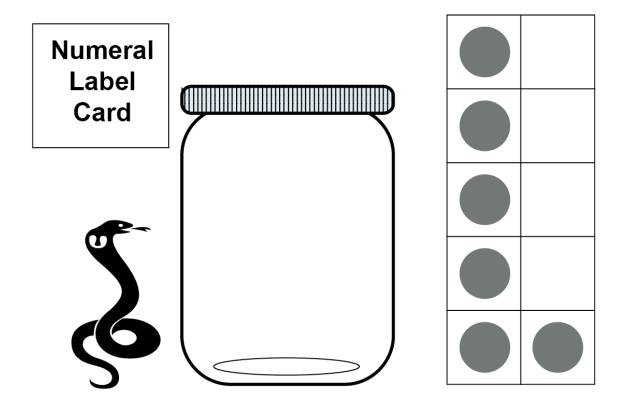




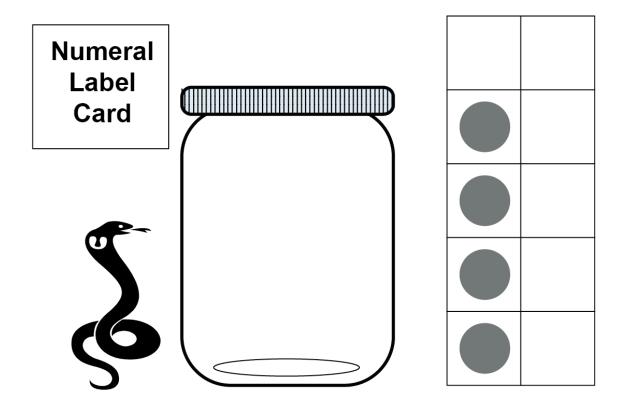




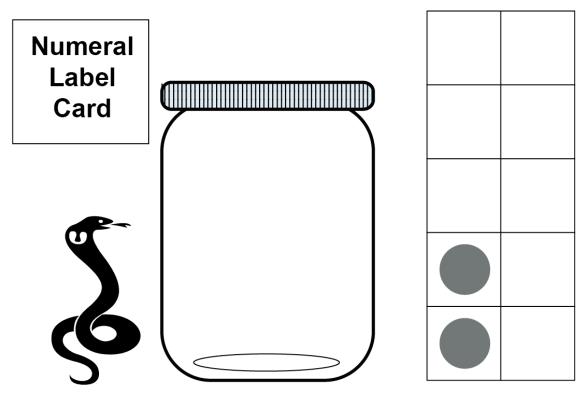




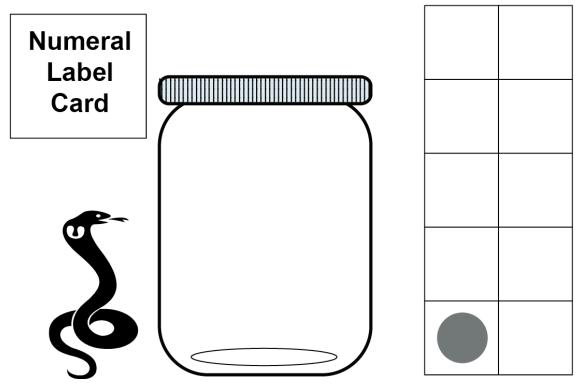
Numeral Label Card

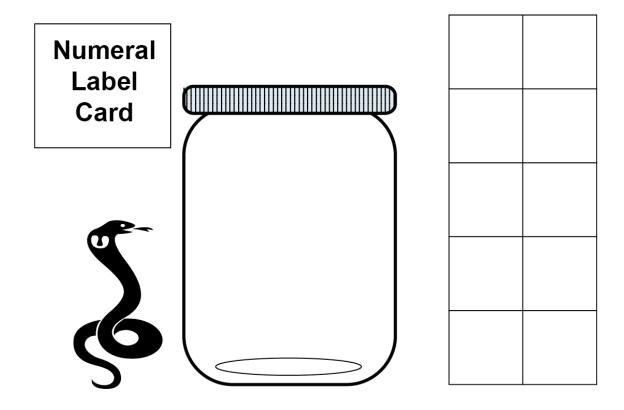


Numeral Label Card



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Note that the for Country of the last ructions: Lam	0						
1	1 2 3 4						
6	7	8	9	10			

#### Session 10: Joining and Separating Quantities to 10

#### **At A Glance**

Children practice mental math as they listen to the teacher drop 3 magnets and then 2 more on top of a cookie sheet that is hiding behind a screen. The teacher prompts children to figure out how many drops he/she made altogether. The goal is to help them to develop visualization strategies for fluently adding within 5 (K.OA.5). Then, the teacher presents two scenarios involving the joining and separation of sets that children act out and then compare to visualize and make sense of how quantities change as a result of various operations (i.e., addition and subtraction) (K.OA.1). Finally, during small groups, the teacher presents additional word problems that children act out using objects and then represent on paper using numbers, pictures, and words. The focus is on representing addition and subtraction using a variety of strategies represented with objects and pictures (K.OA.2).

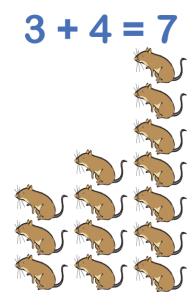
#### **Common Core State Standards**

#### **Operations and Algebraic Thinking**

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

- K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
- K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
- K.OA.5 Fluently add and subtract within 5.
- MP 4 Make sense of problems and persevere in solving them.

- Cookie sheet
- 5 magnets
- Large screen
- Mouse Count by Ellen Stoll Walsh
- Hula hoop
- Chart paper
- Markers
- Cubes or other counters for modeling word problems
- Paper and drawing utensils



- Hide a cookie sheet and a set of magnets behind a screen.
- Instruct children to listen carefully to determine how many magnets you drop onto the cookie sheet. Slowly drop 3 magnets, directing children to lift 1 finger for each drop they hear.
  - Ask, "How many magnets do you think are on the cookie sheet? How do you know?"
- Remove the cookie sheet from behind the screen and display it so that children can confirm their estimate.
- Once children are certain that there are three, tell them to make a mental picture of the number of magnets they see before you hide the sheet and add some more.
- Remind children to continue holding up the 3 fingers they were showing on one hand as
  they lift additional fingers to keep track of the new total. Hide the sheet behind the screen
  and say, "Remember three ..."
- Slowly make two additional drops on the sheet and tell children use their hands/fingers to show how many magnets there are now.
- Remove the sheet from behind the screen to confirm their mental calculation.
- Model the "counting on" strategy.
  - For example, move the 3 magnets slightly to the side and say, "Three ..." Then, count on from that number as you touch each of the two additions one by one: "... four, five ... there are five altogether."



- Inform children that they are going to be actors as they assist you in reenacting one of the scenes from *Mouse Count*.
- Designate a hula hoop as the jar and tell children that you will play the role of the snake.
- Select three volunteers to perform the first scene.
  - Move each child into the hoop as you act out the following scene: "The snake found
    3 mice, little warm and tasty. He counted them into the jar. Mouse count! One ... two
    ... three."

- Now invite four additional volunteers to act out Scene 2.
  - "But 3 mice were not enough, so the snake found 4 more mice—little, warm, and tasty."
  - Move each of the four additional actors inside the hula hoop as you continue the story:
     "Four, five, six, seven."
- Solicit responses from the audience, prompting them to determine the new total.
  - "How many mice are inside the jar altogether?"
- Count all of the actors sitting inside the hoop to confirm that there are seven.
- Inform children that you and the 7 actors will now reenact a new version of the story.
  - "The snake was really sleepy and decided to take a nap. As he was sleeping, 2 mice escaped from the jar."
- Select two actors to stand up and leave the jar.
- Solicit responses from the audience in reference to the new total.
  - "How many mice are inside the jar now?"
- Count each child sitting inside the hula hoop to confirm the audience's response.
  - "After the 2 mice escaped, 5 mice were still inside before the snake woke up."
- Direct the 7 actors to stand and take a bow as the rest of the class applauds.

- Begin a discussion to challenge children to think about how the number of mice inside the
  jar changed as a result of two distinct actions—one which involved a joining together of two
  sets and the other a separating of one quantity from a larger sum.
- Illustrate the scenes from the two stories on a sheet of chart paper. Make three boxes to represent each scene—the beginning (start), middle (change), and end (result).
- Refer to the sequence of events from the first story you illustrated on the chart.
  - "How did the number of mice change from start to finish? Did the number get smaller or larger? What happened or what did the snake do to make the number of mice inside the jar get larger?"
- Write an equation to represent the joining action modeled in the story (3 + 4 = 7).
- Then, refer to the sequence of events from the second story—the one that involved the separation of a smaller quantity form a larger sum.
  - "How did the number of mice change from start to finish? Did the number get smaller or larger? What happened to make the number of mice inside the jar smaller?"
- Write an equation to represent the separating action modeled in that story (7-2=5).

# Beginning Middle End Beginning Middle End 3 + 4 = 7 7 - 2 = 5

- Explain that during center time, you will gather together a small group of children and tell two different number stories—one about joining and another about separating.
- Adjust the numbers you insert in the stories to accommodate the range of children's counting skills. Observe the strategies they use to act the number stories.
  - Does the child directly model each scene with the counters and then count each distinct quantity before determining the total or difference?
    - **Example for joining:** The child counts out a set of 3 counters, another set of 4, and then joins the two sets, starting at 1 and counting all.
    - **Example for separating:** The child makes a set of 7 counters, counts out/removes a set of 2, and then counts the remaining 5 to determine how many are left.
  - Does the child count on/backward from the first number, keeping track of the number of counts forward/backward to determine the total or difference?
    - **Example for joining:** The child makes a set of 3 and then counts on as he/she adds the additional 4 (e.g., "three ... four, five, six, seven").
    - **Example for separating:** The child counts out 7 and then counts backward as he/she removes the two that escaped (e.g., "seven ... six, five ... there are five.").
- As you work with small groups, allow the other children to select among activities/games that were introduced in past sessions—Going Up, Going Down; Rescue 10 Mice; Collect 6 Mice; My 5 Picture; Counting Jar Books; and Counting Jar Job Cards (0–10).

# **Session 11: Counting Beyond 10**

# **At A Glance**

Children revisit the listen-and-count exercise you introduced in Session 10 during which they used auditory strategies to keep track of a growing number of magnets dropped on top of a cookie sheet. In this session, however, the teacher models a counting backward scenario whereby children make a mental representation of the number of pennies removed from the cookie sheet and dropped into the jar. The focus is on helping them develop mental math strategies to fluently subtract within 5 (K.OA.5). Next, the teacher introduces the book *Caps for Sale* and prompts children to compare the number of caps presented in the story to the benchmark number 10. At center time, children then use this information to represent the number of monkeys in the story. The focus is on representing quantities more than 10 (K.CC.5) and using counting and matching strategies to compare (K.CC.6).

## **Common Core State Standards**

#### **Counting and Cardinality**

#### Count to tell the number of objects.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

#### Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies<sup>1</sup>.
 <sup>1</sup>Include groups with up to ten objects.

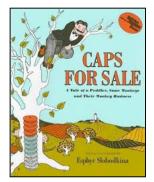
#### **Operations and Algebraic Thinking**

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.5 Fluently add and subtract within 5.

#### MP 4 Model with mathematics.

- Cookie sheet
- Jar of pennies
- Screen
- Caps for Sale by Esphyr Slobodkina
- Linking cubes
- Paper
- Writing utensils



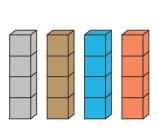


- Display a jar of pennies and the cookie sheet you introduced during Session 10. Hide the cookie sheet and the jar behind a screen.
- Slowly drop 4 pennies from the jar one at a time on top of the sheet. Instruct children to listen carefully, lifting 1 finger for each drop.
  - Ask, "How many pennies did you hear fall onto the cookie sheet? How do you know?"
- Display the configuration of pennies you dropped on the sheet and confirm that there are four
- Instruct children to keep their 4 fingers raised as they listen to you remove some of the pennies.
- Remove 2 pennies from the sheet and drop them one at a time back into the jar. Remind children to listen carefully, lowering 1 finger for each penny you remove.
  - "How many pennies are on the sheet now? How many pennies did I place back inside the jar? How did you count in your mind to figure out how many I dropped back inside the jar?"
- Remove the cookie sheet from behind the screen to confirm the new total.
- Recap/summarize the counting backward strategy you want children to visualize.
  - "First there were 4 [show 4 fingers] and then I counted backward—[begin to lower 1 finger for each penny removed] three and two ... two pennies are left on the sheet."



- Introduce *Caps for Sale*. If the class is unfamiliar with the book, conduct a brief picture walk to establish some background information (e.g., setting, characters) useful for comprehension.
- As you read the book, pause in sections to highlight quantitative features related to the way the vendor stacks and keeps track of the number of caps on his head.
  - "How many red caps does he have? What do you notice about the number of caps in each color group?"

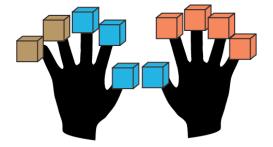
- After reading the story, use linking cubes to demonstrate the way the vendor arranges the caps in groups. Call on four volunteers to count out 4 cubes of each color—red, blue, brown, and grey (substitute black or purple cubes for grey).
  - "Which group of caps did the vendor stack on top of his head first? Which group came second/third/last (on top)? How many altogether?"

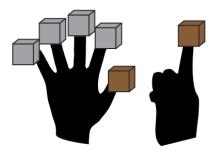




- Compile each group of cubes (caps) into one stack in the order specified in the story.
- Prompt children to count out loud as you point to each cube to verify the total. Emphasize the last counting word (16) as the number that tells how many caps the vendor had on top of his head.

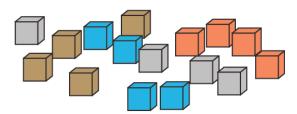
- Stand the stack of cubes on the carpet to prompt children to visualize the magnitude of 16.
- Call on two volunteers to stand. Inform the rest of the class that the volunteers will use their fingers to play the part of the monkeys in the story.
- Instruct the first volunteer to raise all 10 fingers so that you can place a cube on each one. Once you have placed a cube on each finger, count to verify that only 10 cubes have a match so far.
- Begin matching the remaining cubes to the second volunteer's set of fingers—all 5 fingers on one hand and only 1 finger on the other. Count the number of fingers that you filled on the second volunteer's set of hands.
- Write the equation 10 + 6 = 16 so that children can visualize 16 as 10 ones and 6 additional ones.





- Inform children that during center time, each child will create a pictorial representation to show the number of monkeys in the story.
- As the rest of the class works at other math stations, observe a small group of children completing the pictorial representations.
- Provide counters for children to use to represent each monkey in the illustration.
- Pay attention to the strategies each child uses to determine how many monkeys to draw.
  - Does the child count out a set of 16 counters and then remove 1 counter at a time as they draw each monkey?
  - Does the child draw a set of monkeys and then count to be sure there are 16? If so, how does he/she determine how many more he/she needs to make or if he/she needs to scratch off extras?
- When children finish their drawing, instruct them to use one-to-one correspondence to compare the stack of 16 cubes you counted to represent the caps from the story to the number of monkeys in their drawing.





# **Session 12: Finding the Missing Part of 10**

# **At A Glance**

Children reenact the scene from *Caps for Sale* where the monkeys steal a cap from the vendor. The teacher uses the attendance chain/sign-in to determine how many paper plate caps are needed so that each child can play the part of a monkey. The focus is on using counting strategies to make an equivalent set for numbers greater than 10 (K.CC.6). Then, the teacher introduces a part-part-whole word problem whereby children use a variety of strategies to determine the missing part of a set of 10. During center time, children practice determining other two-part combinations of 10 as they solve additional word problems and play a game in which they bowl and count the number of pins (out of 10) they knock down. The objective of both activities is to use pictures, objects, and equations to determine for any number 1–9 which number makes 10 when added to that number (K.OA.4).

# **Common Core State Standards**

#### **Counting and Cardinality**

#### Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies<sup>1</sup>. <sup>1</sup>Include groups with up to ten objects.

# **Operations and Algebraic Thinking**

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

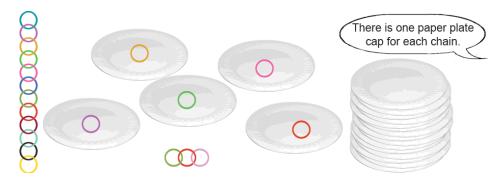
K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

MP 1 Make sense of problems and persevere in solving them.

- Paper plates
- Attendance chain or morning sign-in
- Cubes
- Chart paper
- 10 bowling pins or empty water bottles partially filled with sand
- Blackline master,
   "Bowling for 10 Score Sheet"
- Dot stickers
- Paper and writing utensils



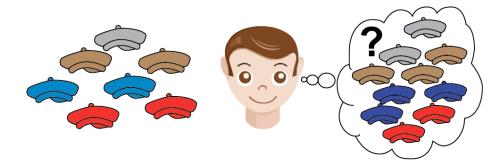
- Inform children that they will help you reenact a scene from *Caps for Sale*, which you introduced during Session 11. To do so, explain that you need to know the number of children present since each of them will play the part of a monkey and, therefore, need a cap.
  - Ask, "How many caps do I need? How do you know? How could we find out?"
- Refer to the morning sign-in sheet or the attendance chain as you count to determine how many paper plate caps to stack on top of your head.
- Instruct children to go around the circle and count off to make sure that the last number the final child says matches the number of paper plates you just counted.
- Place the paper plates on your head and reenact the part of the story where the vendor takes a nap while the monkeys steal his caps.



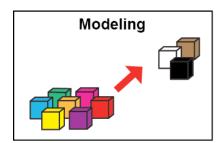
- Instruct each child to come up one at a time and remove a plate from the top of your head. Confirm that every child has a cap.
- Act angry and throw your cap on the carpet, which signals children to do the same just as the monkeys did in the story.
- As you gather the caps, count and stack each one back on top of your head. Confirm that you still have the same number of caps you started off with.
- Have children applaud their role as monkey actors and good counters.

- Gather suggestions from children about what the vendor from the story could do anytime he takes a nap to be sure of how many caps he has so that when he wakes up, he knows how many he should look for in case any monkey took some while he was asleep.
- If no one suggests, tell children that the vendor could write the number down on a sheet of paper or make a picture to show how many if he does not know how to read numerals.
- Present the following word problem: "Suppose the vendor has only 10 caps left to sell and decides to take a nap. Then, when he awoke and counted to be sure he still had 10, he discovered he only had 7. How many caps are missing?"

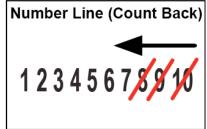
- Pose questions to help children visualize and recall what happened in the word problem.
  - "How many caps did the vendor have before he went to sleep? How many caps did he have when he woke up? Would he have more or fewer caps when he woke up if some of them were missing?



- Encourage (but do not insist) children to use cubes or their fingers to determine how many caps were missing in the stolen cap scenario presented.
- Direct children to pair up with a partner and share how they solved the word problem.
- As children share, pay attention to and look for the following strategies:
  - Direct modeling/counting: The child counts out 10 counters. Then, he/she counts 7 to represent how many caps were not stolen. Finally, he/she counts the leftover cubes to determine how many caps are missing.
  - Counting on: The child uses his/her fingers instead of the cubes to count on from 7 (e.g., seven ... eight, nine, ten) lifting 1 finger for each count forward. The child understands that the 3 fingers he/she lifted represent the missing caps.
- Make a pictorial representation on chart paper of the different ways you observed children solve the word problem.
- On the chart, include a number line model that illustrates a "counting back" strategy. For example, start at 10 on the number line and scratch off numerals as you count 3 numbers back—10, 9, 8. Connect the numerical model to the informal strategies children shared (e.g., count and circle the remaining 7 numerals and circle the 3 numerals you crossed out, emphasizing that they represent how many caps were missing).

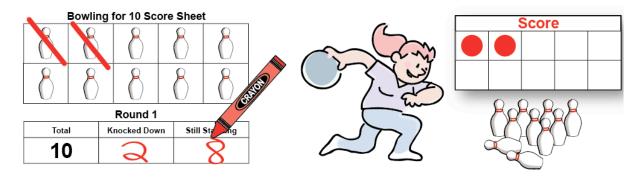




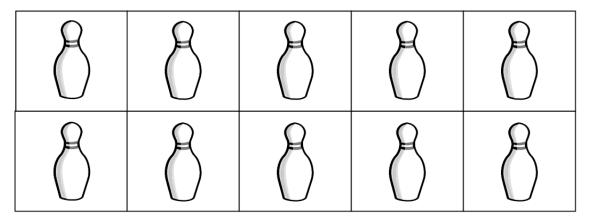


Before dismissing children to centers, demonstrate how to play "Bowling for 10."

- Arrange 10 bowling pins (empty water bottles partially filled with sand to provide weight) in a 4-3-2-1 descending row configuration. Adhere 10 dot stickers to the carpet to indicate where to place each pin. Children play in pairs.
  - 1. One child stands behind a line and rolls a ball toward the pins.
  - 2. The other player monitors the pins and removes the ones that were knocked down.
  - 3. Both players count and represent the results of the roll on the Bowling for 10 Score Sheet, crossing out one square for each pin that was knocked down and writing a corresponding numerical equation (e.g., 10 4 = 6).
  - 4. Then, the players rearrange the pins (making sure all 10 are standing) and switch roles (e.g., roller and counter).
- As children work at math stations, call together a small group to continue solving additional "stolen cap" word problems, each starting out with a sum of 10.

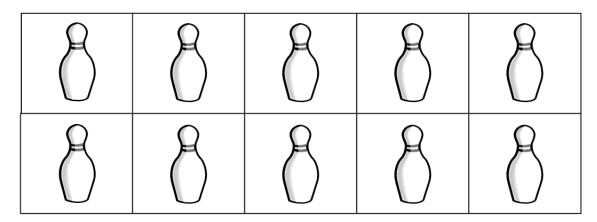


# **Bowling for 10 Score Sheet**



# **Round 1**

Total	Knocked Down	Still Standing
10		



# Round 2

Total	Knocked Down	Still Standing
10		

# **Session 13: Combining and Adding Within 10**

# **At A Glance**

This session begins as the teacher records and orders the scores children generated during the bowling game they played in Session 12 from highest to lowest on a sheet of chart paper. The purpose of this warm-up activity is to help children think about and compare the different two-part combinations that make a 10 (6 and 4, 5 and 5) (K.OA.4). Next, the teacher introduces a 5-by-5 coordinate grid whereby children combine two numbers—the number of moves over and the number of moves up—to determine how many squares to travel altogether to reach his/her assigned coordinate. For example, to reach the square that is 3 spaces to the right and 2 spaces up the grid, the child moves a total of 5 spaces. During center time, children play a similar game with a set of colored cubes. The focus is on using strategies (counting all and counting on) to determine the sum of any two numbers up to 10 (K.OA.2).

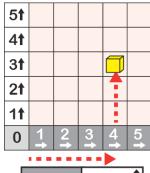
## **Common Core State Standards**

**Operations and Algebraic Thinking** 

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

- K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
- K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
- MP 2 Reason abstractly and quantitatively.

- Children's Bowling for 10 Recording Sheets from Session 12
- Chart paper
- Marker
- Sticky notes
- Linking cubes
- Blackline master, "Chase the Monkeys Coordinate Game Board"
- Blackline master, "Coordinate Cards"



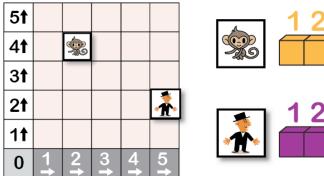


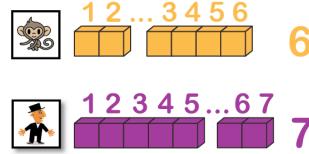
- Encourage children to share some of their scores from the Bowling for 10 game they played during Session 12.
  - Ask, "How many pins were knocked down? How many were left standing?"
- Generate a list on chart paper of all the combinations of 10 children scored during the game, starting with the highest possible score at the top of the list and moving down in descending order (10 and 0, 9 and 1, 8 and 2, 7 and 3, 6 and 4, 5 and 5, 4 and 6, 3 and 7, 2 and 8, 1 and 9, and 0 and 10).
- Determine whether anyone made a perfect score, knocking down all 10 pins with one roll.
  - "If you were able to knock down all of the 10 bowling pins, how many pins would remain standing? How do you know?"
- If no one responds, explain that no pin is left standing if all 10 are knocked down. Highlight the numeral 0 you use to represent no pin left standing.

Total	Knocked Down	Still Standing
10	10 •••••	0
10	9 •••••	1 •
10	8 •••••	2 ••
10	7 •••••	3 •••
10	6 •••••	4 ••••
10	5 ••••	5 ••••
10	4 ••••	6 •••••

- Create a 5-by-5 matrix/array of squares on a separate sheet of chart paper. Label each square at the bottom of the grid with a numeral, starting at the far left with 1 and moving right all the way to 5. Do the same for the vertical ascension of squares on the grid, starting at the bottom of the far-left side with 1 and moving up to 5.
- Explain to children that they will play a game in which the vendor from *Caps for Sale* tries to catch one of the monkeys who stole his caps.
- Draw a picture of the vendor on a sticky note and a monkey on a separate note. Stick both notes on the 0 coordinate of the grid.
- Display a coordinate card and make the monkey go that many spaces over (e.g., 2) on the horizontal number line and up that many squares on the grid (e.g., 4) right above the numeral where he landed on the horizontal run.
- Invite children to stand and pretend to make the same movements. Model the movements for them.

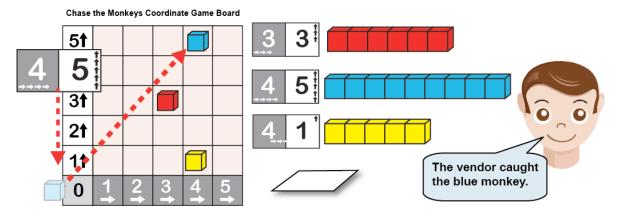
- 1. First, turn so that your right is the same as children's right.
- 2. Then, make as many hops to the right as the monkey did. Count out loud as you hop (e.g., hop to the right ["one"] and another hop to the right ["two"]).
- 3. Next, pretend to climb up the grid as many squares as the monkey did, alternating your hands up and down and counting out loud as you climb (e.g., right hand up ["one"], left hand up ["two"], right hand up ["three"], and left hand up ["four"]).
- Repeat the same procedure (counting and hopping/climbing) as you move the vendor to chase the monkey.





- Instruct children to sit down and help you count the number of squares the monkey traveled altogether over to the right and up the grid.
- While you count, model a "counting on" strategy, counting forward ("one, two") on the horizontal run and then continuing without starting over at one ("three, four, five, six") as you turn and count up the grid.
- Do the same for the vendor's run and then compare which player moved the most spaces over and up the grid.
  - "Who had to go the farthest? How do you know?"
- Demonstrate a game called "Monkey Chase," which the children will play during math stations/center time. Display the game board on the carpet and shuffle the deck of coordinate grid cards, turning them face down. Take a red, yellow, blue, and black cube and remove three cards off the top of the deck of cards. The red, yellow, and blue cubes are the monkeys, and the black cube represents the vendor.
  - 1. Assign a coordinate card to each monkey cube.
  - 2. Count forward (left to right) and up (bottom to top) as you place each cube on the grid according to the coordinates on the card.
  - 3. Use cubes to determine which color traveled the most squares over and up the grid.

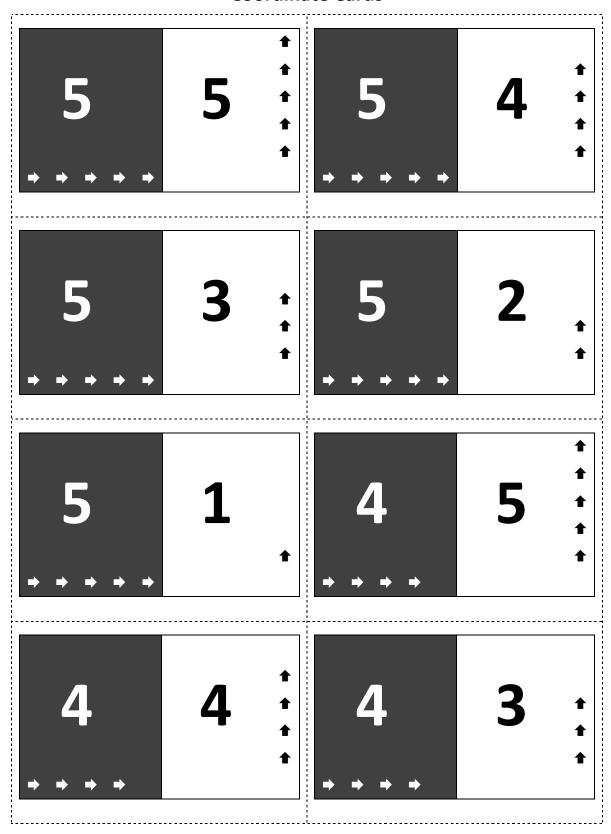
- 4. Shuffle the three cards you used to move the monkeys on the board. Turn the cards over and remove one off the top to determine where the vendor (the black cube) goes.
- 5. The monkey with the same coordinates as the vendor gets captured.

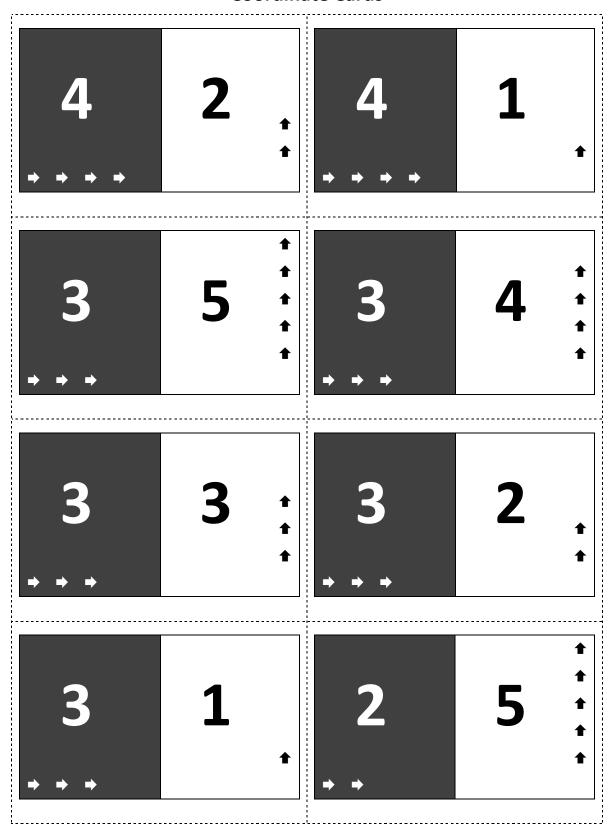


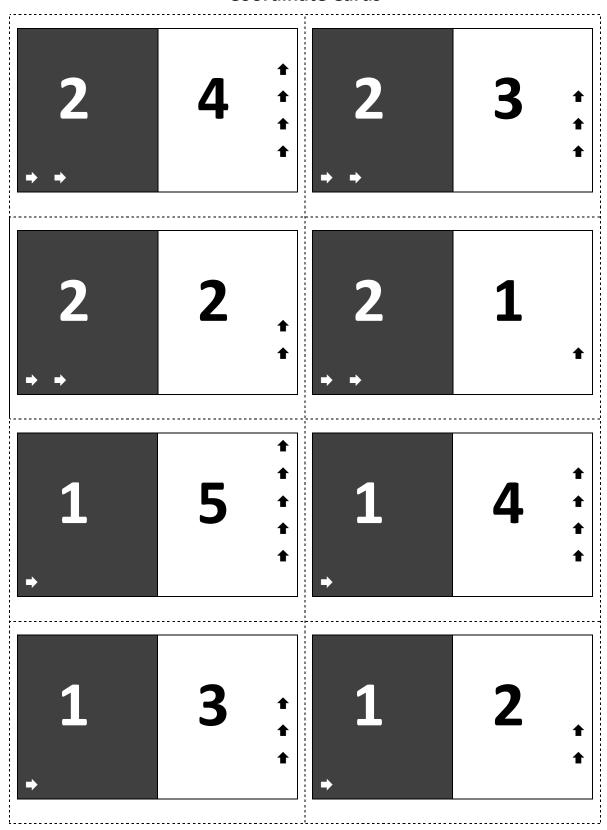
• Closely monitor small groups of children as they play the game, prompting them to think about how many moves over *and* up they need to go for each monkey.

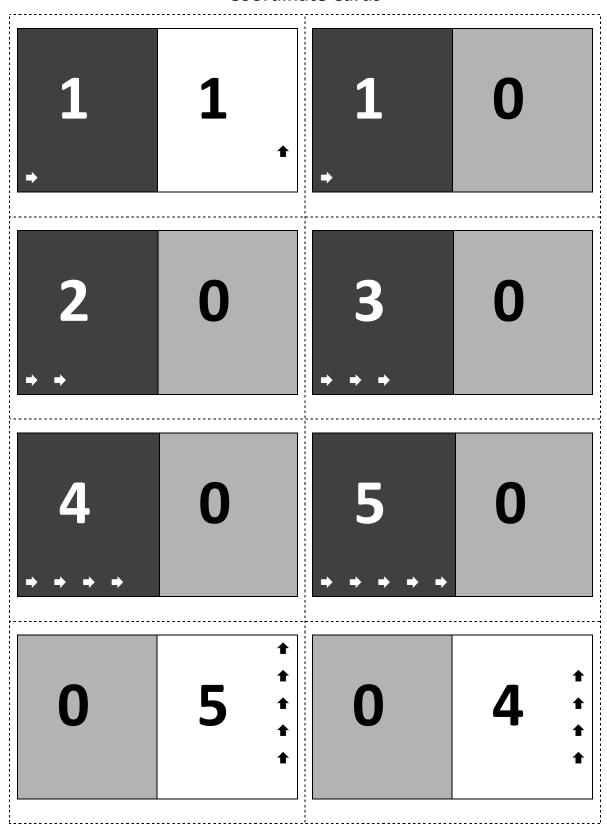
# **Chase the Monkeys Coordinate Game Board**

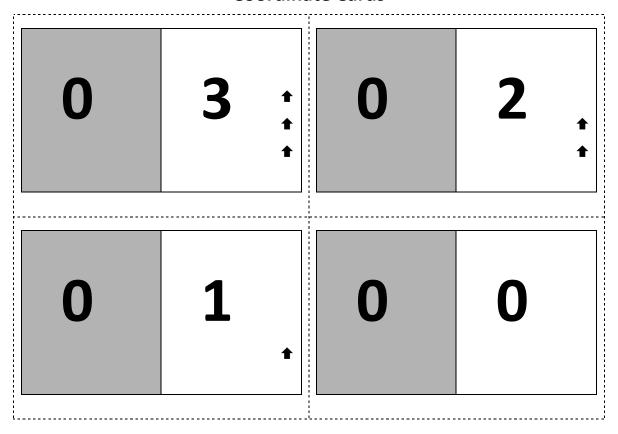
5₊					
4.					
3₊					
2.					
1.					
0	1	2	3	4	5











# Session 14: Adding and Subtracting Within 10

# **At A Glance**

Children revisit the 5-by-5 coordinate grid introduced in Session 13 whereby they use a mental "counting on" strategy to combine a pair of numbers (3 and 2) with a sum of 5 (K.OA.5). The teacher then introduces a missing part problem with a sum of 10 for which children must figure out the difference—the distance between the greater number (10) and the smaller number (4). The focus is on using strategies to find the number that makes a 10 for any number 1–9 (K.OA.4). During center time, children play a new game, rolling a 0–5 dot cube to make a collection of 20 cubes. The objective is to help children practice counting quantities to 20 and decompose numbers to 5 to split a roll as they finish one collection of 10 and begin another (K.OA.3).

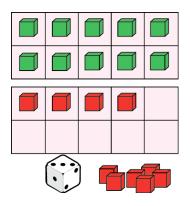
## **Common Core State Standards**

**Operations and Algebraic Thinking** 

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

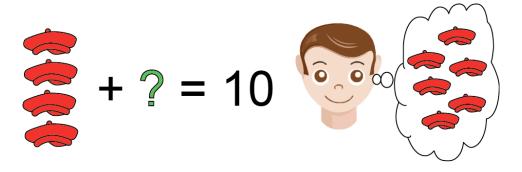
- K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
- K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
- K.OA.5 Fluently add and subtract within 5.
- MP 1 Make sense of problems and persevere in solving them.

- 5-by-5 coordinate grid chart introduced in Session 13
- Linking cubes
- Paper
- Writing utensils
- Chart paper
- Blackline master, "Collect 20 Caps" game board
- Blackline master, "0–5
   Dot Cube"

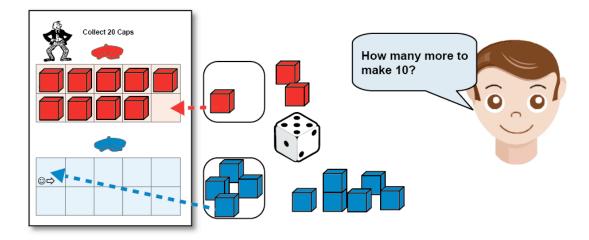


- Post the 5-by-5 square coordinate grid you created during Session 13.
- Display a coordinate card that shows the ordered pairs 3 (horizontal, left to right) and 2 (vertical, bottom to top).
- Instruct children to stand and demonstrate with their bodies how to move on the grid in order to reach the coordinates shown on the card.
- Turn with your back facing children so that your left matches their left.
- Direct children to hop and count over and then count on as they climb up (e.g., hopping 3 steps to the left and counting by ones, "one, two, three ...," and then alternating their hands up and down and counting on as they stop and pretend to climb up, "... four, five."
- Emphasize the sum of the two separate moves—over and up.
  - Say, "3 hops over and 2 climbs up makes 5 moves altogether."

- Prompt children to recall how many caps of each color the vendor from *Caps for Sale* carried around on his head.
- Make a stack of 4 red cubes to represent 1 of the 4 equal groups of caps.
- Display the Combinations of 10 chart you created at the beginning of Session 13.
- Encourage children to refer to the chart as you ask the following:
  - "How many more red caps does the vendor have now? What would the vendor need to do in order to have 10 red caps instead of 4?"
- Provide guiding questions that lead children to develop a mental picture of what they
  already know about the two numbers presented in the scenario and what they need to do
  in order to figure out the difference.
  - "Is 10 greater than or less than 4? How do you know? How many more caps would the vendor need to add to the 4 he already has?"



- Allow children to work with a partner using cubes or fingers or drawing a picture to figure out the problem introduced in the Engage section.
- As children solve the problem, observe and make note of the strategies they use.
  - 1. **Direct modeling (counting all):** The child works backward, counting out a set of 10 cubes first and then removing 4 to represent the number of caps the vendor already has. Next, the child counts the 6 cubes that remain to determine how many more the vendor needs to get.
  - 2. **Counting forward (counting all):** The child counts out 4 cubes and then makes a separate pile as he/she counts forward, adding a cube for each count until he/she reaches 10. The child then counts the number of cubes in the separate pile to determine how many more the vendor needs to get.
  - 3. **Counting on:** The child says 4 and then raises 1 finger for each count forward to 10 (e.g., "four ... five, six, seven, eight, nine, ten.") The child knows that the number of fingers he/she lifted (6) represents the number of caps the vendor needs to get.
- After children have finished solving the problem, allow volunteers to share their work.
- Make a pictorial representation on chart paper of the different strategies you observed, such as those outlined above.
- Before dismissing children to math stations/centers, demonstrate how to play a new game, "Collect 20 Caps."
  - 1. Display the game board, a set of blue and red cubes, and a 0–5 dot cube.
  - 2. The goal is to continue rolling the cube and adding that many cubes to the total already collected on the game board.
  - 3. Once children have filled the first tens-frame with one color (red), they need to begin collecting blue cubes for the second tens-frame.
- As children work at math stations, pull together a small group and play the Collect 20 Caps game.
- Prompt children to think about the growing total and how many more cubes they need to finish one stack of 10 red/blue cubes.
  - For example, "If you have 7 red cubes now, how many more do you need to have 10?"
     How do you know?"
- If a child has almost completely filled one frame (e.g., 9 red cubes) and rolls a 2 or greater, prompt him/her to think about how to split the roll between the two colors. For example, if the child rolls a 5, he/she needs to get 1 more red cube and then begin filling the second frame with 4 blue cubes.





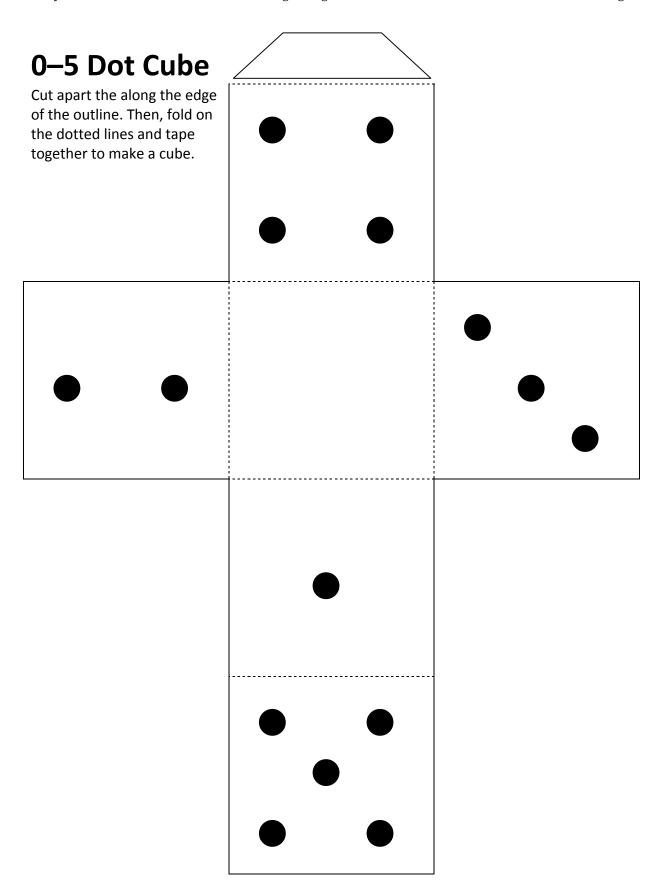
# **Collect 20 Caps**



⊕⇒		



⊕⇔		



# Session 15: Making a Set of 10 and Some More

# **At A Glance**

Children continue to develop fluency adding and subtracting within 5 as they use their fingers to represent an arrangement of 0–5 dots and then count on from that number to make a 5 (e.g., three ... four, five) (K.OA.5). Next, the teacher rolls three dot cubes simultaneously and asks children to estimate whether the combined total is greater than, less than, or equal to 10. After a brief discussion on how to represent and describe numbers 11–19 in a way that focuses on the base-10 structure within those numbers, children continue to practice during center time adding combinations of dots and making representative cube towers. After comparing the cube towers they make to a benchmark of 10, children bring those towers that are greater than 10 to the teacher, who then writes a number sentence to represent the base-10 composition (K.NBT.1).

## **Common Core State Standards**

**Operations and Algebraic Thinking** 

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.5 Fluently add and subtract within 5.

**Number and Operations in Base Ten** 

Work with numbers 11-19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

MP 1 Look for and make use of structure.

- 0–5 dot cube from Session 14
- Linking cubes
- Three (2–6) dot cubes from blackline master
- Chart paper
- Marker



- Display the 0–5 dot cube you used to play the Collect 20 Caps game introduced during Session 14. Roll the cube and direct children to use their fingers to indicate how many dots they see on the top face when it lands.
- For each roll, challenge children to use a "counting on" strategy to figure out how many more fingers they need to lift to make 5.
  - For example, if the cube lands on 3 dots, children show 3 fingers and then count on—saying the counting words *four*, *five*—as they lift 2 additional fingers to make 5.
- Continue rolling until children have been able to practice/model all possible combinations (0 and 5, 1 and 4, 2 and 3, 3 and 2, 4 and 1, 5 and 0).

- Introduce three new dot cubes with arrangements 2–6. Explain what happens when you roll the cube and it lands on "Roll Again."
- Roll the three cubes simultaneously. Roll any cubes again if they land on "Roll Again."
- Prompt children to look carefully at the combination of dots.
  - Ask, "If you were to count all of the dots, how many do you think there might be?"
- Accept any response, explaining that the number children say might not be exactly how many dots there are altogether, but a close *estimate*.
- Display a tower of 10 cubes and challenge children to compare that number to the combination of dots.
  - "Do you think there would be less than 10, more than 10, or about as many as 10?
     How do you know? What could you do to make sure?"



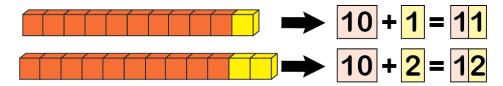




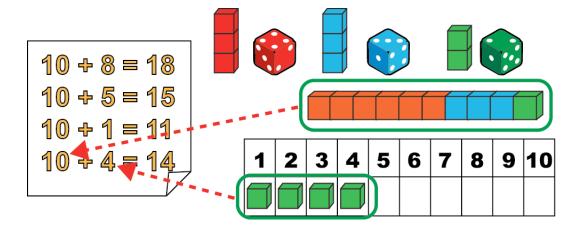


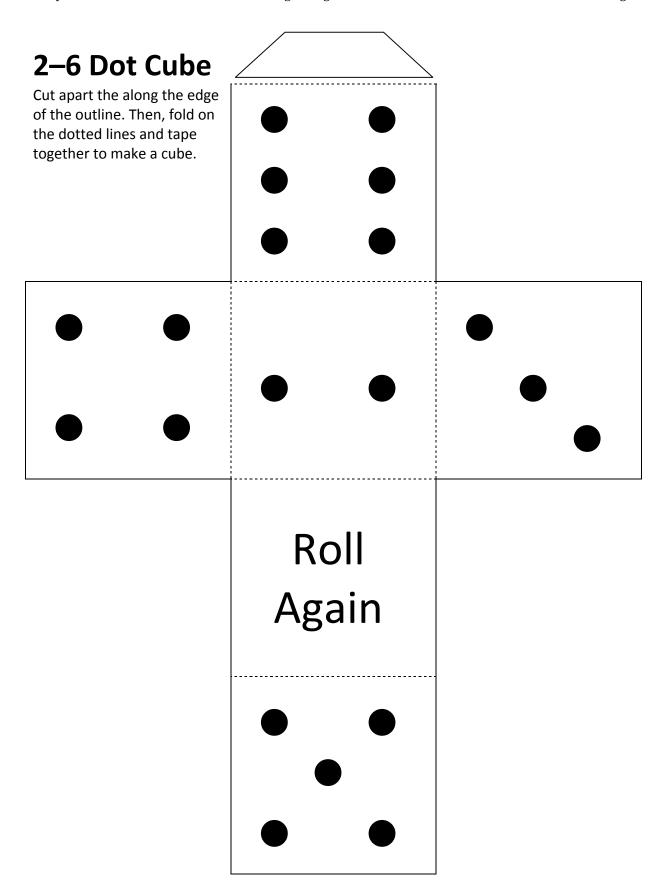
- Call on three volunteers to each build a cube tower that matches the number of dots on his/her assigned dot cube. Join the cubes together and compare the combined tower to the tower of 10.
- If the tower is *less* than 10, highlight the difference and challenge children to use their fingers to show how many more cubes you need to add to the tower to make 10. If the tower is more than 10, discuss how many you need to remove to make 10.

- Once you have constructed a train of 10 cubes, add a cube that is a different color to the right.
  - "How many cubes tall is the tower now? What number comes after 10?"
- Write the equation 10 + 1 = 11 on a sheet of chart paper. Discuss the number 11.
- Explain that in countries like Korea children have a special way of counting numbers that come after 10.
  - o "The word for 11 in Korean is yeol ha na, which means ten-one."
- Add another cube to the right of the train so that there are 10 of one color (to the left) and 2 of a different color (to the right). Write the equation 10 + 2 = 12 on the chart below the number sentence for 11.
- Tell the class that Korean children call the number that comes after 11 *yeol dul*, which means 10 and 2.
  - "What is the English word for the number that comes after 11?"
- Explain that 12 can be thought of as a group of 10 and 2, just like Korean children say.



- Inform children that during center time, they will play the "Triple Dot Cube Roll" game just as you previously modeled.
- Instruct children to continue making additional triple dot cube rolls and matching connecting cube towers until they have a combination that is greater than 10.
- Tell children to bring you the tower they make that is greater than 10 so that you can continue recording on chart paper the "10 and some more" combinations.
- When children bring you their towers, have them decompose/break apart the tower into a
  10 and then arrange the extras on a 1–10 number line to determine how many cubes over
  10 they went. For example, if the child brings a tower of 18 (the highest number they can
  roll with triple sixes), he/she takes off the extra 8 cubes over 10 and arranges them from
  1–8 on the number line.
- Record a number sentence that represents the "10 and some more" combination (e.g., 10 + 8 = 18).





# Session 16: Ordering and Comparing Numerals 1-20

# **At A Glance**

Children revisit the sign-in attendance routine as they enter the room in the morning, focusing on how the order of their position on the list corresponds to the numeral next to their name. The teacher then counts out and labels a set of sticky notes with a corresponding numeral to represent the number of children present. The purpose of this activity is to practice rote counting using written numerals 1–20 to keep track and represent the count (K.CC.3). At math time, children help the teacher arrange the numbered sticky notes in vertical stacks of 10 to visualize how the numerals 11–19 are logically ordered to show a relationship to 10 with respect to the 1–9 sequence of sequence of single digits. During centers, children create a vertical 1–20 number line, select a numeral card, and then count up to 20 from that number. The focus is on counting on from any number to 20 (K.CC.2).

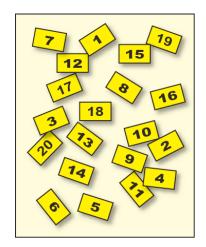
## **Common Core State Standards**

## **Counting and Cardinality**

Know number names and the count sequence.

- K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
- K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).
- MP1 Look for and make use of structure.

- Chart paper
- 1½-in. by 2-in. sticky notes
- Markers
- Blackline master,
   "Numeral Cards 1–20"
- Blank Calendar blackline master



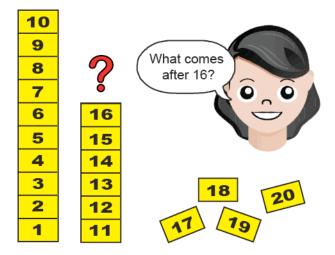
- As children arrive in the morning, have them sign in on blank chart paper, writing their name next to a numeral that represents the order in which they entered the room.
- At morning meeting time, take attendance by having children count off in the order their name appears on the sign-in list.
- Attach one sticky note randomly on a blank sheet of chart paper one by one for each child as they count off.
- Count the number of notes to verify that the total matches the number of children who counted off during the attendance routine. As you count each note, label it by writing a numeral that matches the order in which it is counted.
- Explain to children that you will use the notes for a new activity introduced during math time.

- Refer back to the sheet of numbered sticky notes you created during morning meeting.
- Inform children that you are very tired and need to take a nap. As you pretend to sleep, have children sneak up one-at-a-time in the order they are sitting around the circle and take a sticky note that has a numeral that matches the one next to their name on the attendance sign-in sheet.
- After each child has taken a note, pretend to wake up and act mad, shaking your fist and throwing your pretend hat to the ground. (Note: Remind the class that your throwing tantrum is the signal for them to throw their sticky notes to the ground.)



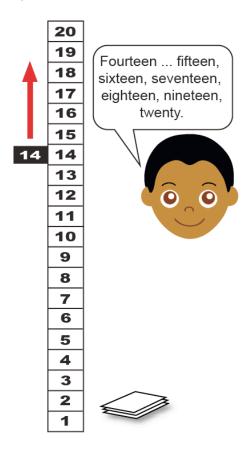
- Randomly collect each note one at a time (not in numerical order) and post them back on the blank chart.
- Count out under your breath (in a frustrated tone) as you collect each note.

- Explain to children that you will organize the notes in stacks of 10, going in numerical order from bottom to top for each stack.
- Call on individual children in the order their name appears on the sign-in sheet to help you order the notes according to your stacking plan.
- Remind children to consult the numeral next to their name on the sign-in sheet as they take their assigned note and help you arrange it in numerical order.
- Once you reach 10, stop and make a new stack of notes next to the first one.



- When all of the notes have been ordered on the chart, draw children's attention to the patterned arrangement of digits in each stack/column.
  - Ask, "How are the numerals in both stacks alike/different?"
- Direct children's attention to the stack with numbers 11–20.
  - "What changes as you move up the stack? What stays the same?"
- If children fail to notice, point out how the last digit in each numeral matches the digit in the stack opposite to it (numbers 1–10) on the left.
- Highlight the repetition of the numeral 1 in the fist digit place of numbers 11–19. Remind them about how children from Korea count, saying "ten-one, ten-two, etc.," instead of reciting a nonpattern sequence of counting words characteristic of the English language (e.g., "eleven, twelve, etc.").
- Explain that during math center time, children will work with a partner at one station arranging numeral cards 1–20 vertically in numerical order, starting at the bottom and moving to the top.

• Once children complete the arrangement, they can play a variation of the Up and Down the Elevator game (introduced in Session 4) in which both partners shuffle a deck of 1–20 numeral cards, remove one off the top, and then match that numeral to a numeral on the 1–20 number line. Children count on from that number to 20 (e.g., if children draw a 14, they count up from 14 to 20).



Numera 1–	1	
2	3	4
5	6	7
8	9	10

Numeral Cards 11–20		11
12	13	14
15	16	17
18	19	20

	-	T	T	
Saturday				
Friday				
Thursday				
Wednesday				
Tuesday				
Monday				
Sunday				

#### Session 17: Counting and Representing Sets 11-20

#### **At A Glance**

The teacher takes attendance by having children look at the location/order of their assigned numeral displayed on a vertical (1–20) number line and stand as he/she points to and calls that number. The teacher emphasizes that the last child who stands has a numeral that represents the number of children present at school (K.CC.5). After leading a brief discussion about how children can use tools such as a number line to help them read numerals and make sets of objects, the teacher introduces a set of Animal Cube Puzzle Counting Cards for quantities 11–20. Children work with a partner to fill these cards with sets of cubes that have been sorted and grouped by color in stacks of 10. They must use up one stack of 10 to fill an outline before borrowing extra cubes of another color to complete the puzzle. The goal is to challenge children to use precision to decompose numbers 11–19 into 10 ones and some more ones (K.NBT.1).

#### **Common Core State Standards**

#### **Counting and Cardinality**

#### Count to tell the number of objects.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

#### **Number and Operations in Base Ten**

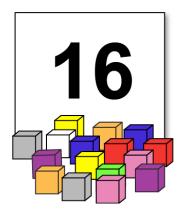
#### Work with numbers 11-19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

#### MP 7 Attend to precision.

#### **Materials**

- Numeral cards 1–20
- Chart paper
- Marker
- Index cards
- Linking cubes grouped in stacks of 10 by color
- Blackline master, "1–20 Number Line"
- Animal Counting Cube Puzzle Cards (11–20) backline master
- Blackline master, " '10 and Some More' Cards (11–20)"



#### Activate

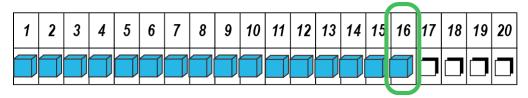
- Instruct children to take a numeral card as they gather for morning meeting time.
- Tell children to look carefully at a sheet of chart paper where you will proceed in numerical order, writing each numeral that was distributed to them.
- Take attendance by having the child with the numeral that you are writing on the chart stand and say the name of the number on the card he/she is holding.
- Start with the numeral 1 at the bottom of the chart and move up until all the numerals have been written.
- Confirm that the last child who stood up has the numeral that tells how many children are at present at school.
- After taking attendance, instruct children to sit down with their numeral cards and look carefully once again at the vertical number line you created on the chart.
- Point to the numeral 12 on the chart and ask the child with that card to identify the name of the number. If the child does not remember the name, allow him/her to solicit help from a friend.
- Direct all children with a numeral card that is lower than 12 to stand. All those with a numeral higher remain sitting.
- Count children who are standing, emphasizing that the last counting word you say matches the numeral you are pointing to on the chart.
- If time permits, practice identifying other numerals and making sets of children that represent that number.



#### Engage

- Ask children to recall how many caps the vendor in *Caps for Sale* carried on his head.
- Gather suggestions from children about what the vendor could do to remember how many caps he had on top of his head without having to draw a picture of each one.
- If no one suggests, propose that the vendor could write down the numeral of the last counting word he says after he finishes counting the caps. Then when he forgets, he can simply read the numeral, which will help him remember and see a mental picture of the number of caps he should have in his inventory.

- Call on a volunteer to write the numeral 16 (the number of caps the vendor had in the story) on an index card.
- Discuss what children could do if they did not know how to read the numeral but wanted to count out a set of caps that matches it.
- Display a horizontal number line (1–20) on the carpet. Ask another volunteer to find the numeral on the number line that is identical to the number the previous volunteer wrote on the index card.
- Model how to begin at the numeral 1 on the number line and move forward, assigning one cube per numeral until you reach 16.



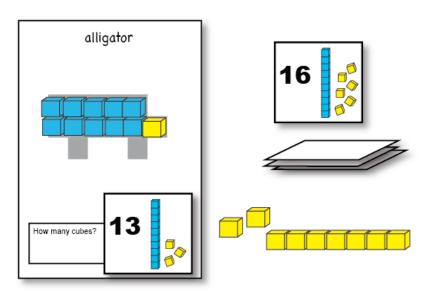
Count the cubes to confirm that there are 16.

• Explain that during math stations/center time, children will practice labeling sets of objects they count with a corresponding numeral.

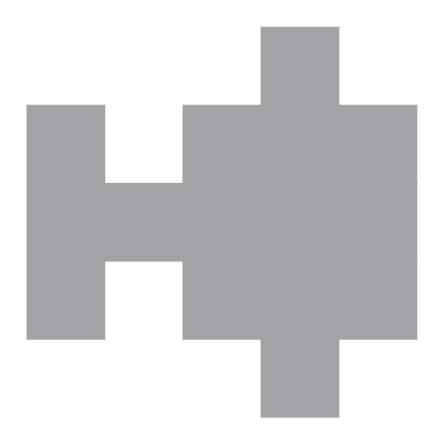
#### Develop

- Gather a small group of children as the rest of the class works at other math stations set up around the room.
- Distribute a tower of 10 linking cubes to each child. Ask, "How many of each color are there?" Count to confirm that each tower has exactly 10.
- Introduce the Animal Counting Cube Puzzles and 11–20 numeral cards. Display the giraffe puzzle and model how to fill it by arranging linking cubes face up inside the outline.
- After using up one tower of 10, point to the one empty square space remaining inside the outline.
  - "How many more cubes do I need to complete the puzzle?"
- Borrow one cube from another tower of 10 and emphasize how 11 is one more than 10.
- Assign pairs of children a puzzle that contains a maximum of 15 cubes (lizard [12], alligator [13], tiger [14], and fish [15]). Allow them to work with their partner filling their assigned puzzle. Make sure each pair chooses whose cube tower they will use first and whose tower they will use to borrow extra cubes.
- When children use up one tower of 10 cubes, challenge them to look at the remaining empty squares inside the outline to determine how many extra cubes they need to complete the puzzle.

- Display the 11–15 numeral cards and guide each pair to figure out which card shows how many cubes they used to fill their outline. Lead children to see that the numeral corresponds to the language used to describe how many (e.g., one 10 [point to the representative numeral—the 1 in 14] and four more [point to the 4] is called *fourteen*).
- Introduce the remaining Animal Counting Cube Puzzle Cards for numbers 16–20 as children gain fluency counting and representing numbers up to 15.



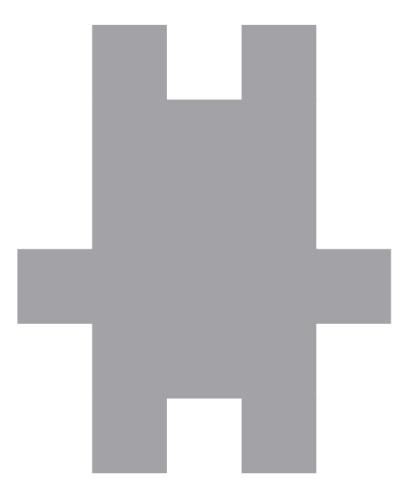
# fish



## turtle



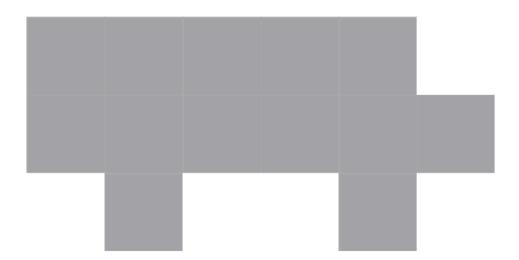
# bear



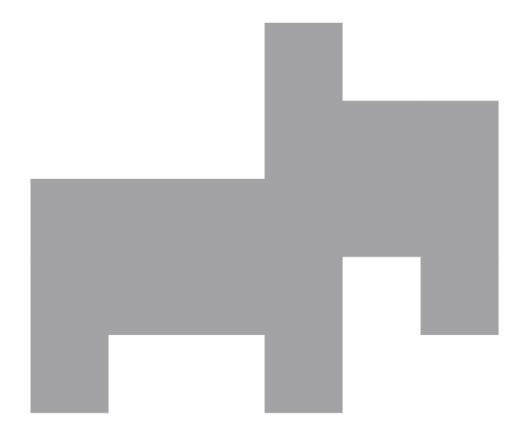
# giraffe



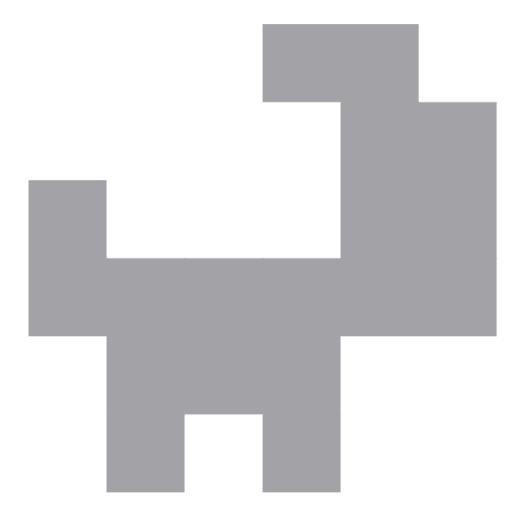
# alligator



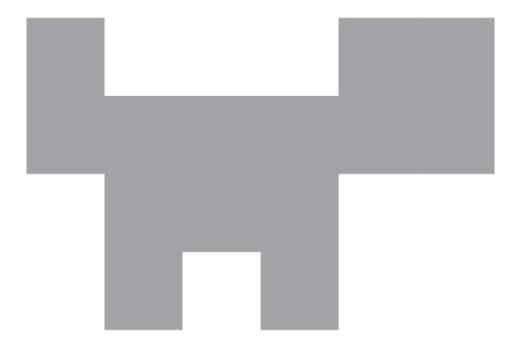
# elephant



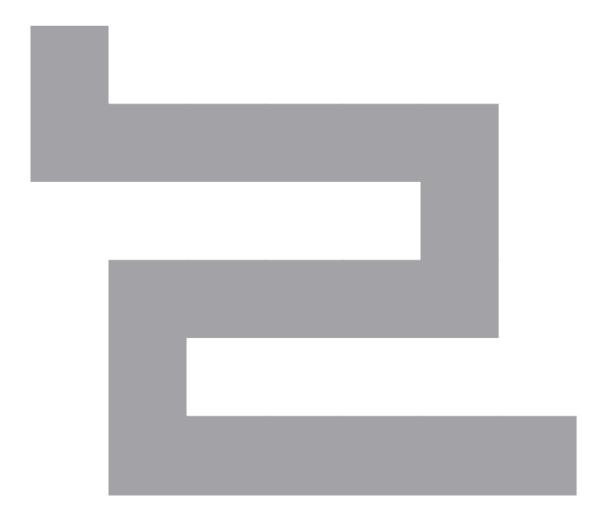
# antelope



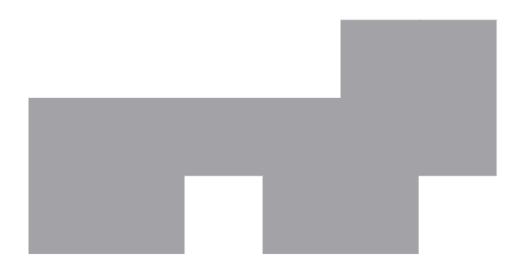
# tiger

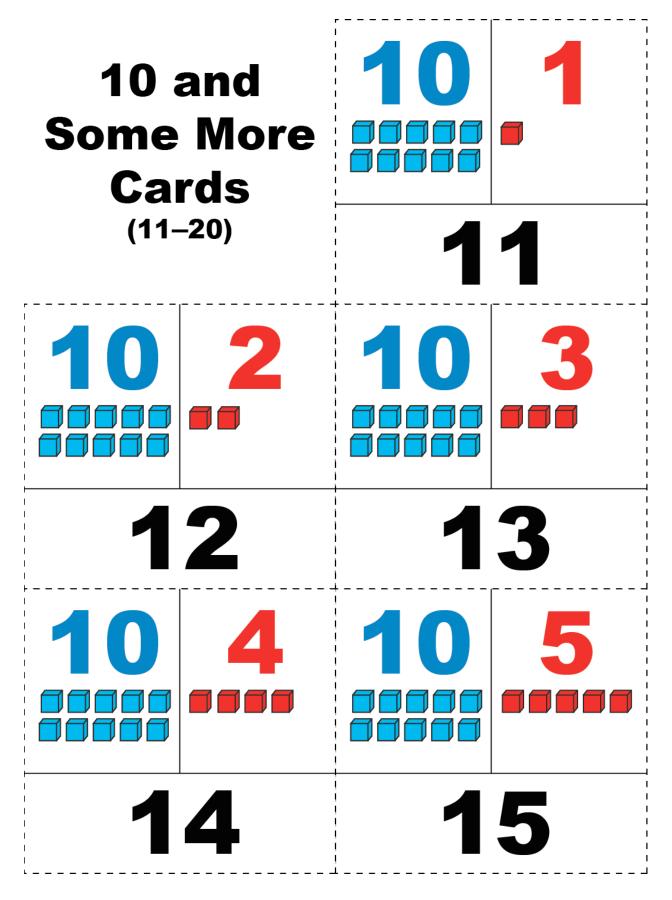


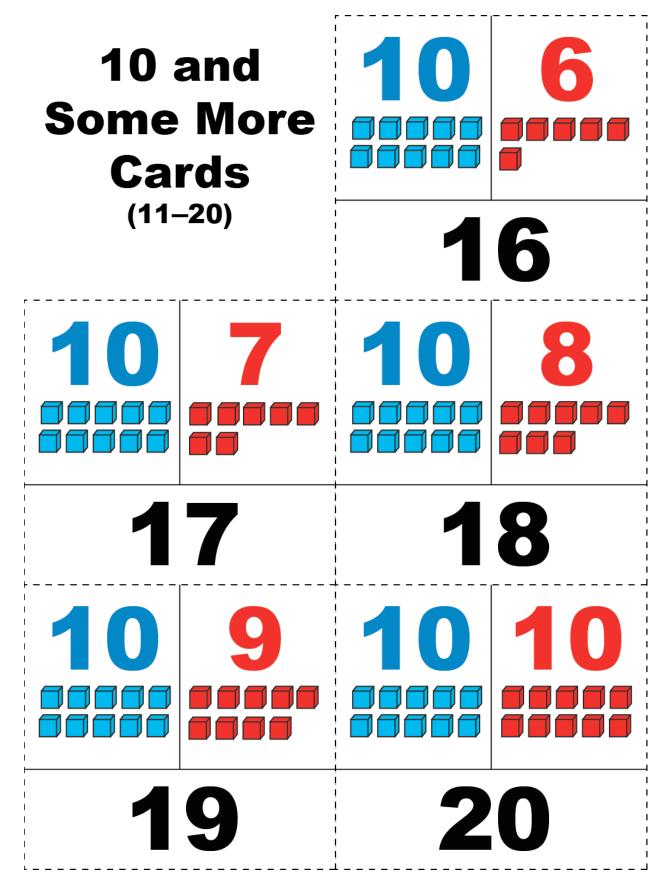
## snake



## lizard







# 1-20 Number Line

10	20	
6	19	
∞	18	
_	17	
9	16	
2	15	
4	14	
8	13	
7	12	
	11	

### Session 18: Representing Numbers 1-20

#### **At A Glance**

Children take attendance by counting off one by one as they go around the circle. The teacher then places a set of numeral cards (10–15) face down in the middle of the circle. After selecting one of the numerals from the center, children continue going around the circle counting off up to 20 from that chosen number. The focus is on counting forward beginning from a given number instead of having to begin at 1 (K.CC.2). Next, volunteers are selected to take a handful of cubes until the class reaches a grand total of 20. The teacher demonstrates different ways to arrange the cubes so that they are easier to count (K.CC.5). After voting on which arrangement is easier to count, children go to centers/math stations, one of which involves a dot cube game focused on counting, collecting, and representing totals up to 20 using pictures and numbers (K.CC.3).

#### **Common Core State Standards**

#### **Counting and Cardinality**

Know number names and the count sequence.

- K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
- K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

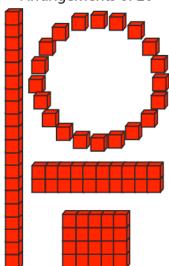
#### Count to tell the number of objects.

- K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
- MP 3 Construct viable arguments and critique the reasoning of others.

#### **Materials**

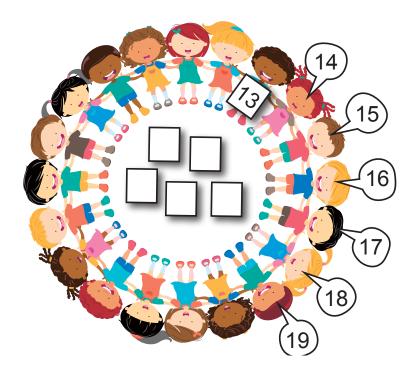
- Numeral cards 1–20
- Linking cubes
- Chart paper
- Markers
- Blackline master, "Quadruple Roll Recording Sheet"
- Four 0–5 dot cubes (for four individual players)

Arrangements of 20



#### **Activate**

- Tell children to stand and form a circle. Have the class count off as it goes clockwise round the circle taking attendance.
- Arrange a set of numeral cards (10–15) face down in the center of the circle area. Invite
  the last child to count off during the attendance routine to select one of the cards and
  identify the numeral.
- Direct the class to proceed clockwise around the circle, starting from the child with the selected numeral card, and count on from that number. For example, if the child selected the numeral 11, he/she says that number, and the count continues going clockwise from where he/she is located in the circle.
- When the count stops, the child who said the last counting word (20) selects another card, and the class continues the counting-on game from that number.
- The counting circle stops when all of the cards in the middle have been selected.

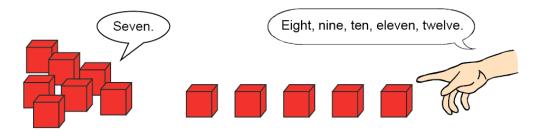


#### Engage

- Invite a volunteer to grab a handful of linking cubes out of a tub of about 100. Inform the class that you want to have exactly 20 cubes.
- Challenge children to make a rough estimate about how many cubes there might be in that one handful.
  - Ask, "Do you think [volunteer's name] might have grabbed exactly 20 cubes? What could we do to be sure how many there are?"

Count the cubes to verify how many you have collected so far.

- Call on a second volunteer to grab another handful of cubes.
- Remind children how many cubes there were from the first handful and model how to count on from that number.

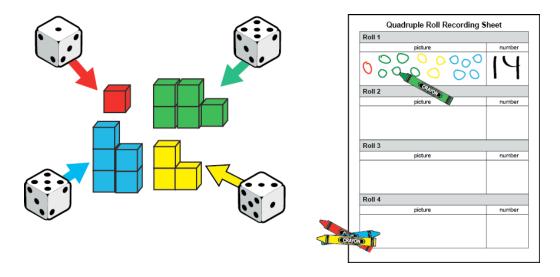


- Continue to select additional volunteers to grab one handful of cubes until you have reached the target number 20. If the last handful goes over, stop at 20 and return the remaining cubes to the bin.
- Count all of the cubes in the collection to verify that you have reached your goal of 20.

#### Develop

- Gather suggestions from the class about how to arrange the cubes so that keeping track of how many were counted/not counted is an easier task.
- If no one suggests, try out each of the following arrangements: a straight line, circle, and various rectangular arrays, including 5-by-4 and 2-by-10 configurations.
- After making each arrangement, count the cubes to emphasize that the number has not changed.
- Make a pictorial representation of each arrangement on chart paper.
- Dismiss children to centers by allowing them to vote on which arrangement of 20 is easier to count.
- As the rest of the class is working at centers/math stations, gather a group of four children. Demonstrate and then play a new game called "Quadruple Roll."
  - 1. All four players begin the game by counting out a set of 20 cubes.
  - 2. Each player rolls a 0–5 dot cube and places that many cubes in the center.

- 3. One player counts how many cubes were collected altogether as a group and represents that amount on the Quadruple Roll Recording Sheet using both pictures (e.g., hash marks) and numbers. The cubes from that roll are then bundled together in a stick and removed from the center.
- 4. The game continues until all 4 players have counted and represented the number of cubes in the center.



## **Quadruple Roll Recording Sheet**

Roll 1		
Pictu	re	Number
Roll 2		
Pictu	re	Number
Roll 3		
Pictu	re	Number
Roll 4		
Pictu	re	Number

## **Session 19: Counting by Ones to 100**

#### **At A Glance**

The teacher revisits the Arrangements of 20 chart introduced during Session 18. Children then count the number of classmates who voted for each arrangement to determine which one was the class favorite. After discussing what makes the most popular arrangement easiest to count, the teacher prompts children to consider how the counting word sequence beyond 20 proceeds in a predictable order based on the constituent arrangement of digits 1-9 established at the beginning of the count. Children then continue to look for additional patterns in numbers as they observe a hundreds chart and then practice the rote-counting sequence to 100 (K.CC.1) by participating in a dance calibrated to the rhythm of the 1–9 cycle repeated throughout each subsequent decade. The focus is on familiarizing children to the base-10 decade system so that they can count forward from any number within 100 without having to begin at 1 (K.CC.2).

#### **Common Core State Standards**

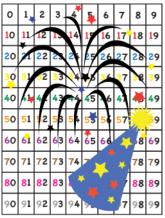
**Counting and Cardinality** 

Know number names and the count sequence.

- K.CC.1 Count to 100 by ones and by tens.
- K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
- MP 7 Look for and make use of structure.
- MP 8 Look for and express regularity in repeated reasoning.

#### **Materials**

- Red/blue number line representation of odd/ even numbers (1–20) developed in Session 18
- Hundreds chart
- Blackline master, "0–99
   Numeral Cards"
- Dancing the Decade Picture Cards blackline master

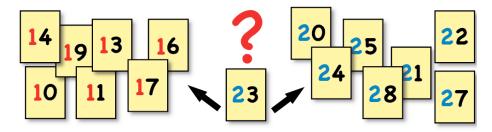


#### Activate

- Display the Arrangements of 20 chart from which the children voted for their favorite arrangement during Session 18.
- Count the number of cubes represented in each arrangement so that children can verify that a large amount such as 20 conserves it number regardless of the way it is arranged.
- Count the number of children who voted for each arrangement and compare the results.
  - Ask, "Which arrangement is the most popular? How do you know? Why do you think
    it is easier to count 20 objects when the items are arranged that way?"
- Challenge children to think about how the structure of the counting word sequence beyond 20 closely follows the 1–9 digit order established at the beginning of the count.
  - "If we continued to count after 20, what counting word would we say next? What would come after 21? Do you notice a pattern?"
- Upon reaching 29, stop and prompt children to determine which counting word comes next—the start of a new decade, 30.

#### Engage

- Gather children in front of a classroom hundreds chart display. Encourage them to study
  the chart closely in order to identify any patterns in the way that the numbers are
  arranged.
  - "What stays the same about the numbers as you look across each row? What changes? Do you see a pattern? What do you notice about the numbers going down?"
- Display two sets of numeral cards on the carpet. All cards in one set should be numbers in the teens and the other set of cards should have numbers in the twenties. (Be sure, however, not to include all numbers in each representative decade—set aside some of the numeral cards 10–19 and 20–29 for children to sort into a respective group.)

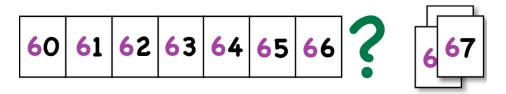


- Direct children to look carefully at both sets to determine the sorting rule.
  - "How are all of the numerals in this set alike/different?"

- Display one of the numeral cards you did not include in the teens group and select a volunteer to determine in which group the card belongs.
  - "Why does this numeral belong in this group and not with the other cards?"
- Allow other volunteers to sort the remaining discarded cards into a respective group.

#### Develop

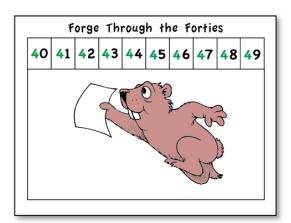
- Assign one set of 0–99 numeral cards representative of each decade from the hundreds chart display to pairs of children (or groups of three if the class is sufficiently large).
- Instruct children to work with their partners sequencing their assigned decade cards in numerical order.
- Once all the children have completed the sequencing activity, call on each pair (or group) in the order their decade follows in the 1–100 counting sequence to arrange their cards accordingly on the hundreds chart (0–9, 10–19, 20–29, 30–39, 40–49, 50–59, 60–69, 70–79, 80–89, 90–99).



- Explain to children that the digit 0 is not a counting word. Rather, it represents an empty set.
- Direct children to stand and perform the following Dancing Through the Decades chant/dance as they rote count to 100 by ones.
  - o (0–9) Dig Through the Digits: Make a digging motion with your arms.
  - (10–19) Tiptoe Through the Teens: Tiptoe with your finger on your lips as you quietly recite the counting sequence. Remind children that they have to be careful as they count in this decade because the number words are strange.
  - o **(20–29) Twirl Through the Twenties:** Distribute scarves to each child. They twirl the scarves with their hand in the air.
  - o (30–39) Thumb Through the Thirties: Make up and down movements with your hands with thumbs facing up.
  - (40–49) Forge Through the Forties: Pretend to carry a heavy load behind your back as you walk in place appearing to go forward.
  - o **(50–59) Fiddle Through the Fifties:** Hold a pretend fiddle between your chin and arms and move an invisible arc across, up, and down.
  - *(60–69) Sing Through the Sixties:* Hold a pretend microphone to your mouth and sing the counting sequence.

- (70-79) Serenade Through the Seventies: Get down on one knee and pretend to strum a guitar as you count.
- o **(80–89) Exercise Through the Eighties:** Bend your arms up and down pretending to lift a barbell.
- o **(90–99) Nibble Through the Nineties:** Pretend to bite off a piece of cheese and chew as you count.
- o 100!: Shout and applaud.





• During math/center time, interview each child, asking him/her to count as high as he/she can. If the child makes it to 40, stop and congratulate him/her on his/her counting skill. Then, prompt the child to count on from various numbers (e.g., "What if you started counting from 76, what number would come next?"). Encourage the child to continue counting from that designated number. The goal is to be sure the child is able to successfully transition between each successive decade (e.g., he/she should say *ninety* after *eighty-nine*, not *eighty-ten*.)

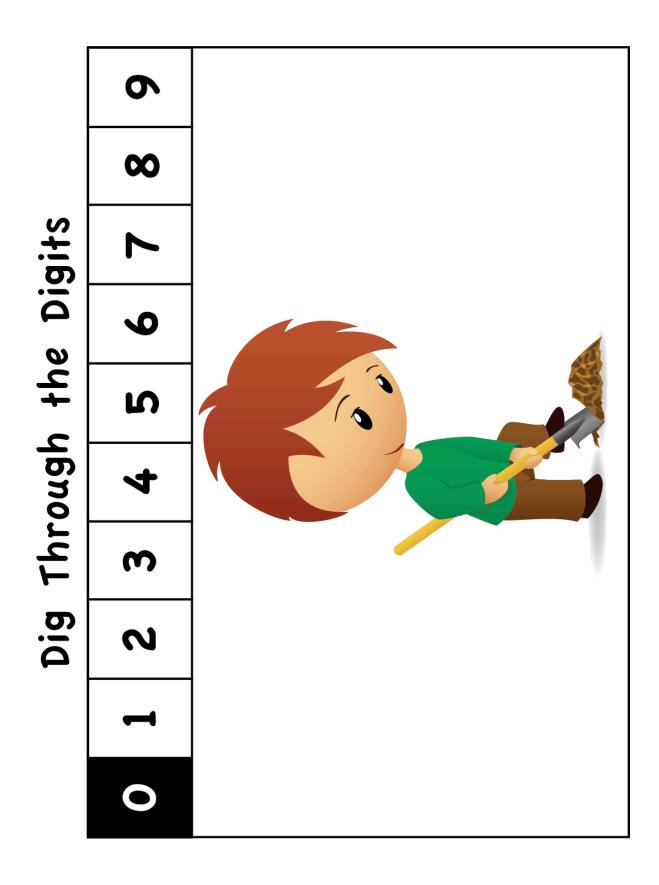
0	1	2	3	4
5	6	7	8	9
<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	19

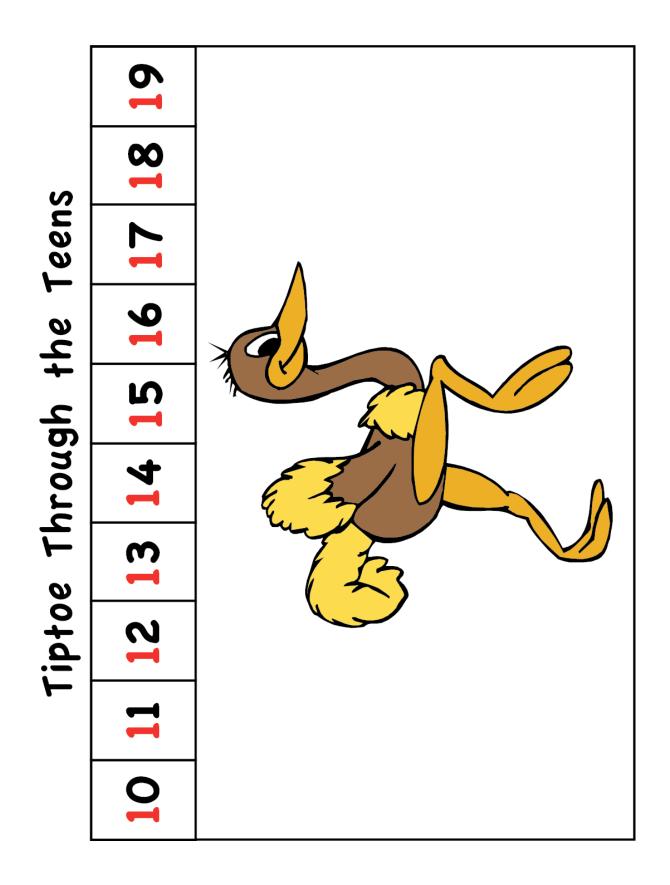
20	21	22	23	24
<b>25</b>	<b>26</b>	27	28	29
30	31	<b>32</b>	33	34
35	36	<b>37</b>	38	39

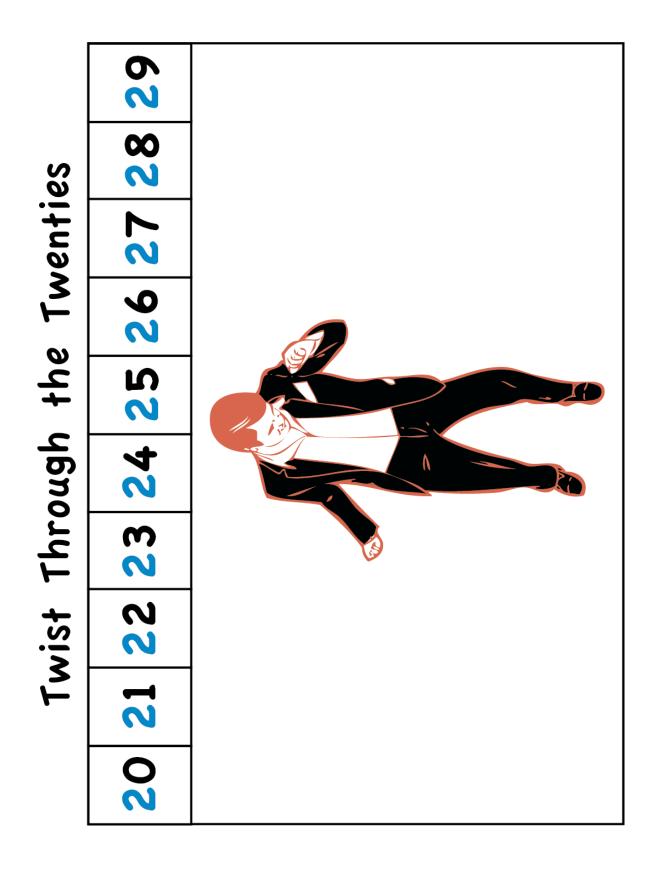
40	41	<b>42</b>	43	44
45	<b>46</b>	<b>47</b>	48	49
<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>
<b>55</b>	<b>56</b>	<b>57</b>	58	<b>59</b>

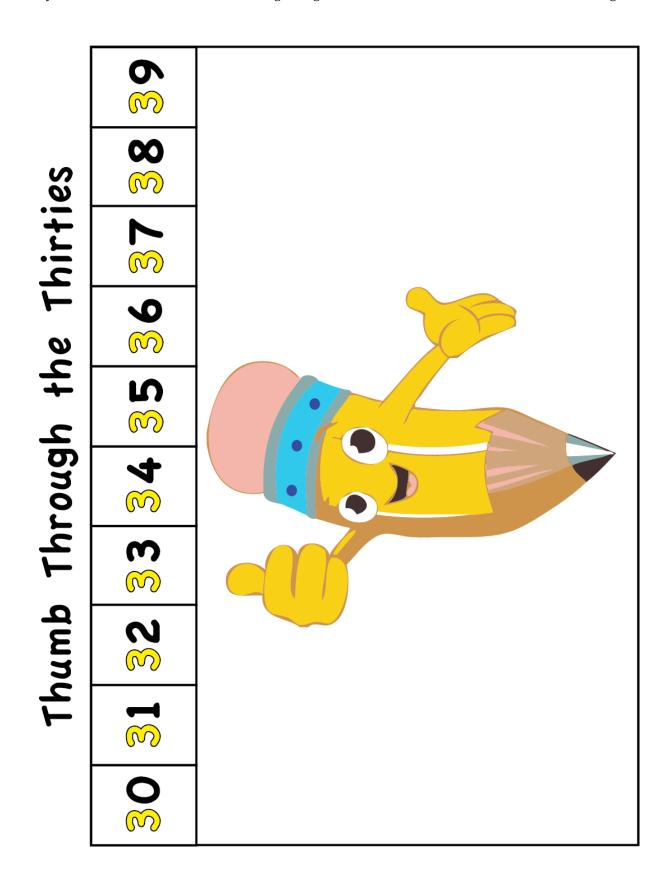
60	<b>61</b>	<b>62</b>	<b>63</b>	64
65	66	67	<b>68</b>	69
<b>70</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>
<b>75</b>	<b>76</b>	<b>77</b>	<b>78</b>	<b>79</b>

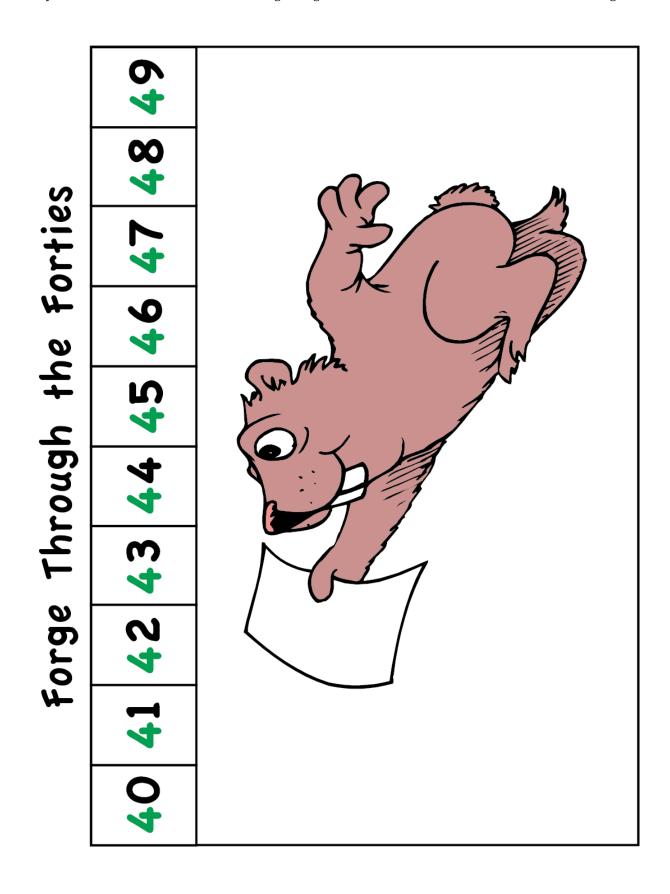
80	81	<b>82</b>	83	84
85	86	87	88	89
90	91	92	93	94
95	96	97	98	99

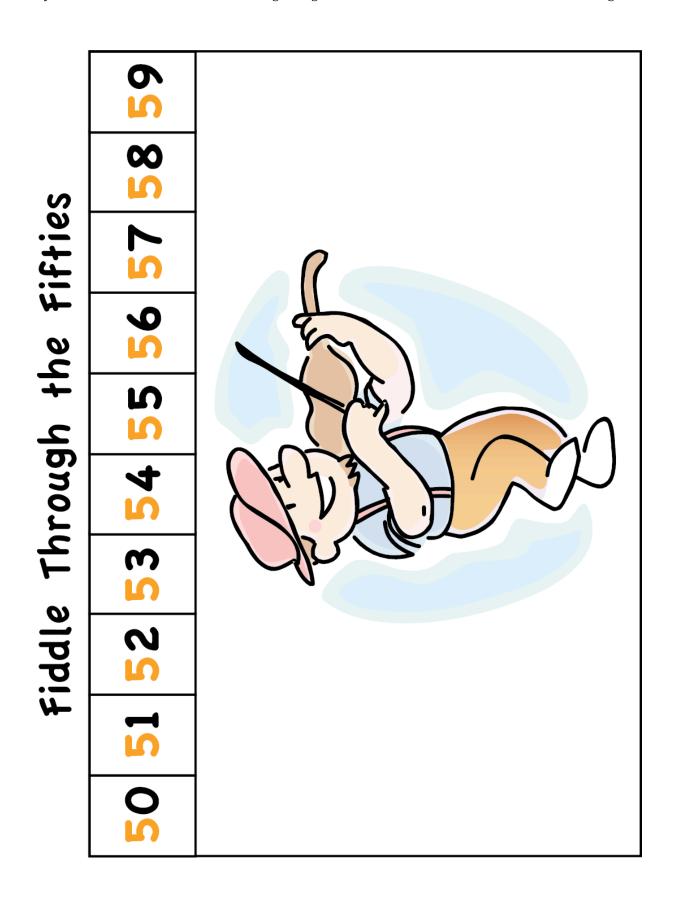


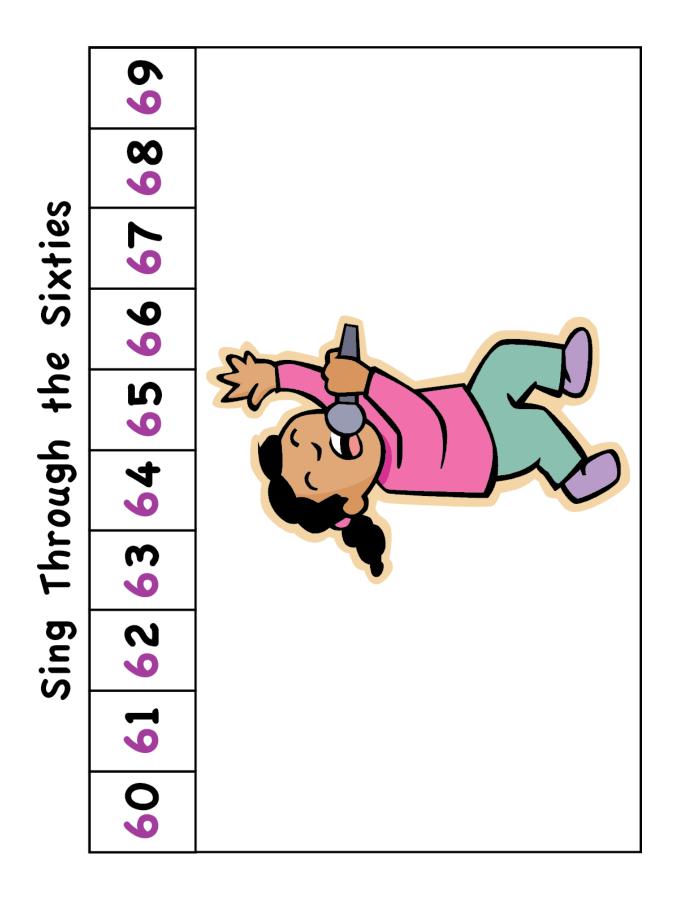




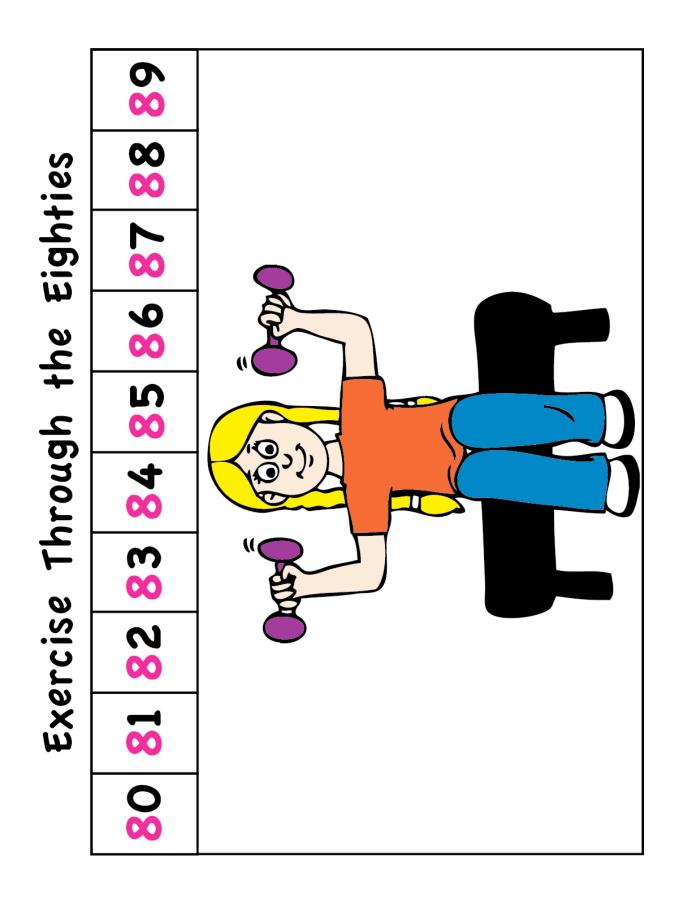


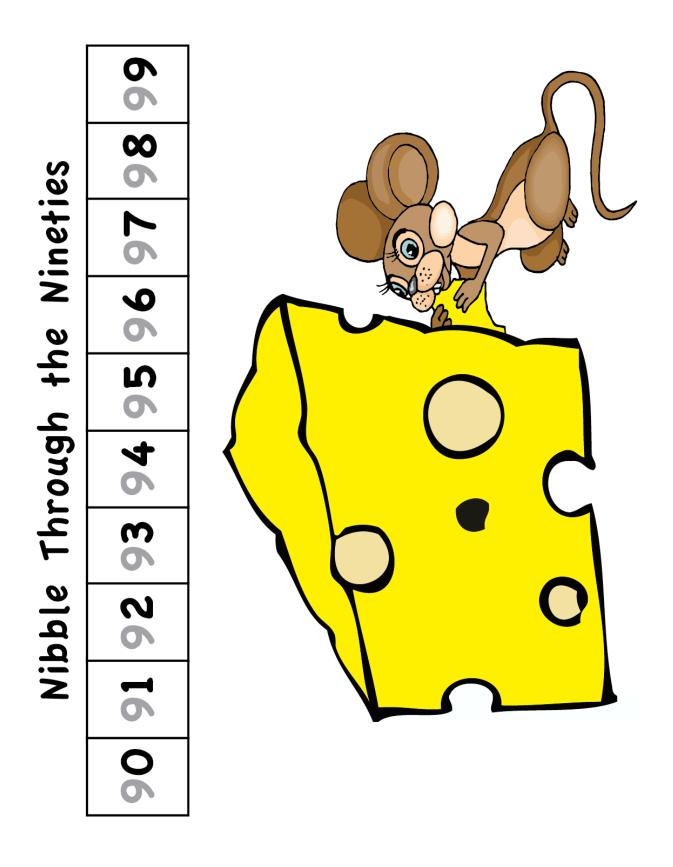




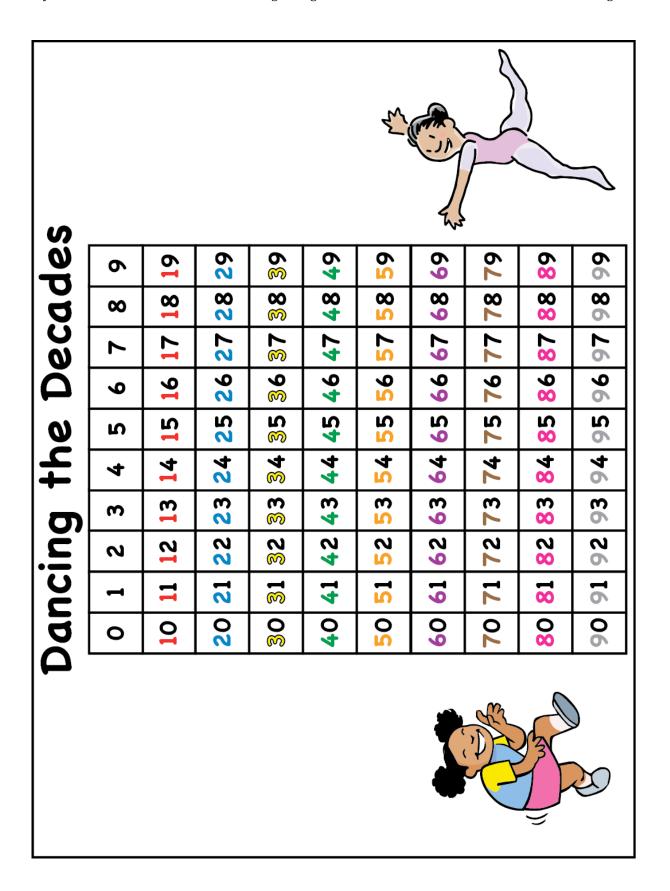












# **Session 20: Counting by Tens to 100**

## **At A Glance**

Children practice counting by ones to 100 as they revisit the Dancing the Decades routine introduced in Session 19. At various times throughout the day, the teacher prompts the class to perform any of the 10 decades in isolation to facilitate automaticity and fluency rote counting to 100 (K.CC.1). Children then assist the teacher in counting a set of 50 pennies, first by ones and then by tens as volunteers use their fingers to keep track of the large count. Children then practice the rote-counting sequence to 100 as they play a jumping game with a partner. After one player jumps over a series of numbers arranged by decade, the next player counts on by ones from the number where his/her partner left off (K.CC.2). Finally, at the close of the unit, children represent their own story about selling a set of 20 items of their choice. The teacher prompts them to think about how to arrange the collection of 20 items in such a way that it is easy to count (K.CC.5).

#### **Common Core State Standards**

#### **Counting and Cardinality**

Know number names and the count sequence.

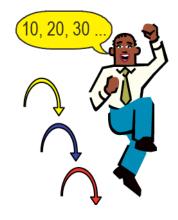
- K.CC.1 Count to 100 by ones and by tens.
- K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

#### Count to tell the number of objects.

- K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects..
- MP 7 Model with mathematics.

### **Materials**

- Dancing Through the Decades picture cards (from Session 19)
- 50 pennies
- Empty jar
- 10 sentence strips (each with a set of numerals within one assigned decade ordered and arranged vertically up and down the strip)
- 0–5 dot cube
- Paper
- Writing utensils
- Counters

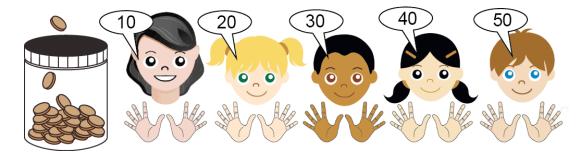


### Activate

- Invite children to stand and perform the Dancing through the Decades chant and movement exercise they rehearsed during Session 19.
- Use the picture cards that accompany the dance to cue children on how to move their bodies for each count they make through a respective decade (e.g., children make one twirl for each counting word they say through the twenties, 20–29).
- Vary the activity by having children dance only through certain decades as they perform various daily routines, such as going back and forth to the cafeteria or transitioning between activities.
  - For example, "Let's exercise through the eighties before we start reading today's story."
- When performing a decade dance out of sequence, encourage children to explain where they are in the 1–100 counting sequence in relation to other decades.
  - "Which decade comes before/after the eighties? Through which decade do we forge/sing, etc.?"

### Engage

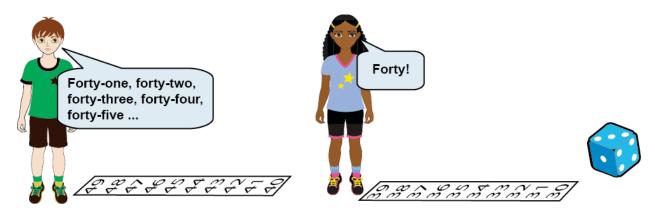
- Review what children remember about the price the vendor from *Caps for Sale* charged for one cap—50 cents.
  - "How much is 50? When do we get to the fifties and fiddle during our Dancing Through the Decades routine? Which decades come before the fifties?"
- Display a jar of 50 pennies, explaining that the quantity is equivalent to the price of one cap.
- Remove the pennies from the jar and scatter them on the carpet so that children can visualize the magnitude of 50 when it is uncontained and each constituent unit is visible.
- Invite children to count out loud as you count by ones to 50, dropping 1 penny at a time back inside the jar. Before beginning the count, call on a volunteer to use his/her fingers to keep track of the count.
- During the count, stop upon reaching 10 when the volunteer has no more fingers to lift.
   Call on another volunteer to continue the finger inventory until 20 when he/she has no more fingers to show.
  - "How many pennies have we counted so far? How many sets of hands have we used to keep track of the count? How many volunteers have we gone through?"
- Continue the penny count up to 50, stopping after each accumulation of 10 fingers (30 fingers, 3 volunteers; 40 fingers, 4 volunteers; 50 fingers, 5 volunteers).



- After counting the pennies, explain that counting to 50 would be faster if you just counted by decades.
- Instruct each finger-counting volunteer to raise his/her set of fingers as you tap him/her on the shoulder and demonstrate how to count by tens to 50—ten, twenty, thirty, forty, fifty.

## Develop

- Arrange a set of 10 sentence strips (each containing a series of numbers representing a
  decade in the 1–100 counting sequence) in a straight line on the carpet. Leave enough
  space between each strip for a child/person to stand.
- Start at 1 and demonstrate how to jump over each strip and count by 10s.
- Introduce a new math station game called "Jump the Decades" whereby children work with you and a partner practicing the jumping and counting by tens exercise you just modeled.
- One player is the tens counter, and the other player is the ones counter.
  - 1. The tens counter starts the game by rolling a 0–5 dot cube to indicate how many sentence strips (decades) to jump over.
  - 2. After jumping and counting that many decades forward, the ones counter starts where the first player stopped and begins to count by ones down the next sentence strip decade. For example, if the first player stopped at 30, the second player starts at 31 and count forward by ones until reaching the next decade, 40.
  - 3. The tens counter then rolls the dot cube again and proceeds to count by tens as he/she jumps over that many strips/decades.
  - 4. If the tens counter fails to make it to 100 after the second round of jumps, the ones counter goes next and counts by ones to the next decade.
  - 5. Upon reaching 100, the players switch roles as the tens and ones counters and go down the hundreds path for a second round.



- When necessary, be present at the Jump the Decades center to assist children if they forget a counting word or have trouble transitioning between certain decades.
- As children become more confident and self-regulating at the Jump the Decades center, begin an end-of-unit assessment task in small groups in which the objective is to create and illustrate one's own story about selling a set of 20 items of his/her choice (balls, shirts, etc.).
- Allow children to use counters to represent the items they want to sell before beginning to illustrate the story with pictures and numbers on paper.
- Prompt children to consider how to arrange the merchandise they want to sell in a way that is easy to count, such as arranging the items in a line, circle, rows and columns, etc.