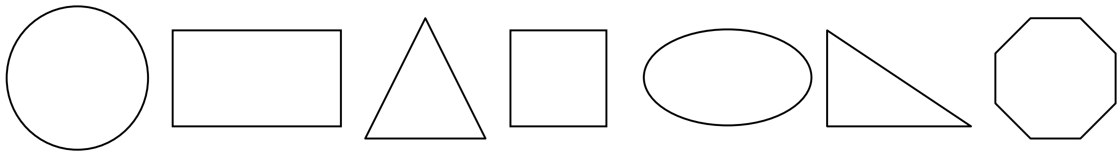
Don’s Shapes

These are Don’s favorite shapes.

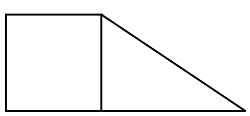


Name this shape from Don’s favorites.

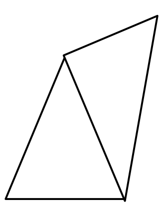
1. ________________

Don made these figures from 2 of the shapes.

Name the two shapes Don used.

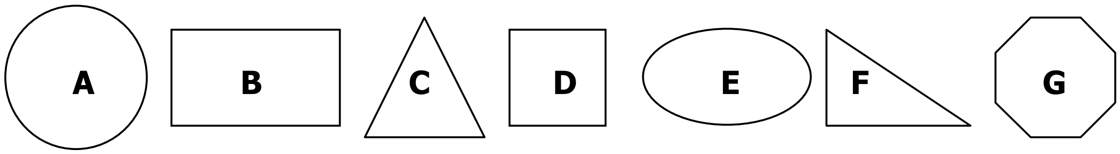


2. __________________ and _________________



3. __________________ and __________________

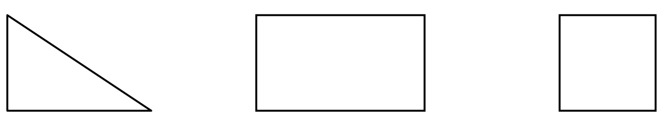
Here are Don’s favorite shapes again.



4. Which of Don’s shapes have straight sides? Write the shape’s letter in the box it belongs.

Have straight sides	Don’t have straight sides

Look at these three shapes.



5. How is the 1st shape different from the other two?

____

____

____

7

Don's Shapes
Mathematics Assessment Collaborative
Performance Assessment Rubric Grade 2

	Don's Shapes: Grade 2	Points	Section Points
	The core elements of the performance required by this task are: • Describe and classify two-dimensional shapes according to their attributes and /or parts of their shape • Develop an understanding of how shapes can be put together or taken apart to form other shapes • Communicate reasoning using words, numbers or pictures Based on these credit for specific aspects of performance should be assigned as follow:		
1	**Rectangle**	**1**	**1**
2	**Square** and **triangle** Accept rectangle and triangle.	**1**	**1**
3	**Triangle** and **triangle**	**1**	**1**
4	Gives correct answers to include all: Straight sides: **B,C,D,F,G** Not straight sides: **A,E**	**2**	**2**
5	Gives correct explanation such as: The first shapes has three sides but the others have 4 Gives an attribute of only one set such as: It is a triangle.	**2** **(1)**	**2**
Total			**7**

2nd grade – Task 3: Don's Shapes

Work the task and examine the rubric.
What do you think are the key mathematics the task is trying to assess?

____

____

____

____

Identifying students who understand and classify shapes by their appearance, rather than by their geometric attributes or names

Look at the student work for questions 2 and 3. Many students could name one of the two shapes, but not the other. Which students answered incorrectly and used words like "slide", "pyramid", "half triangle" or "half kite" to describe the right triangle? Were there students who described the square as a "cube"?

- What do these responses tell us about the way students are understanding shapes?
- How can we help students to focus on the defining attributes of a shape, instead of how suggestive their appearance is to other objects?
- In what ways are students confusing two-dimensional and three-dimensional objects? Why might this be a natural confusion in this context?

Look at the student work for question 4. Which students classified "Shape G" as "Not Straight Sides"? Which students struggled with placing "Shape C" or "Shape F", by leaving them off entirely or classifying one or both of them as "Not Straight Sides"? Which students could correctly place "Shape C" as "Straight Sides", but placed "Shape F" as "Not Straight Sides"?

Shape G Not Straight	Shape C Not Straight	Shape F Not Straight	Shape C AND Shape F both classified as Not Straight

Look at the student work for question 5. Which students described the first shape as "slanted", "goes down", or "has a diagonal". Did they see the rectangles as having "straight sides"? How many students made an overly general statement, such as "they are different" or "they are not the same"?

Questions to consider:
How does this understanding of the sides of a triangle, based on the appearance of slope, help us understand why a student might consider Shapes C and F to "not have straight sides"? Why might a student consider Shape C to have straight sides, but not Shape F? Which students seem to be able to identify an equilateral triangle as a triangle, but not a right triangle as a triangle? What does this tell us about their understanding of the definition of a triangle? In what ways can we help move a student who makes blanket statements about size or appearance towards making specific statements about geometric attributes?

Identifying students who need language support to make a comparison

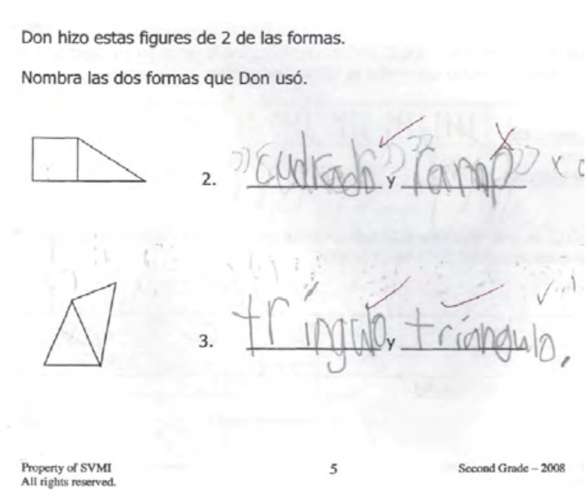
Look at student work for question 5. How many students received only partial credit here by making an accurate statement based on one or more attributes of the triangle? How many students received only partial credit here by making an accurate statement based on one or more attributes of the rectangles?

> Questions to consider:
> How can you encourage students to make comparison statements using academic language such as, "The triangle has three sides, but the square has four sides"? Both the attribute being compared and the comparison itself should be explicitly stated. Simply stating that the "the triangle has three sides" is trued based on the attributes, but does not make a comparison. How can we push beyond teaching the words of the math, and into helping students develop the forms and structures of language that help them talk about the math in explicit, accurate, and meaningful ways?
>
> How could sentence stems be used to give them practice with these types of language structures? In what ways can we be explicit about what it means to compare, analyze, summarize, hypothesize, or explain in a math classroom?

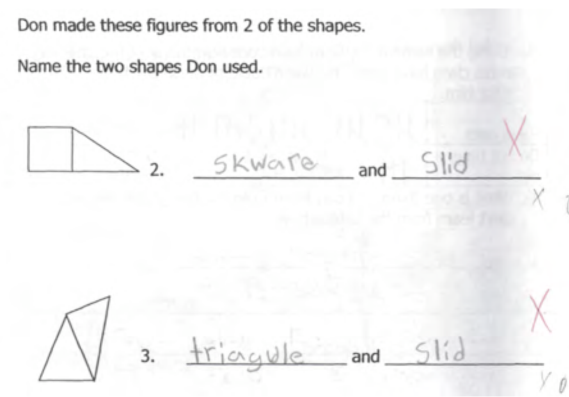
Students working at the lowest levels on this task were working with shapes based on their appearances. They may change the way they name or define a shape because of it's orientation or because it appears similar to another object. **Students A – F** demonstrate some developing thinking in this area.

Student A and **Student B** are not yet focusing on the attributes of the triangle shape. They are able to identify some triangles as triangles, but others are renamed as "ramp" or "slide" based on perceived changes in the shape when it is moved in space.

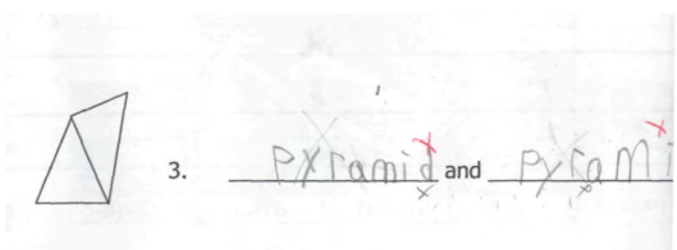
Student A

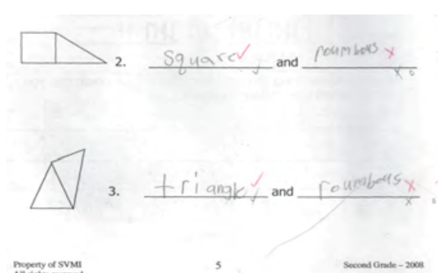


Student B



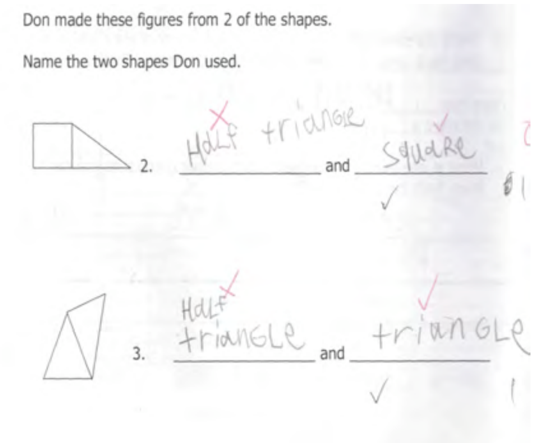
Student C is identifying the triangles in question 3 as pyramids, because the shape is suggestive of a three-dimensional pyramid. Unlike **Student A and Student B,** this student believes that the "pyramid" maintains is basic "pyramidness" even when it is changed in space. Likewise, **Student D** believes that the misidentified "rhombus" does the same. What is it about the two triangles **Student D** identifies as "rhombus-like"? Does is appear similar to the blue pattern block piece? In what ways does it harm or help students to be introduced to the names of shapes without attaching the meaning of the defined attributes inherent in a shapes's name?



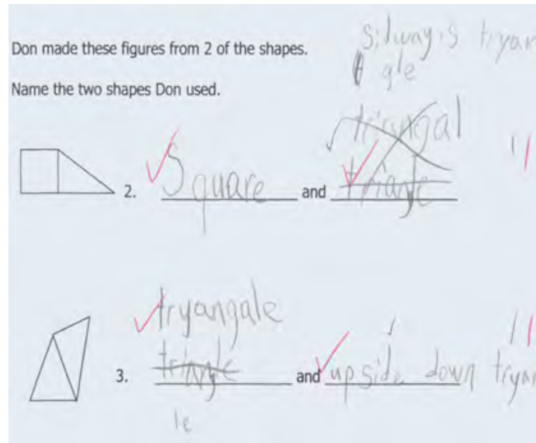


Students E and F believe that the right triangles need to be distinguished with adjectives to reflect their orientation (sideways or upside down) or their appearance (half, because it appears to be an equilateral triangle that has been cut in half down the center).

Student E

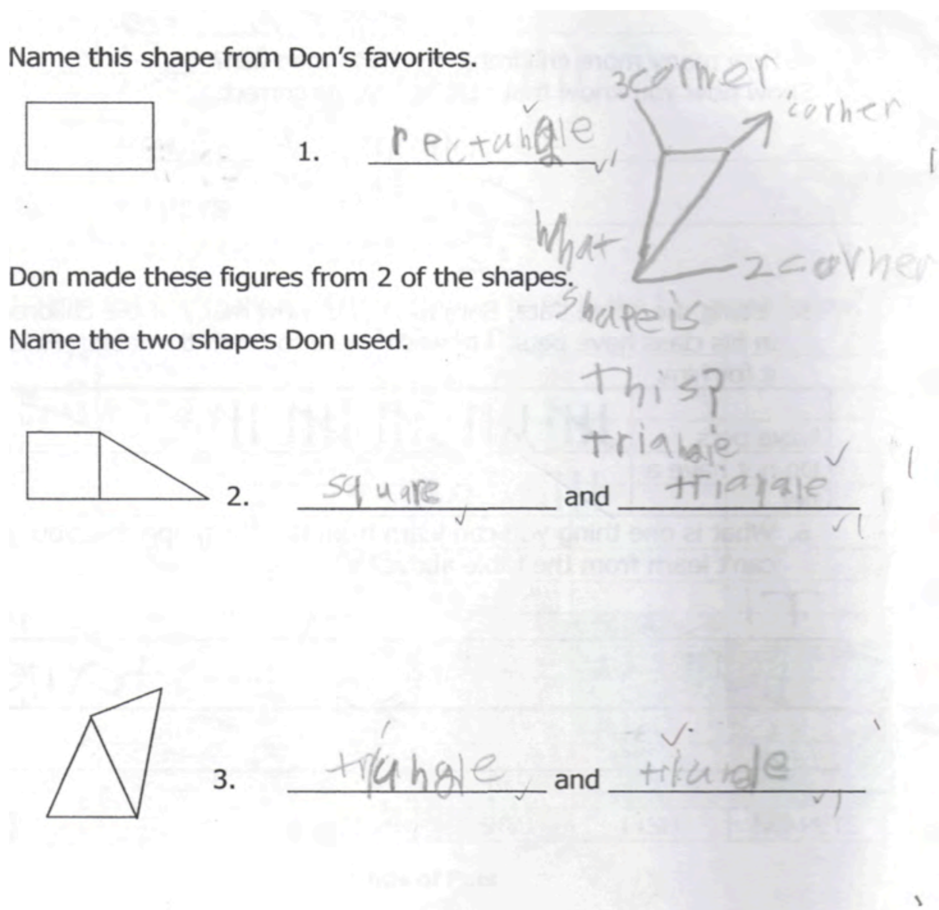


Student F



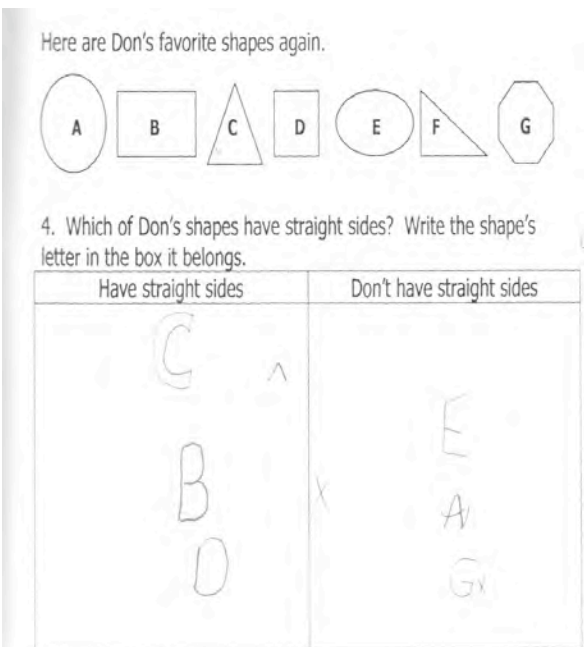
Student G is demonstrating with a drawing what it might look like to use an understanding of the attribute of "number of vertices" to determine that this shape must be a triangle, regardless of the orientation.

Student G

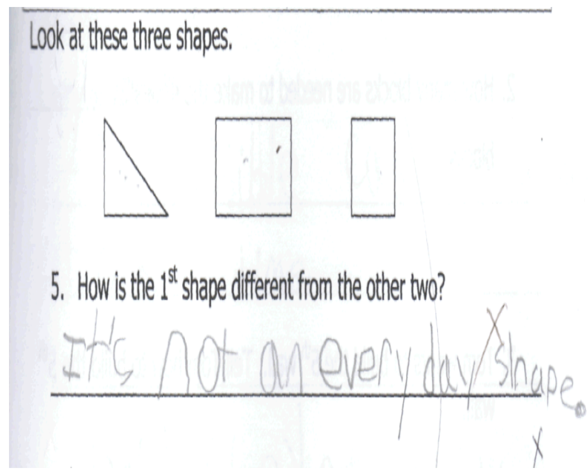


Student H and I together present an understanding, or perhaps a set of ideas, that was very common in the student work for this task. **Student H** has classified Shape G incorrectly, as a shape that doesn't have straight sides. Shape G appears suggestive of a circle for many students. Furthermore, this student has left Shape F off the sort completely. Many students only listed C, B, and D as the shapes that have straight sides. Is it possible that Shape F is completely dismissed or forgotten about because students are so used to seeing the "traditional" shapes and orientations represented by Shape B as a rectangle, Shape C as a triangle, and Shape D as a square? **Student I** seems be demonstrating the same misconception as **Student H**. Does classifying the right triangle as "not an everyday shape" reflect this student's understanding that the rectangle and square are both the "correct" shape because they are the traditional shapes and orientations of so many elementary classrooms?

Student H

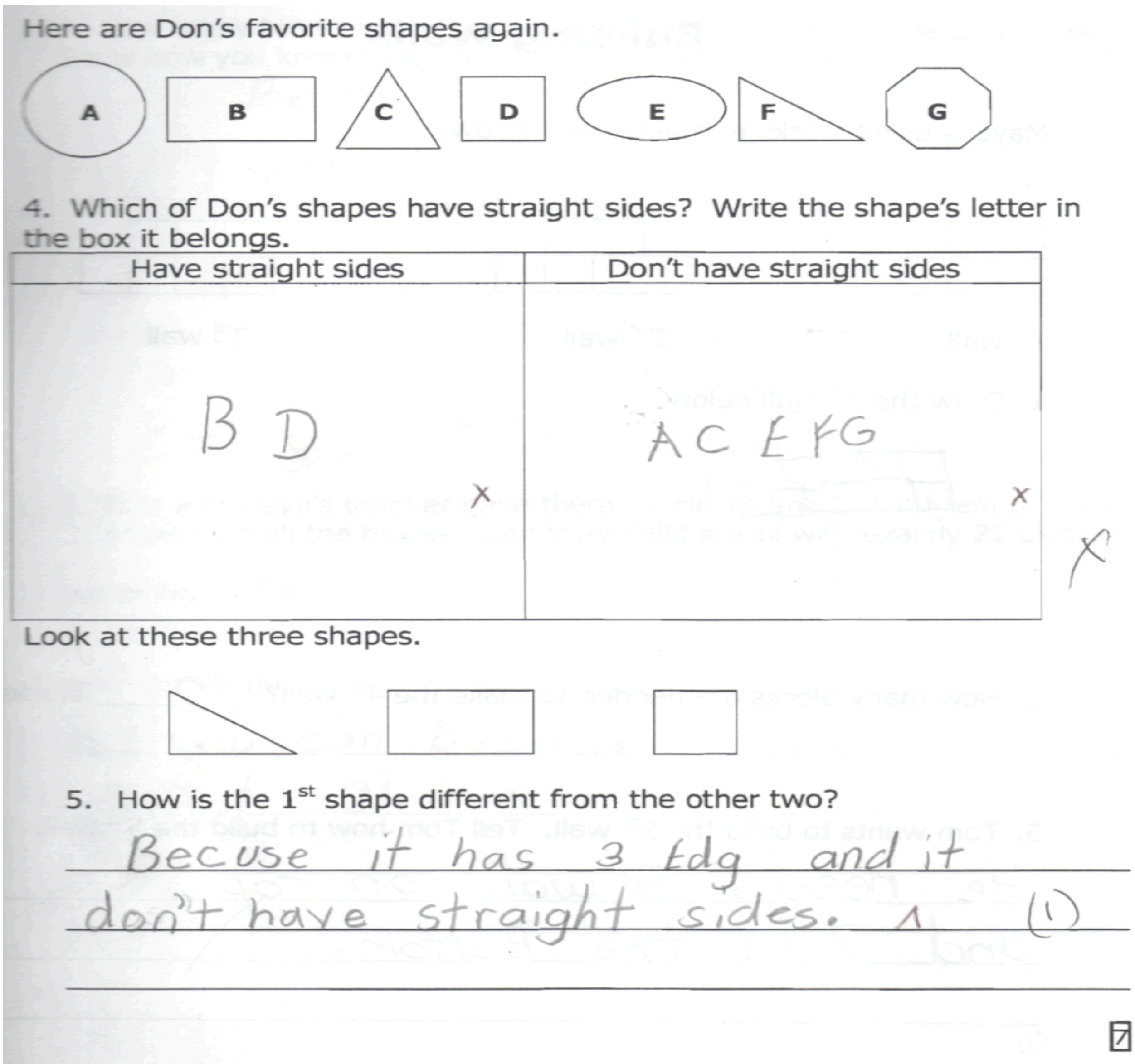


Student I



Student J has incorrectly sorted both Shape C and Shape F as “not having straight sides”. This was a common mistake for struggling second graders, and **Student J’s** written comparison in question 5 may offer some insight into this misconception. The student writes that the triangle “doesn’t have straight sides”. Other students express this same confusion, writing comparisons about the triangle having a “diagonal” or being “slanted”. The definition and usage of the word “straight” here seems to shed some light on the thinking about the incorrect sort for the triangle shapes in question 4.

Student J


Here are Don’s favorite shapes again.

4. Which of Don’s shapes have straight sides? Write the shape’s letter in the box it belongs.

Have straight sides	Don’t have straight sides
B D x	A C E F G x

x

Look at these three shapes.

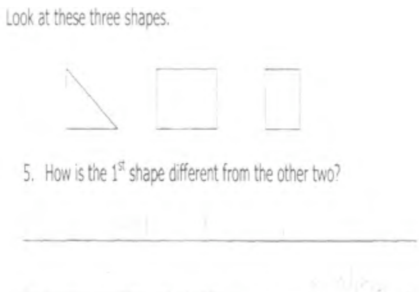
5. How is the 1st shape different from the other two?

Becuse it has 3 Edg and it don’t have straight sides. ^ (1)

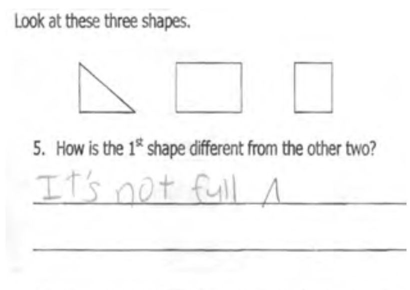
7

Students K, L, and M represent the thinking of many struggling students who tried to answer question 5 by writing about the appearance of the shapes, rather than any particular defining attributes of the shapes. **Student K** writes that the triangle is "weird looking" and smaller. **Student L** calls the triangle "not full" because it appears to be halved. **Student M** writes that the triangle is different because it doesn't look like a box. *Have these students demonstrated an ability to discriminate between these shapes, and describe them, based on some aspect of their appearance? And if so, in what ways could the introduction and practical use of a more formal language help these students to clarify their understandings and move them toward focusing on attributes?*

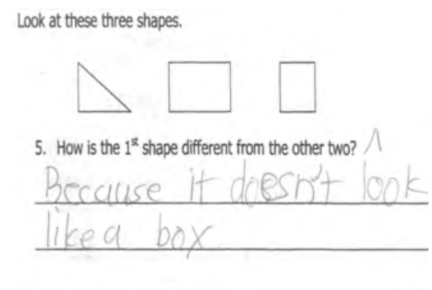
Student K



Student L

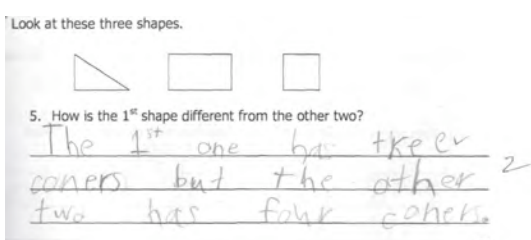


Student M

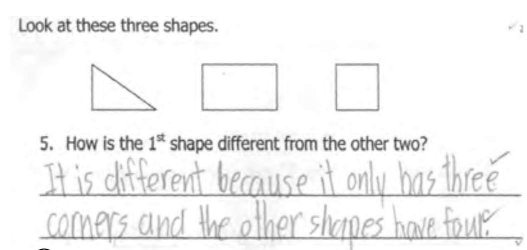


Students N – Q represent students who are able to make a complete and accurate comparison statement, or who are on their way to being able to make a comparison. What explicit language, vocabulary, and structures are successful **Students N and O** using to make the comparison? Looking at the work for **Students P and Q**, what evidence do you see that their incomplete statements can be turned into comparison statements? What explicit instruction might these students need? What experiences will help them continue to make sense of the defining attributes of shapes, and how those attributes relate to other shapes?

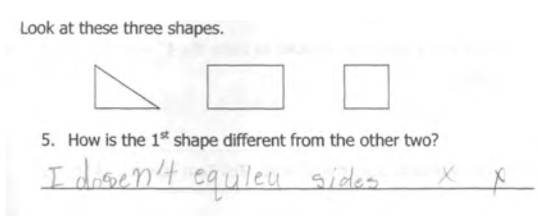
Student N



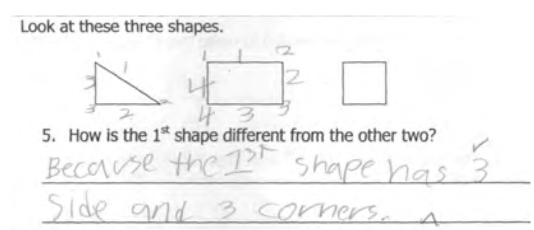
Student O



Student P



Student Q



2nd Grade Task 3 Don's Shapes

Student Task	Name, describe and classify two-dimensional shapes. Identify two shapes that form another shape. Sort shapes based on attributes. Write a complete comparison statement of two or more shapes based on one or more attributes.
Core Idea 4: Geometry and Measurement	**Students will recognize and use characteristics, properties, and relationships of two-dimensional shapes.** • Describe and classify two- and three-dimensional shapes according to their attributes and/or parts of their shapes. • Develop an understanding of how shapes can be put together or taken apart to form other shapes.

Mathematics in this task:

- Identify and name two-dimensional shapes
- Identify the two two-dimensional shapes used to form another shape
- Classify two-dimensional shapes according to the attribute of "having straight sides" and "not having straight sides"
- Make a comparison statement between a set of two-dimensional shapes using one or more attributes of the shapes as the basis for the comparison

Based on teacher observations, this is what second graders know and are able to do:

- They could identify and name a rectangle, triangle, and square
- They were able to identify the square and triangle that formed the bigger shape
- They were able to identify the two triangles that formed the bigger shape
- Many were able to classify shapes based on having straight sides or not

Areas of difficulty for second graders:

- Many incorrectly placed one or both triangle shapes, and/or the hexagon shape, when classifying the two-dimensional shapes based on having straight sides or not
- They had trouble making a complete and accurate comparison statement between two or more shapes.

Strategies used by successful students:

- Look at the attributes of shapes, rather than the overall appearance of the shapes, to name and classify them
- Use an accurate *The triangle has ____ but the other shapes have ____* sentence form to make a comparison statement based on attributes.

MARS Test Task 3 Frequency Distribution and Bar Graph, Grade 2

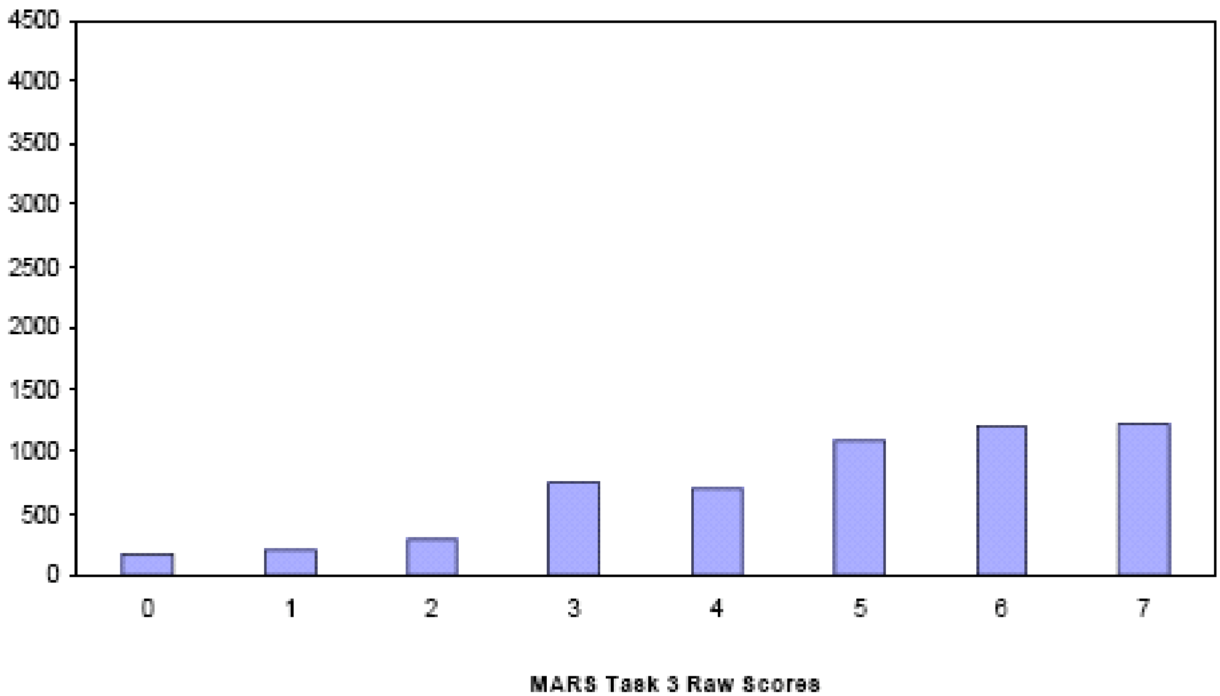
Task 3 - Don's Shapes

Mean: 4.82 StdDev: 1.86

Table 12: Frequency Distribution of MARS Test Task 3, Grade 2

Task 3 Scores	Student Count	% at or below	% at or above
0	165	2.9%	100.0%
1	197	6.5%	97.1%
2	283	11.5%	93.5%
3	746	24.8%	88.5%
4	703	37.3%	75.2%
5	1093	56.8%	62.7%
6	1202	78.3%	43.2%
7	1219	100.0%	21.7%

Figure 21: Bar Graph of MARS Test Task 3 Raw Scores, Grade 2



Don’s Shapes

Points	Understandings	Misunderstandings
0	All the students in the sample with this score attempted the task. They might be able to name one of the two shapes that form the larger shape, but they could not satisfy all the demands of any one part.	Students at this level were not able to identify and name a rectangle. They saw only one of the smaller shapes in the forms made of two shapes. When comparing shapes, they used overly general statements, such as “They are different”.
1 - 2	Students in this group could identify and name a rectangle. They might be able to name one or both of the shapes that form the larger shape.	Students at this level made more than one classification error on problem 4. They named shapes based on their appearance, calling a right triangle a “slide” or a square a “box”.
3	Students could identify and name a rectangle. 80% of these students could identify both of the shapes that form the larger shapes. 90% of these students were able to correctly classify shapes A and E as not having straight sides.	Students were not able to attend to the constraints in problem 4; most could not classify and sort all the shapes. 25% of the students placed only one shape in each category. Another 25% inaccurately placed Shape F into the “does not have straight sides” category, perhaps because it has the appearance of a “diagonal” or “slanted” line.
4	Students could identify and name the basic shapes, and recognized them when they were used to form a larger shape. Two-thirds were able to make an accurate statement about the triangle based on the attribute of number of sides or number of vertices. 20% could correctly classify all shapes.	80 % of these students could not accurately classify all the shapes. Around half of them could not accurately sort one or both of the triangles in question 4, and 1 in 4 of them could not accurately sort the octagon. Almost half of these students made a statement based on appearance, rather than attribute, when asked to compare the shapes in question 5.
6	Students could name and identify the simple shapes. These students could accurately classify the shapes in question 4. They understood that a right triangle is a triangle because of its attributes, regardless of its orientation on the page or that it doesn’t look like an equilateral triangle. 90% of these students could make an accurate statement of the attributes of *either* the triangle or the other two shapes in question 5.	Students were not able to make a complete and explicit comparison statement using the attributes of *both* the triangle and the other two shapes in question 5.
7	Students were able to identify, name, classify and sort the two-dimensional shapes. They focused on the attributes of the shape, rather than their overall appearance. Students made explicit comparison statements in question 5.	

Teaching Implications

Two Dutch researchers, Pierre van Hiele and Dina van Hiele-Geldof, have provided an insight into the differences in geometric thinking that can help inform our work with young children. Their model includes a five-level hierarchy of understanding spatial ideas, but for the purposes of our work with young children, we will focus on the first two levels:

Level 0 ~ Visualization

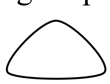
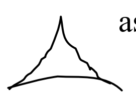
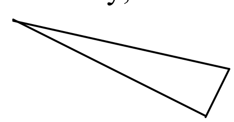
At this level, children see a geometric figure as a whole and would describe a shape by what it looks like. For example, a rectangle might be described as a door.

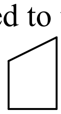
Level 1 ~ Analysis

At this level, students can describe the properties of the figure. For example, they might say that a rectangle has four sides and the opposite sides are parallel.

Visualization and analysis are the foundation for understanding geometry, and in the article “Begin With Play” (Pierre M. Van Hiele, "Developing Geometric Thinking Through Activities That Begin with Play," *Teaching Children Mathematics* 5, no. 6 (February 1999): pp. 310-16) van Hiele explains that young children must first learn about shapes through many informal experiences. It is through these “play” experiences that children begin to make sense of two-dimensional and three-dimensional forms.

“As children work with blocks of various kinds, as they examine and analyze boxes and containers, they become more and more discriminating […] They learn to recognize the attributes of shapes, to notice how shapes are alike and different. They learn to identify and sort by such attributes as the number of sides, the number of corners, the number of faces, the relationship of lengths of sides, whether the figure has straight lines or curves, whether it rolls or doesn’t roll, whether it has symmetry or not. They learn what changes and what stays the same when direction or position or size is changed.” (from Understanding Geometry, by Kathy Richardson, Lummi Bay Publishing, 1999)

The question for us is, in what ways does limiting the primary classroom geometry experience to simply naming shapes lead to misconceptions in their thinking? In one study, children identified shapes such as  or  as triangles, for example, but  as not. (Richardson, 1999)

We want children to understand that the name of shape actually *defines* it. A triangle is a triangle not because they can identify this shape  as a triangle, but because they can identify *any* closed, two-dimensional shape that has three straight sides and three vertices as a triangle, regardless of its orientation or size. We see this a lot with pattern blocks, where students learn the correct mathematical *name* of the red piece as “trapezoid”. But they don’t learn the *definition* and *attributes* attached to that name, only that this is the name of a particular piece. So, when a student sees the shape  he or she does not also recognize it as a trapezoid.

The language of geometry becomes meaningful when students are able to use it to label the attributes that define a shape. According to Richardson (1999), “they will be more apt to focus on the attributes if they are asked to describe the attributes in their own words using whatever language makes sense to them. The first step in language development should be learning to see, to notice, to discriminate; the next step should be determining which shapes with similar attributes go together and why […] Learning the language prematurely can only cause confusion and misconceptions and keep children from looking closely at important attributes.” (p. 7)

Ideas for Action Research

John van de Walle offers some activities for helping students focus on both attributes and the language attached to those attributes in his book, "Teaching Student-Centered Mathematics, K-3" (Pearson, 2006).

- **Secret Sort** (p. 194) The secret sorting activity can be used to introduce a new property. Using two-dimensional shape cut-outs (BLM 20 – 26 from van de Walle, or your own), sort the shapes so that all have at least on right angle. When students discover your rule, you have an opportunity to talk more about that property.
- **What's My Shape** (p. 195) Make two sets of 2-D Shapes on paper. Cut one set and glue them inside a folded half-sheet of construction paper to make "secret shape" folders. A student leader holds one "secret shape" folder and asks the other students in his or her group to find the shape that matches the shape in the folder. They can only ask questions that may be answered with "yes" or "no". They can sort the working shapes as many times as they need to in order to help them narrow the possibilities of which shape can be in the "secret folder". Students who are working to figure out what's inside the "secret folder" may not point to a shape and ask, "Is this it?" The final piece is then compared to what's in the folder. (The more shapes in the collection that resemble the secret shape, the more difficult the task.)
- **Mystery Definition** (p. 207) On the overhead, present a set of 5 or so two-dimensional shapes that have something in common (example, a set of 4 sided shapes in which all the sides are equal, and show a variety of rhombi, including a square). Make another set of 5 or so two-dimensional shapes that do not have the shared attribute of the first set (either because it doesn't have three sides, or not all the sides are equal). Make a third set of 5 or so shapes, some of which have the attribute and some of which do not. Students must decide which shapes in the third set go with the shapes in the first set.
  "*All of these* have something in common" is the first set.
  "*None of these* has it" is the second set.
  "*Which of these* also has it" is the third set.

When doing these activities, listen carefully to the *natural* language that students are using to talk about the shapes and attributes. After their definitions have been discussed and compared, are there more formal definitions that will add to or clarify the ideas they are developing? In what ways can you begin to introduce and have students use more formal definitions and mathematical vocabulary, and how can that be done to enhance their learning, rather than impede or overwrite their thinking?

In what ways can students make their thinking and explanations more precise by using written explanations? Where in the above activities can students begin to write about the similarities and differences in the shapes and attributes they have been comparing? What language *structures* can you provide to help support their efforts? When reviewing student work, does it reflect that they understand the explicit nature of a direct comparison, and that it's focused on one attribute and requires a statement of both sets in relation to that attribute?

Performance Assessment Task
Don's Shapes Grade 2
The task challenges a student to demonstrate understanding of the attributes of two-dimensional shapes. Students must be able to identify shapes and make comparisons between and among shapes. Students must make sense of composite shapes when decomposed; they must be able to identify the shapes when the composite shape is decomposed.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Reason with shapes and their attributes.** 2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
Common Core State Standards Math – Standards of Mathematical Practice
MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
2	2008	7	4	75 %