Problem of the Month: *Cutting a Cube*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving, and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The POM is structured with a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially as the key challenge for grades kindergarten and one. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview
In the Problem of the Month *Cutting a Cube,* students use two- and three-dimensional geometry to solve problems involving cubes and nets. The mathematical topics that underlie this POM are the attributes of polygons - faces, edges, vertices – as well as spatial visualization, counting strategies, classification and geometric solids.

The problem asks the students to examine a cube to analyze the attributes of a cube and how a cube can be cut into a flat pattern, as well as what flat patterns can be made into cubes. In the first level of the POM, students are presented with a model of a cube. Their task is to recognize and identify the attributes of a cube. In Level B,

students are presented with situations that involve determining the least number of cuts it takes to divide a cube into a single flat pattern or net. The students explain why it takes 7 cuts to make a cube into a net. In Level C, students explain that any arbitrary 7 cuts do not determine a unique net, and they show multiple examples of nets that can be folded into a cube. In Level D, students determine all the unique nets that fold into a cube and explain a valid process for determining all the unique nets that fold into a cube. In Level E, students draw all the unique hexominoes and explain a valid process for determining all the unique hexominoes.

Mathematical Concepts

Spatial visualization plays an important part in real-world experiences. Whether designing the most complex structures created by designers, architects, and construction workers or arranging the furniture in a room, spatial awareness and visualization are essential. In this POM, students explore various aspects of spatial visualization, including designs in both two and three-dimensional space. This involves examining flat patterns as well as solid objects and understanding the relationship between the two objects. Students use polygons and develop understandings of their attributes both in the plane and on the surface of polyhedra. In addition to exploring the geometric aspects of this POM, students seek to find patterns, count numbers of possibilities, and justify their answers. The mathematics involved in these aspects of the problem is often called discrete mathematics.

Problem of the Month
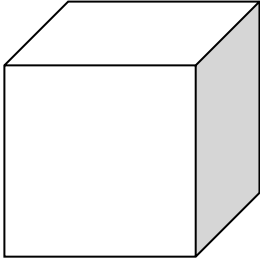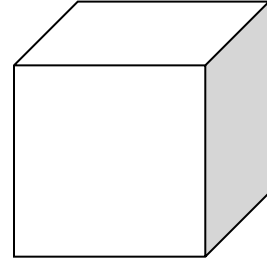
Cutting a Cube

Level A

A cube is a very interesting object. So we are going to examine it.
Without holding a cube, try to picture it in your mind. How many sides (faces) does a cube have?

How many corners (vertices) does a cube have?

How many lines (edges) does a cube have?

What can we say about the size of the sides (faces) and the lines (edges)?

When you have made your guess (conjecture), then hold a cube and check (verify) your answers to the questions listed above.

How might you be able to remember the parts (attributes) of a cube?
Explain.

Level B

A cube is like a box. You might think of it as a special type of cardboard box. We could cut up a cardboard box and make it into one large flat piece of cardboard. We often do that when we want to recycle the cardboard. The easiest way to cut a cardboard box is to cut along the lines (edges). How many cuts does it take to make the box into one flat piece? In other words, what is the least number of lines (edges) that need to be cut so that the cardboard is in one flat piece? Remember all the sides of the cardboard must remain attached in one single flat piece. What is the least number of cuts that need to be made? Explain how you determined your answer.

Why do you think your answer is correct?

Write a note to a friend to convince your friend that your solution will always work for every cube.

Level C

When you cut a cube into one flat piece, we call that piece a **net**. The reason we call it a net is because we can trace the pattern of the flat piece on a piece of paper or cloth material. If we cut out the pattern, we can fold it back over the cube, surrounding it like a net.

The nets that cover a cube can be cut into different patterns. One net looks like a cross. It has four faces in a column and two more faces on either side of that column. How would you cut the cube (which edges) to make the net into a cross pattern? Is there more than one way to cut the cube to make a cross?
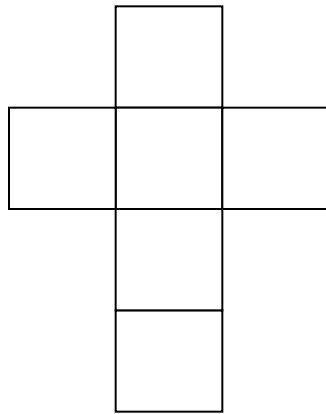
Find some different net patterns that would also cover a cube. Determine how you would have to cut the cubes to make them into new net patterns. Explain your methods.

Are there ways to cut the cube so that it won't make a net? Explain your thinking.

Sometimes you might think two nets are different but, if you move one around, it then looks exactly like the other net. How can you tell if two nets are different? Explain and define the difference.

Level D

We want to find all the nets that can be folded into a cube. For this investigation, we will define two nets as being the same if we can turn (rotate), move (translate), or flip (reflect) the net and the two nets cover each other exactly.

How many unique nets fold into a cube? Draw all possible nets that can be folded into a cube.

How did you go about determining the number of nets?

How do you know that you have found all the unique nets that fold into a cube?

Convince a skeptic that you have found all the possible nets of a cube.

Level E

There are some patterns of six squares that do not fold into a cube. For example, a pattern of six faces arranged in three columns of two squares all attached together cannot fold into a cube.
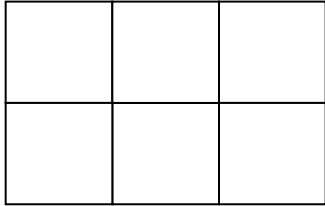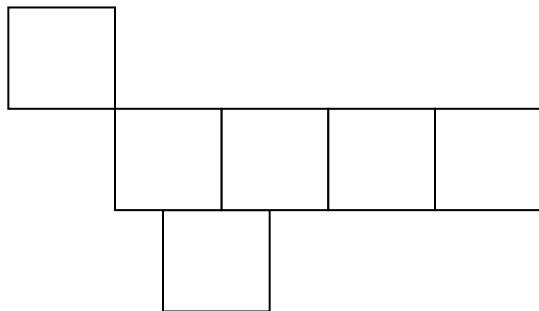

We call these patterns of six attached squares **hexominoes**. The word is like dominoes, except it has six squares instead of just two squares. A hexomino has six squares, and all squares must share at least one side with another square, and all the vertices of the squares must coincide. Arrangement A below is a hexomino, while B is not.

A.
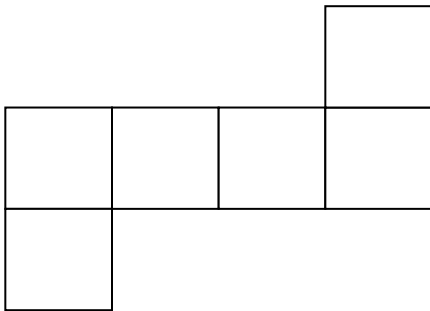
B.

Find all the configurations of hexominoes. These include all the nets that fold into cubes and all the other hexominoes that can't fold into cubes.
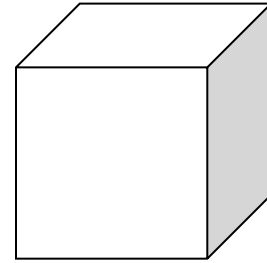
• Draw all unique hexominoes.

• How did you go about determining the number of unique hexominoes?

• How do you know that you have found all the unique hexominoes?

• What percentage of hexominoes are nets that fold into cubes?

• Convince a skeptic that you have found all unique hexominoes.

Problem of the Month
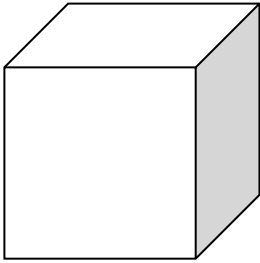
Cutting a Cube

Primary Version Level A

Materials: One large cube for the teacher to use during discussion and small cubes for every student to hold and examine in their groups.

Discussion on the rug: The teacher holds up one large cube. **"A cube is a very interesting object. So we are going to examine it. What does examine mean? Who does examinations? What do you think are the parts of the cube we can examine?"** The teacher asks questions to have the children think about parts, especially faces and corners, and maybe lines or edges.

In small groups: Each student is holding a cube. The teacher asks the following questions, only going on to the next question if students have success.

"We are going to examine our cube.
1. How many flat sides does a cube have?
2. How many corners does a cube have?
3. How many lines or edges does a cube have?"

At the end of the investigation have students either discuss or dictate a response to this summary question: **"How can you remember the parts of a cube?"**

Problem of the Month
Cutting a Cube
Task Description – Level A
This task challenges students to explore various aspects of a cube without looking at a cube. Students are asked to examine a cube and to determine the number of faces ("flat sides"), corners (vertices), and edges a cube has. Additionally, students are asked to synthesize this discussion by explaining and/or writing a response to the question, "How can you remember the parts of a cube?"
Common Core State Standards Math - Content Standards
Geometry   **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**   K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.   K.G.2 Correctly name shapes regardless of their orientations or overall size.   **Reason with shapes and attributes.**   1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others.   Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.   **MP.5 Use appropriate tools strategically.**   Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.   **MP.6   Attend to precision.**   Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Problem of the Month
Cutting a Cube
Task Description – Level B
The problem of the month asks students to use spatial reasoning to determine the least number of cuts necessary to make a cube flat.   Additionally, students are asked to explain how their answer was determined, how they know it is correct, and to write a letter to a friend to convince them that their solution will always work for every cube.
Common Core State Standards Math - Content Standards

Geometry
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
K.G.2 Correctly name shapes regardless of their orientations or overall size.
K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three- dimensional ("solid").
Reason with shapes and attributes.
1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3   Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
MP.5   Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
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Problem of the Month
Cutting a Cube
Task Description – Level C
The problem of the month asks the students to use spatial reasoning to determine different net patterns to cover a cube. Students will be able to explain that any arbitrary 7 cuts do not determine a unique net. Additionally, students are asked to have an understanding of the similarities and differences of determined nets - what makes nets the same and what makes them different?
Common Core State Standards Math - Content Standards
Geometry **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. K.G.2 Correctly name shapes regardless of their orientations or overall size. K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three- dimensional ("solid"). **Reason with shapes and attributes.** 1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. 2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.1. Verify experimentally the properties of rotations, reflections, and translations…
Common Core State Standards Math – Standards of Mathematical Practice
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Problem of the Month:
Cutting a Cube
Task Description – Level D
The problem of the month asks students to use spatial reasoning to find all the unique nets that can be folded into a cube and to justify how they know that all of the nets have been found. Students are asked to explain a valid process for determining all the unique nets that fold into a cube.
Common Core State Standards Math - Content Standards
Geometry **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. K.G.2 Correctly name shapes regardless of their orientations or overall size. K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three- dimensional ("solid"). **Reason with shapes and attributes.** 1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. 2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.1. Verify experimentally the properties of rotations, reflections, and translations… **High School – Geometry - Geometric Measurement and Dimension** **Visualize relationships between two-dimensional and three-dimensional objects** G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects**.**
Common Core State Standards Math – Standards of Mathematical Practice
MP.3   Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.5   Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. **MP.6   Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Problem of the Month
Cutting a Cube
Task Description – Level E
The problem of the month asks students to use spatial reasoning to find all the configurations for hexominoes – patterns of 6 squares that do not fold into a cube. Students are asked to explain a valid process for determining all the unique hexominoes and to convince a skeptic that all the unique hexominoes have been found.
Common Core State Standards Math - Content Standards

Geometry
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
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K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three- dimensional ("solid").
Reason with shapes and attributes.
1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes..
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Understand congruence and similarity using physical models, transparencies, or geometry software.
8.G.1. Verify experimentally the properties of rotations, reflections, and translations…
High School – Geometry - Geometric Measurement and Dimension
Visualize relationships between two-dimensional and three-dimensional objects
G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects**.**

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month
Cutting a Cube
Task Description – Primary Level
This task challenges students to explore various aspects of a cube. Students are asked to examine a cube and to determine the number of faces ("flat sides"), corners (vertices), and edges a cube has. Additionally, the students are asked to synthesize this discussion by explaining and/or dictating a response to the question, "How can you remember the parts of a cube?"
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Geometry   **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**   K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.   K.G.2 Correctly name shapes regardless of their orientations or overall size.   **Reason with shapes and attributes.**   1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
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