Problem of the Month: Cubism

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:

In the Problem of the Month *Cubism,* students use spatial visualization and properties of geometry to solve problems and create models and representations of three-dimensional objects. The mathematical topics that underlie this POM are the attributes of two and three-dimensional geometry, spatial visualization, isometric and orthogonal drawings, counting principles and geometric justification.

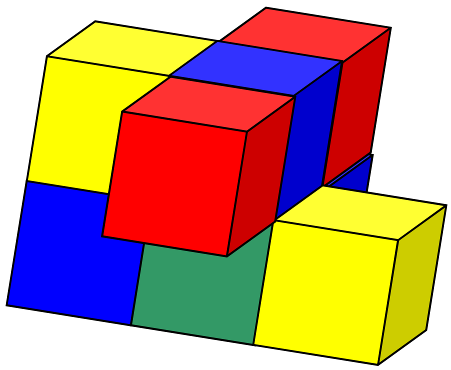
The problem asks students to explore three-dimensional objects in various problem situations. In the primary level of the POM, students are presented with a three-dimensional object created from multilink blocks. The students are then asked to visualize the object and determine the number of cubes. The second part of the task involves building a three-dimensional object out of cubes by examining drawings of the object in both a flat pattern view of the bottom of the object as well as a three-dimensional isometric view. In level A, students are presented with a more complex object made of cubes. The perspective of the picture of the object is not sufficient to determine whether the object has a hidden cube. The students are asked to determine and justify the number of unit cubes in the structure. In level B, students are presented with a picture of a three-dimensional object with many cubes some seen and others assumed/imagined. The students are asked to determine the number of unit cubes in the object and explain their reasoning. They are also asked to draw a representation of the object using isometric dot paper. In level C, students are given a large cube comprised of 27 red unit cubes. The large cube is painted on the outside with yellow paint covering the surface area. The students are asked to find the number of differently colored cubes that make up the larger cube. In level D, the students are given five orthogonal views of an object. The students are asked to determine the composition of the three-dimensional structure – building it and drawing it on isometric paper. In level E, students are presented with Rubik's Cube. Students are asked to determine justify the number of possible states the cube could attain.

Mathematical Concepts:
In this POM, students explore three-dimensional geometry. Students use spatial visualization, properties of geometry, models and representations, mathematical reasoning and counting principles to solve problems. Students interpret and represent with isometric and orthogonal drawings. They explore, visualize, count, conclude, generalize, and justify while solving non-routine problems.

Problem of the Month

Cubism

Level A
Pablo built the figure below using cubes.

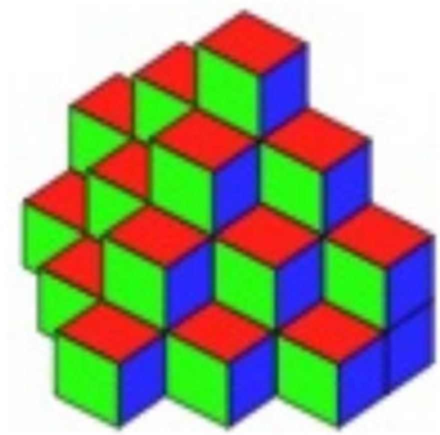


How many cubes did Pablo use?

Describe the cubes he used in terms of position and color.

How do you know you are correct? Explain your reasoning.

Level B
Pablo sees a picture of the cube structure below:



He builds the structure only using unit cubes. Build an identical structure using cubes. How many cubes did you use? Explain how you knew how many cubes to use and how they were arranged.

The drawing shown above is called an isometric drawing because it shows a three-dimensional perspective of the structure. Drawing isometric images is an important skill. Isometric dot paper can be used to assist you in drawing the image. Use isometric dot paper to draw the same image.

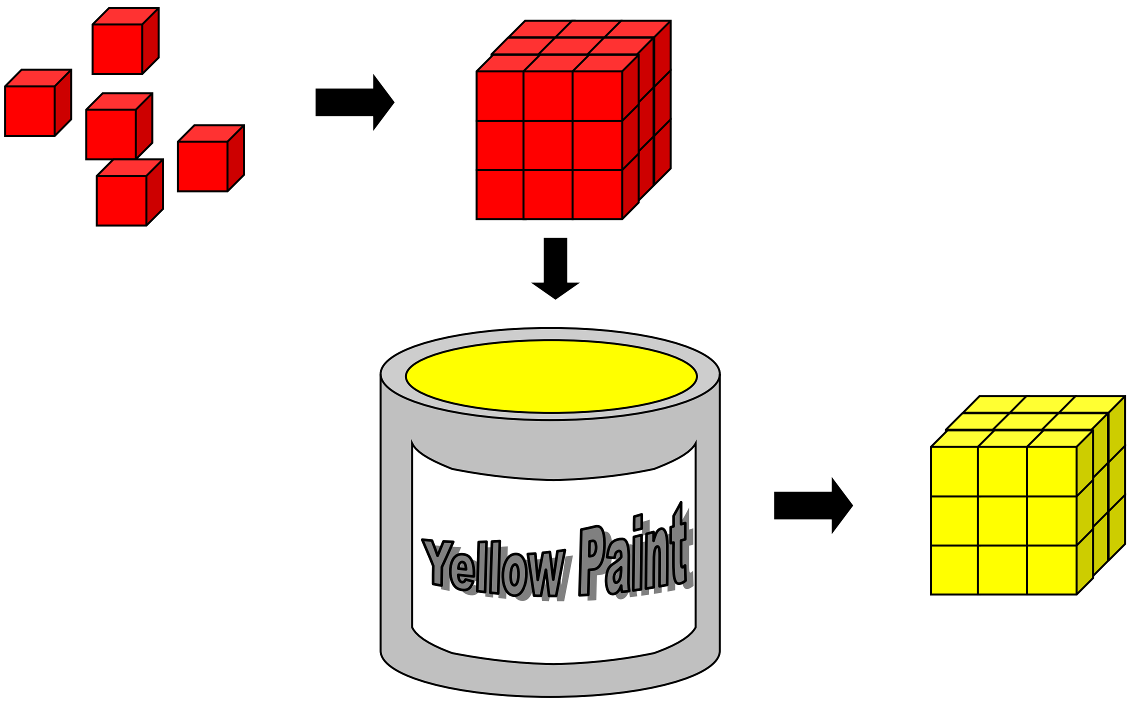
Use isometric dot paper to draw the same structure from a different perspective.

Suppose each unit cube is colored so the opposite faces of the cube are the same color. How many unit faces that show on the surface of the structure are red? How many are green? How many are blue? What is the surface area of the structure?

Pablo thinks again and says, “The structure might have more cubes than I can see from this perspective.” How many more cubes can the structure have? Explain your reasoning.

Level C

Paloma starts with a number of red unit cubes (1 inch by 1 inch by 1 inch). She uses the unit cubes to form a larger cube (3 inches by 3 inches by 3 inches). Paloma now gets a bucket of yellow paint and she dips the whole large cube in the paint covering the outside yellow.



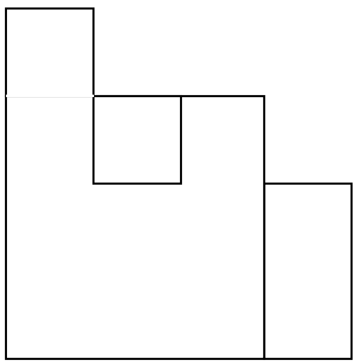
Now that the paint has dried on the larger cube, determine the color of the faces of each of the unit cubes that make up the larger yellow covered cube. Describe the number of unit cubes that make up the larger cube. For each unit cube, describe the color of each face and the position of the unit cube on the larger cube.

Suppose you started out with a larger red cube that was (6 inches by 6 inches by 6 inches). If you followed the same process of covering the outside with yellow paint, describe the number of unit cubes that make up the larger cube. For each unit cube, describe the color of each face and the position of the unit cube on the larger cube.

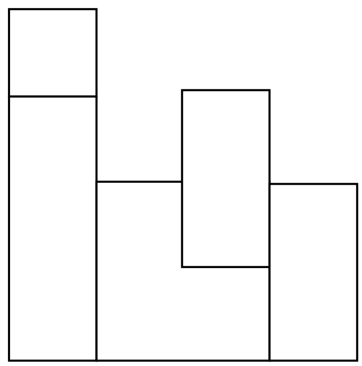
Consider any size cube made up of red unit cubes that were then dipped and colored yellow. What generalization can be made about the different sets of colored cubes? Explain.

Level D

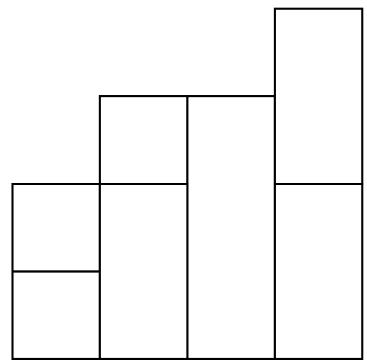
Isometric drawings show perspective. Another type of drawing used in blueprints for architectural purposes are orthogonal drawings. Orthogonal drawings only show two-dimensions. Below are five orthogonal drawing of the same structure (built from cubes). The different views are sometimes called elevations. The solid lines drawn inside the figure indicate surface edges and imply varying depth across the elevation.



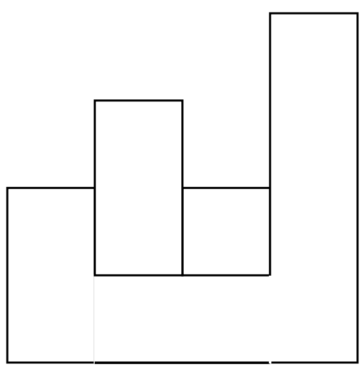
Left View



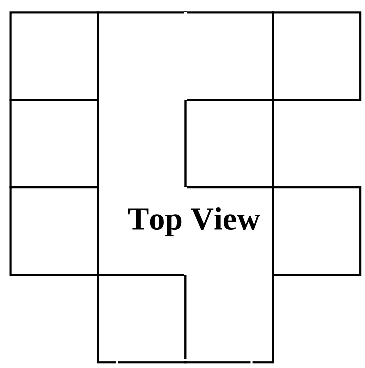
Front View



Right View



Back View



Top View

Front

Determine the composition of the three-dimensional structure. Build it using unit cubes. Calculate the volume and surface area of the structure. Using the isometric dot paper, draw the structure from two different perspectives. Explain how you determined the composition of the structure.

Level E

The Rubik's Cube is a 3-D mechanical puzzle invented in 1974 by Hungarian sculptor and professor of architecture Ernő Rubik. Originally called the "Magic Cube," the puzzle was licensed by Rubik to be sold by Ideal Toys in 1980 and won the German Game of the Year special award for Best Puzzle that year.

The classic Rubik's Cube is comprised of 27 unit cubes arranged as a 3-by-3-by-3 cube. Each of the six faces consists of nine unit squares. The squares' colors are traditionally white, red, blue, orange, green, and yellow. Each square color appears nine times around the cube. A pivot mechanism enables each face to turn independently, thus mixing up the colors.

The Ideal Toy Company stated on the package of the original Rubik's Cube that there were more than three billion possible states the cube could attain. That was a significant under-estimation. Determine the actual number of possible states there are. Explain your solution.

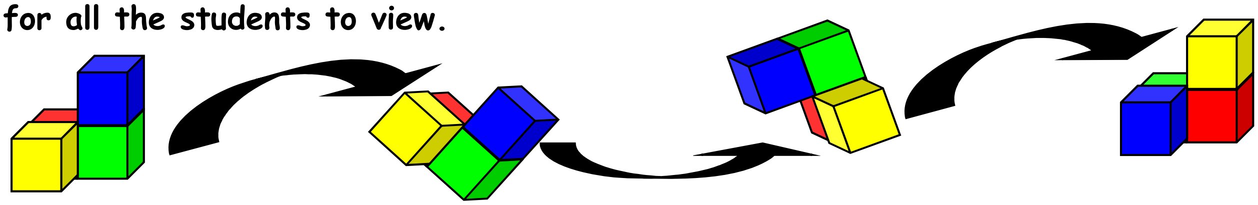
Problem of the Month

Cubism

Primary Version Level A

Materials: Multi-link cubes in bags (4 red, 2 yellow, 2 green, 3 blue), paper, and pencil.

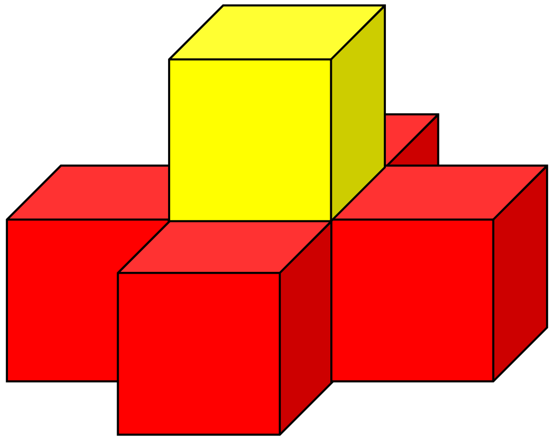
Discussion on the rug: Teacher says to the class, "I have made a figure out of cubes." The teacher shows the class the figure to the class, rotating it for all the students to view.



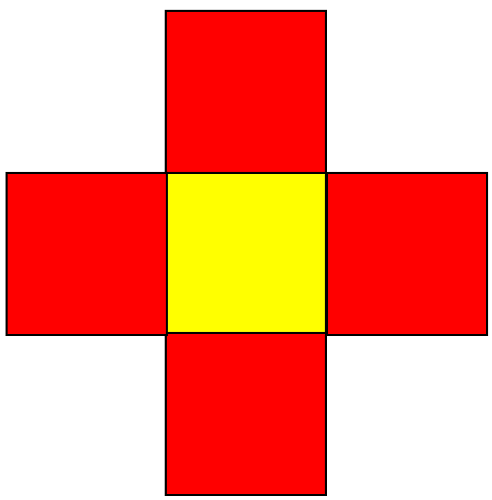
The teacher asks the students to determine the number of cubes in the figure. The teacher says to the class, "Show on your fingers how many cubes there are." The class responds. The teacher then asks a student to explain how they know. The teacher says, "How can we build this figure?" The teacher allows the students time to build the figure as s/he continues to hold it up for the class to see. "Make sure you use the same color cubes," the teacher reminds the class. Then, students explain how they know their figure is the same and how they put it together. Students demonstrate in front of the class. (The process may be repeated with a different figure until the students have comfort with the procedure of visualizing and building figures.)

Back at their desks: Each student has access to a bag of cubes, the primary version handout, pencil, and paper. The teacher instructs the class to look at the pictures of the figure in the handout. The students are told that these pictures are two different views of the same figure. One view is looking from the side, and the other is looking straight at the bottom. The students are given time to build a figure that matches the picture. At the end of the investigation, have students either discuss or dictate a response to the prompt: "How many cubes are in the figure? What are the colors of the cubes? How are they put together? Tell me how you built your figure."

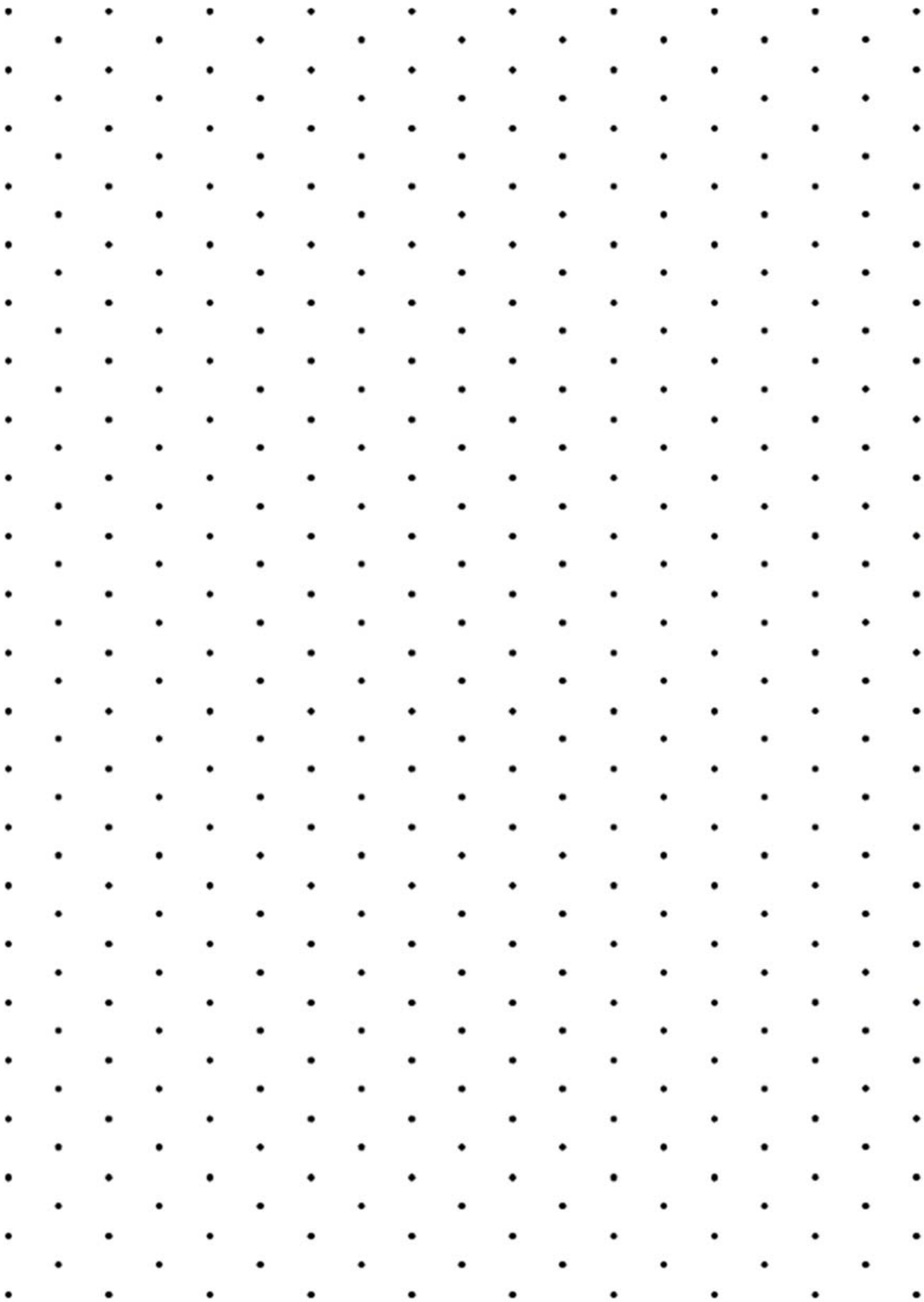
Primary Version Level A - Handout



Side View



Bottom View



Problem of the Month
Cubism
Task Description – Level A
This task challenges students to visualize a three-dimensional figure composed of cubes. The students are to identify the number of cubes in the figure. The students are to describe the cubes they used in terms of position and color and to justify the number of cubes that are in the figure.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Identify and describe shapes.** K.G.1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. K.G.2. Correctly name shapes regardless of their orientations or overall size. K.G.3. Identify shapes as two-dimensional or three-dimensional ("solid"). **Analyze, compare, create, and compose shapes.** K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts. K.G.5. Model shapes in the world by building shapes from components and drawing shapes. K.G.6. Compose simple shapes to form larger shapes. **Reason with shapes and their attributes.** 1.G.1. Distinguish between defining attributes versus non-defining attributes; build and draw shapes to possess defining attributes.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP. 3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Cubism

Task Description – Level B

This task challenges a student to visualize and analyze a three-dimensional figure composed of cubes drawn from an isometric perspective. The picture is a three-dimensional object with many cubes, some seen and others assumed/imagined. The students are asked to determine the number of unit cubes in the object and explain their reasoning. They are also asked to draw a representation of the object using isometric dot paper.

Common Core State Standards Math - Content Standards

Geometry

Reason with shapes and their attributes.

1.G.1. Distinguish between defining attributes versus non-defining attributes; build and draw shapes to possess defining attributes.

1.G.2. Compose two-dimensional shapes or three-dimensional shapes to create a composite shape, and compose new shapes from the composite shape.

3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month
Cubism
Task Description – Level C
This task challenges a student to determine the surface areas of cubes embedded in cubes. Students are given a large cube comprised of 27 red unit cubes. The large cube is painted on the outside with yellow paint covering the surface area. The students are asked to find the number of differently colored cubes that make up the larger cube.
Common Core State Standards Math - Content Standards
<u>**Measurement and Data**</u> **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** 5.MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure, which can be packed without gaps or overlaps using **n** unit cubes is said to have a volume of n cubic units. 5.MD.4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. 5.MD.5. Relate volume to the operations of multiplication and addition and solve real world and mathematical <u>**Geometry**</u> **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** 7.G.6. Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month:
Cubism
Task Description – Level D
This task challenges a student to use visualization and reasoning to construct and draw a three-dimensional object. The students are given five orthogonal views of and object. The students are asked to determine the composition of the three-dimensional structure – building it and drawing it on isometric paper.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Draw, construct, and describe geometrical figures and describe the relationships between them.** 7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month
Cubism
Task Description – Level E
This task challenges a student to analyze the permutations of the Rubik's Cube. Students are presented with the Rubik's Cube. Students are asked to determine justify the number of possible states the cube could attain.
Common Core State Standards Math - Content Standards
High School - Statistics and Probability - Conditional Probability and the Rules of Probability **Use the rules of probability to compute probabilities of compound events in a uniform probability model.** S-CP.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month
Cubism
Task Description – Primary Level
This task challenges students to visualize a three-dimensional figure composed of cubes. Students are presented with a three-dimensional object created from multilink blocks. The students are then asked to visualize the object and determine the number of cubes. The second part of the task involves building a three-dimensional object out of cubes by examining drawings of the object in both a flat pattern view of the bottom of the object as well as a three-dimensional isometric view.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Identify and describe shapes** K.G.1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. K.G.2. Correctly name shapes regardless of their orientations or overall size. K.G.3. Identify shapes as two-dimensional or three-dimensional (“solid”). **Analyze, compare, create, and compose shapes.** K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts. K.G.5. Model shapes in the world by building shapes from components and drawing shapes. K.G.6. Compose simple shapes to form larger shapes.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.