# Cindy's Cats

This problem gives you the chance to:

• solve fraction problems in a practical context

Cindy has 3 cats: Sammy, Tommy and Suzi.



1. Cindy feeds them on Cat Crunchies.

Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of the box.

What fraction of a whole box do the cats eat, in all, each day?

Show how you figured this out.

2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

Which of the two cats sleeps for longer?

How much longer does it sleep each day?

Show how you figured this out.

3. Cindy's cats often share a carton of cat milk.

Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and Suzi always drinks  $\frac{1}{6}$  of the carton.

What fraction of the carton of cat milk is left over?

Show how you figured it out.

4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.



Sammy used it for  $\frac{1}{4}$  of the times and Tommy used it for  $\frac{3}{10}$  of the times.

How many times did Suzi use the cat door?

Explain how you figured it out.

Task 3: Cindy's Cats	Rı	ubric
The core elements of performance required by this task are:  • solve fraction problems in a practical context  Record on these prodit for appoint appoint of performance should be assigned as follows:	points	section points
Based on these, credit for specific aspects of performance should be assigned as follows		
1. Gives correct answer: 7/8	1	
Show work such as: $1/2 + 1/4 + 1/8 = 4/8 + 2/8 + 1/8 = 7/8$	1	2
2. Gives correct answer: <b>Suzi by 1/10</b> of a day. Accept 10% or $2^4/_{10}$ hours	1	
Shows correct work such as: $3/5 = 6/10$ and so $7/10 - 6/10 = 1/10$	1	2
Accept work in percents.		2
3. Gives correct answer: 1/12	1	
Shows correct work such as: $1/3 + 5/12 + 1/6 = 4/12 + 5/12 + 2/12 = 11/12$ 12/12 - 11/12 = 1/12	1	
Partial credit Gives answer 11/12 and shows correct work.	(1)	2
4. Gives correct answer: <b>45</b> times	1	
Gives correct explanations such as: $1/4 = 25/100$ and $3/10 = 30/100$ So $25 + 30 = 55$		
Therefore Sammy and Tommy used it 55 times. $100 - 55 = 45$ This means Suzi used it 45 times.	1	
Partial credit		
Gives answer such as 9/20, 18/40, or 45% and shows correct work.	(1)	2
Total Points		8

# Cindy's Cats

Work the task and look at the rubric.	What are the key mathematical ideas in this task?

When thinking about this task, students had access to the mathematics using a variety of strategies. Look at student work and see if you can categorize the strategies. Without judging whether the strategy was used with complete success, how many of your students attempted to use:

Common denominators	Decimals	Fractions over 100	Percents	Drawing or using a model	Invented a strategy

Now look at student work for part one. This is the entry level portion for most students.
How many of your students seemed to understand that the operation was
addition?
How many of your students tried some kind of subtraction?
How many of your students have work where the operation is unclear?

Look at the answers in part 1. How many of your students have answers of:

7/8	87%	3/14	1/14	3/4	3/8	Whole number	Other

What do students understand? What are students confused about? In each case is the confusion caused by misunderstood or incorrectly memorized algorithms or procedures? Is the confusion a result of opportunity to learn? Is the confusion caused by not being able to think about the meaning of fractions? Is the confusion caused by not understanding operations?

Suzi	Either	Either	Each of	Either	Tommy	4/10	7/10	Other
1/10	Whole number	3/10	them slept	1 3/10	1/10			
	answer		half a day					

Did any of your students attempt to change the fractions into hours in a day (make it into 24<sup>th</sup>'s)? Were students struggling with identifying the correct operation? Were students struggling with how to compare the fractions?

Many students understood that the operation involved in part 3 involved subtraction, but because they could only think about two fractions at a time they subtracted too early. What strategies do you use in class to help students figure out the correct operation for a problem? What kinds of models might help students make sense of the "action" of the problem?

Now look at student work for the final part of the task. What mathematics did students need to understand in order to work this task? What are the big mathematical ideas? What strategies do you think students have to make sense of this part of the task?

Look at student work. How many of your students put:

45	55	45/100 or 9/20	55/100 or 22/40	4/14	Other

What is the piece each student is missing? How are they showing a different misconception?

What do you think are some of the layers that need to be developed to build a deep understanding of fractions?

What are some of the issues that arise in working in context that don't show up when students work a page of practice problems?

# Looking at Student Work on Cindy's Cats

An important part of solving word problems is understanding what you know and what is the result of each calculation. Student A makes use of clear labels to define the numbers in the problem and show why the calculations make sense. The student uses common denominators throughout. Student A is able to think about multiplication to find 1/4 of 100 and 3/10 of 100.

## Student A

1. Cindy feeds them on Cat Crunchies. Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of

the box. What fraction of a whole box do the cats eat, in all, each day?

Show how you figured this out.

I converted & into &, (for Sammy) then I added & (for Tommy), and I converted & into &, (for Suzi) so I did \( \frac{4}{8} + \frac{1}{8} + \frac{2}{8} = \bigg( \frac{1}{8} \) total.

2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

Which of the two cats sleeps for longer?

How much longer does it sleep each day?

Show how you figured this out.

# Student A, part 2

3. Cindy's cats often share a carton of cat milk.

Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and

Suzi always drinks  $\frac{1}{6}$  of the carton.

What fraction of the carton of cat milk is left over?

Show how you figured it out. I converted & into 1/2 (for Sammy), plus 1/2, (for Tommy) and I also converted & into 1/2, (for Suzi) then I added 4+5+2=1, then I did 12-12 (2 lefforer)

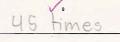
4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.



Sammy used it for  $\frac{1}{4}$  of the times and Tommy used it for  $\frac{3}{10}$  of the times.

How many times did Suzi use the cat door?



Explain how you figured it out.

4 of 100 is 25 (for Sammy), is of 100 is 30.

Student B also uses common denominators, but thinks through the three numbers in parts, showing a substitution of equivalent fractions in almost algebraic form. Notice that in part 4, the student attempts to use common denominators and then seems to switch to a different strategy in the written explanation. Again Student B has very clear labels to describe what each answer represents. What type of discourse or experience helps students to develop the ability to label answers or understand what is being found by doing a calculation?

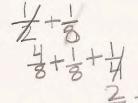
### Student B

1. Cindy feeds them on Cat Crunchies.

Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of the box.

What fraction of a whole box do the cats eat, in all, each day? Show how you figured this out.

The bax



2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

Which of the two cats sleeps for longer?

How much longer does it sleep each day? Show how you figured this out.

3/ 7/10

7-6-10/V

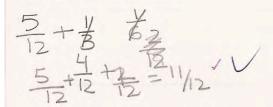
# Student B, part 2

3. Cindy's cats often share a carton of cat milk.

Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and

Suzi always drinks  $\frac{1}{6}$  of the carton.

What fraction of the carton of cat milk is left over? Show how you figured it out.



4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.

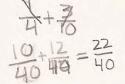


Sammy used it for  $\frac{1}{4}$  of the times and Tommy used it for  $\frac{3}{10}$  of the times.

How many times did Suzi use the cat door?

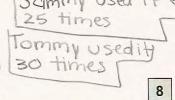
Explain how you figured it out.

Prew that if of 100 was 25





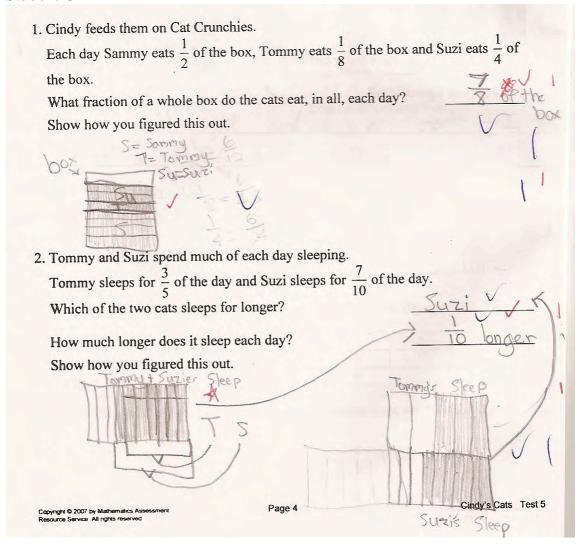






Student C is able to use diagrams to make sense of the size of the fractions and show the action or operation of the problems. In part 2, Student C shows the comparison in two different ways. *Can you describe the mathematics in each diagram?* 

#### **Student C**



# Student C, part 2

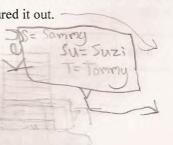
3	Cindy's	cats o	ften s	hare a	carton	of cat	milk
J.	Ciliuy 3	cats o		mai c	t carton	UI Cat	IIIIII.

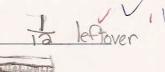
Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and

Suzi always drinks  $\frac{1}{6}$  of the carton.

What fraction of the carton of cat milk is left over?

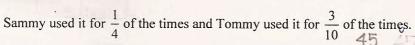
Show how you figured it out.





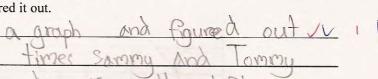
4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.



How many times did Suzi use the cat door?

Explain how you figured it out.



Student D uses percents to think about the situation in part 4.

# Student D

4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.



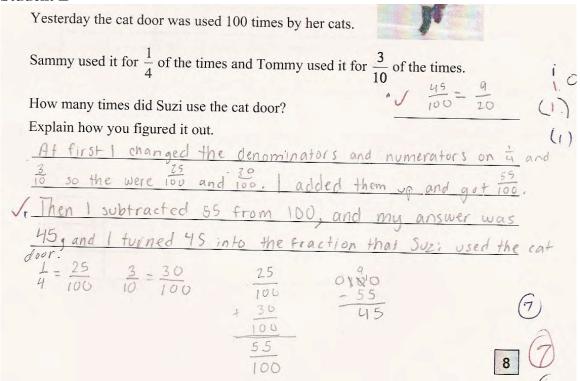
Sammy used it for  $\frac{1}{4}$  of the times and Tommy used it for  $\frac{3}{10}$  of the times.

How many times did Suzi use the cat door?

Explain how you figured it out.  $\frac{1}{4} = 25\%$   $\frac{3}{8} = 30\%$   $\frac{30}{4} = \frac{55}{100}$ 

In part 4, students had a difficult time interpreting their answers. Student E has done all the correct calculations, but can't break down the meaning of the final answer from the number 45/100 to the meaning 45 times out of 100. What kind of question could you pose to the class to get everyone thinking about what the 45/100 represents?

#### **Student E**



Student F is able to use common denominators and goes the extra step to show how to find the denominators. Student F also struggles with understanding the meaning of the calculations. In part two the student is able to make the comparison by finding common denominators, and can see that there is a difference of one part. However, instead of thinking of the one as one part out of ten the student thinks about a day and converts it to 1 hour with an implied out of 24. What kind of discourse can help students think about the whole or what it means to be a fraction of day? Why don't you use the 24 hours? In part 4, the student has trouble thinking about the meaning of operations. The student understands that there is a subtraction, but doesn't understand "the whole" or quantity being subtracted from. The student also realizes that the answer is a whole number (number of times), but doesn't understand what part of the fraction makes sense.

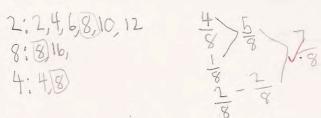
#### Student F

1. Cindy feeds them on Cat Crunchies.

Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of the box.

What fraction of a whole box do the cats eat, in all, each day?

Show how you figured this out.



2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

Which of the two cats sleeps for longer?

Suzi

How much longer does it sleep each day? Show how you figured this out.

5:50 6/7

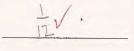
# Student F, part 2

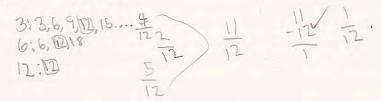
3. Cindy's cats often share a carton of cat milk.

Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and

Suzi always drinks  $\frac{1}{6}$  of the carton.

What fraction of the carton of cat milk is left over?
Show how you figured it out.





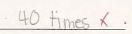
4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.



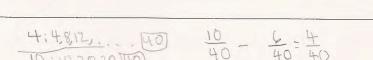
Sammy used it for  $\frac{1}{4}$  of the times and Tommy used it for  $\frac{3}{10}$  of the times.

How many times did Suzi use the cat door?



Explain how you figured it out.

I found out that 4 and 10 were equilent to 40. I changed 4 to 40 and 3 to 50 then subtracted the 3 and I got to



(4)

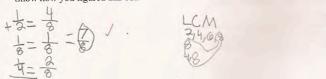
Student G has trouble choosing the operation in part 2 of the task. This is a comparison or subtraction operation and the student makes the common mistake of trying to add. In part 4 the student knows that subtraction is involved by doesn't have the correct whole, 100 times, to subtract from. The student also misses the idea of interpreting the answer from a fraction to number of times. How do we help push students to develop the logical reasoning needed to interpret what is being asked versus what is being calculated? This is important, because the nuances of logic needed to interpret the answer don't come up when students do practice exercises, but only arise through working tasks with rich contexts. Do you think you give students enough opportunity to work and discuss mathematics in context?

# Student G

1. Cindy feeds them on Cat Crunchies.

Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of

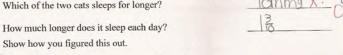
What fraction of a whole box do the cats eat, in all, each day? Show how you figured this out.



2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

Which of the two cats sleeps for longer?

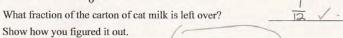


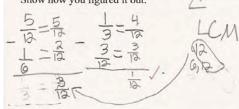
$$-\frac{7}{10} = \frac{7}{10} = \frac{13}{10} \text{ or } |\frac{3}{10} = \frac{13}{10} = \frac{13}$$

3. Cindy's cats often share a carton of cat milk.

Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and

Suzi always drinks  $\frac{1}{6}$  of the carton.





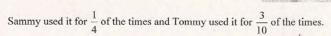


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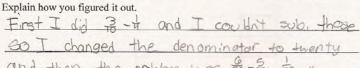
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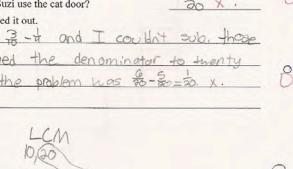
4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.



How many times did Suzi use the cat door?







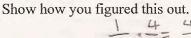
Student H has a total score of 3, but shows a lot of understanding about fractions. The student is able to choose a common denominator and find equivalent fractions in order to add or subtract fractions. The student is able to add and to subtract fractions with unlike denominators. The student is not able to choose correct operations or identify what is being asked for. The student is a great calculator, but doesn't know how to think about solving problems in context.

#### Student H

1. Cindy feeds them on Cat Crunchies.

Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of the box.

What fraction of a whole box do the cats eat, in all, each day?





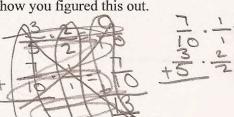
2. Tommy and Suzi spend much of each day sleeping.

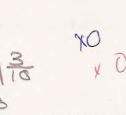
Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

Which of the two cats sleeps for longer?

How much longer does it sleep each day?

Show how you figured this out.





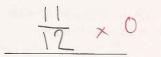
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Page 4

# Student H, part 2

Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and

Suzi always drinks  $\frac{1}{6}$  of the carton.



What fraction of the carton of cat milk is left over?

Show how you figured it out.



4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.



Sammy used it for  $\frac{1}{4}$  of the times and Tommy used it for  $\frac{3}{10}$  of the times.

How many times did Suzi use the cat door?

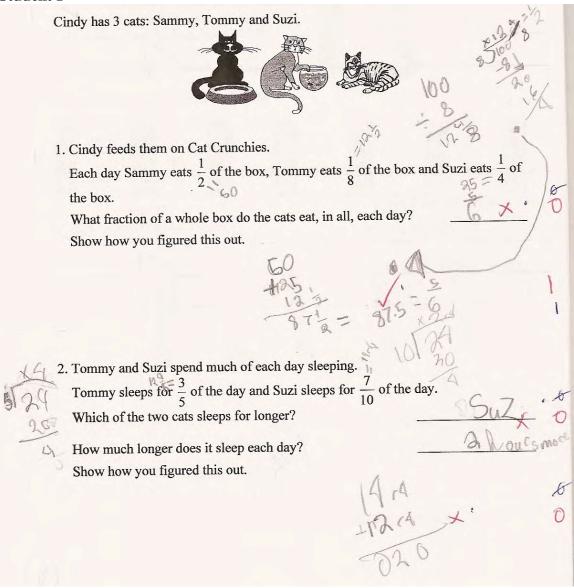


Explain how you figured it out.

1,10=10	A			
3 4 12			Х	0
22 40	7	1	0	3

Student I also has a score of 3, but shows a very different profile of understanding. In part one, the student seems to want all fractions to be parts of 100 and actually makes correct calculations. However, the student tries to relate 87.5% back to a fraction and does that conversion incorrectly. In part 2 the student tries to find parts of the day by changing to a denominator of 24. The fraction conversions are messy and strategy breaks down in trying to deal with the remainders but could have yielded a correct solution. In part 3 the student again tries to make everything into parts of 100, but again has trouble dealing with the remainders. This strategy serves the student well in the final part of the task. What might be some next steps for this student?

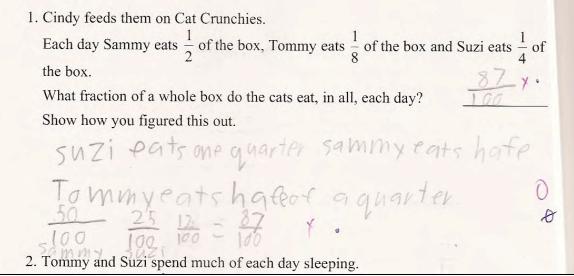
## Student I



Student I, part 2
$6 \frac{16}{100} = \frac{2}{3}$
3. Cindy's cats often share a carton of cat milk.
Sammy always drinks $\frac{1}{3}$ of the carton, Tommy always drinks $\frac{5}{12}$ of the carton, and
Suzi always drinks $\frac{1}{6}$ of the carton. $\frac{22}{\sqrt{5}}$
What fraction of the carton of cat milk is left over?
Show how you figured it out.
22 10 = 20 10 = 20 22 11 5 = 50 23 11 25 = 50
4. Cindy's cats love to jump in and out of their cat door.  Yesterday the cat door was used 100 times by her cats.
20
Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times.
How many times did Suzi use the cat door?
Explain how you figured it out.  I knew == 25 and == 30 So Fadd
them then published from 100 and 1

Student J also tries to work with parts of 100 in part 1, but doesn't understand what to do with the remainder for 1/8. The student did not attempt other parts of the task. What are some of the skills the student has that you can use to build further understanding?

#### Student J



Student K works with the fractions in sets of two. Although the student has a score of 2, the student shows a great deal of understanding in some areas. In part one the student chooses the correct operation for the task, converts to an appropriate common denominator (actually working quite hard because each set requires a different denominator), and adds correctly. The student doesn't simplify the final answer correctly. Thinking about the problem in chunks makes it difficult to choose an operation in part three. While the overall action is subtracting or finding what's left over, this action doesn't happen until after the other quantities are totaled. Then the student needs to put in an implied quantity, the whole. It would be interesting to interview a student with this type of error to see how he or she might solve a similar task involving just whole numbers. Would the student then see the addition? What do you think this student's personal algorithm is for finding denominators? Why does this make the task more difficult? What activities might help the student see the logic and simplicity of finding a smaller common denominator? What questions might you pose to the class?

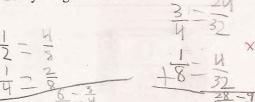
### Student K

1. Cindy feeds them on Cat Crunchies.

Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of the box.

What fraction of a whole box do the cats eat, in all, each day?

Show how you figured this out.



2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

Which of the two cats sleeps for longer?

SUZi to

0

How much longer does it sleep each day?

Show how you figured this out.

$$\frac{7}{10} - \frac{35}{50}$$

$$\frac{3}{5} - \frac{30}{50}$$

$$\frac{5}{50} - \frac{1}{50}$$
Cindy's Cats. Test 5

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3. Cindy's cats often share a carton of cat milk.

Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and Suzi always drinks  $\frac{1}{6}$  of the carton.

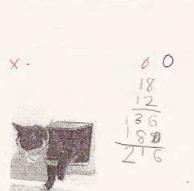
What fraction of the carton of cat milk is left over?

Show how you figured it out.

gured it out. 
$$\frac{5-890}{216}$$
 $\frac{1}{3} - \frac{6}{18}$ 
 $\frac{3}{18} - \frac{33}{216}$ 
 $\frac{1}{6} - \frac{2}{18}$ 
 $\frac{57}{216}$ 

4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.



5<sup>th</sup> Grade Task 3 Cindy's Cats

Student Task	Solve fraction problems in a practical context. Use part/ whole					
	relationships to solve problems.					
Core Idea 2	Understand the meanings of operations and how they relate to each					
Number	other, make reasonable estimates and compute fluently.					
Operations	<ul> <li>Reason about and solve problem situations that involve more</li> </ul>					
	than one operation in multi-step problems.					
	Use equivalent forms to add and subtract commonly used					
	fractions.					
	<ul> <li>Develop and use strategies to solve problems involving number</li> </ul>					
	operations with fractions relevant to students' experience.					

Based on teacher observations, this is what fifth graders knew and were able to do:

- Find common denominators
- Convert to equivalent fractions

## Areas of difficulty for fifth graders:

- Choosing operations in word problems
- Drawing accurate diagrams for fractions
- Interpreting what their calculations represent
- Understanding that fractions need to be converted to common denominators
- Choosing convenient common denominators (e.g. trying to switch to 24<sup>th</sup>'s in part 2 was too difficult or interpreting remainders when trying to convert to 100<sup>th</sup> 's was difficult)

## Strategies used by successful students:

- Using common denominators
- Labeling their work and their answers as they worked through the different steps
- Using percents
- Using accurate models (built with common denominators)

#### MARS Test Task 3 Frequency Distribution and Bar Graph, Grade 5

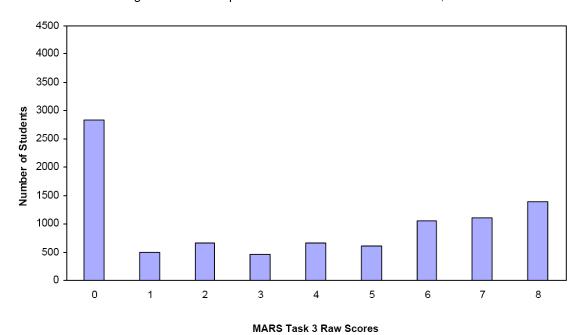
### Task 3 - Cindy's Cats

Mean: 3.67 StdDev: 3.10

Table 27: Frequency Distribution of MARS Test Task 3, Grade 5

T 10	01 1 1	0/ /	0/ /
Task 3	Student	% at or	% at or
Scores	Count	below	above
0	2842	30.7%	100.0%
1	492	36.0%	69.3%
2	668	43.2%	64.0%
3	455	48.1%	56.8%
4	665	55.3%	51.9%
5	615	61.9%	44.7%
6	1045	73.2%	38.1%
7	1101	85.1%	26.8%
8	1384	100.0%	14.9%

Figure 36: Bar Graph of MARS Test Task 3 Raw Scores, Grade 5



The maximum score available for this task is 8 points.

The minimum score for a level 3 response, meeting standards, is 4 points.

More than half the students, 69%, were able to find an answer to part one, but may have struggled with showing their steps. About half the students, 52%, could use common denominators to add 3 fractions in part one of the task and use common denominators to compare fractions in part 2. Almost 40% of the students could also use addition and subtract from 1 whole to find the solution to part 3. Almost 15% could meet all the demands of task including interpreting 45/100 to mean 45 times out of 100. Almost 31% of the students scored no points on this task. 98% of the students with this score attempted the task.

# Cindy's Cats

D: 4	Understandings	Misunderstandings
Points		
0	98% of the students with this score attempted the task.	Students did not understand how to work with common denominators. More than 12% added denominators to get 3/14 or 1/14. 5% added two of the three fractions to get 3/8 or 3/5. Other common errors for part 1 included 3/4, 1, 87%.
1	Most students with this score had a correct answer to part 1, but did not show enough of their work.	Students struggled with making the comparison.7% thought Suzi had 4/5 more and 3% thought Tommy had 4/5 more. 7% knew Suzi had the most but didn't quantify the difference and just wrote 7/10. 13% tried to find the number of hours (1 to 15 hours). Many students struggled with choosing the correct operation, adding instead of subtracting.
4	Students with this score could usually solve all of part 1 and 2, adding fractions with unlike denominators and doing comparison subtraction with fractions.	Students had difficulty identifying the operation in part 3 or understanding that the subtraction was from 1 whole and not other fractions. 5% forgot to subtract and had answers of 11/12. 5% thought the answer was 1/2. Another 5% had answers of 1. Other common errors were 1/4, 5/12, and 2/3.
6	Students could add and subtract fractions to solve parts 1,2, and 3 of the task.	13% calculated 45/100 or 9/20 but did not know how to interpret their answer in part 4. 8.5% did not attempt part 4 of the task. 9% again added denominators to get fourteenths.
8	Students could add and subtract fractions with unlike denominators. They were able to compare fractions and quantify the difference. Students understood the whole and could use it to find out the amount of remaining milk after 3 cats had each had a portion. Students were able to translate a calculation of 45/100 and interpret the answer as 45 times out of the 100 times the door was used. Students were able to understand number operations with fractions to choose the appropriate method for solving a problem in context.	

# **Implications for Instruction**

Students need practice working with fractions in context. They should have a variety of strategies for combining fractions: models, common denominators, changing fractions to decimals or percents. Students should also be able to compare fractions to find out which is larger and subtract fractions from 1 whole.

Some students are still having difficulty choosing operations. Work with bar models might help them to clarify the action of the story problems.

### **Ideas for Action Research**

# Looking at student work:

Often when planning remediation or helping students who are behind, teachers think about the students who are almost there. What are the few steps they need to be successful? But what is it that the students who are at the lowest end of the spectrum need? How are their issues different?

Sit down with colleagues and examine the following pieces of student work. Consider the following questions:

- 1. What are the strengths, if any, that the student has? What does the student understand about the meaning of fractions? What does the student know about procedures with fractions? What are the concepts the students understand about the situation? How might these strengths be used to help build their understanding of the whole situation?
- 2. Is the student making appropriate choices of operations? Do you think the student could pick the correct operation if given a similar problem with whole numbers? What is your evidence? How do students learn to identify the action of the story?
- 3. How did students use representations? Were the representations accurate? Why or why not? What would have helped the student to improve their representation? Could their representation be modified in some way to solve the problem?
- 4. What is the role of labels in understanding what is known and what needs to be found? How does using labels help students to understand what they have calculated and interpret the meaning of the calculation? (This seems especially critical in part 4)
- 5. What misunderstandings does the student have? What skills is the student missing? What does this suggest about a specific course of action to help this student?
- 6. How are the needs of each of these students the same or different?

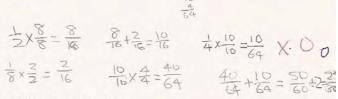
After your have carefully looked at each piece of student work, see if you can devise a plan of experiences/ discussions/ tools that might help these students to make more sense of these situations. While you don't have these exact students in your class, each member of the group will probably have students with similar misunderstandings. Identify students who you think are low and plan different approaches for attacking the problems outlined here. Have each person in the group try out a different course of action and report back on the how the lesson or series of lessons effected the targeted students. See if you can all use some similar starting problems and bring work of the students to share. What types of activities or experiences made the most noticeable improvement in student work?

#### Arnold

1. Cindy feeds them on Cat Crunchies.

Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of the box

What fraction of a whole box do the cats eat, in all, each day? Show how you figured this out.



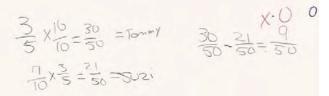
2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

Which of the two cats sleeps for longer?

How much longer does it sleep each day?

Show how you figured this out.



3. Cindy's cats often share a carton of cat milk.

Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and

Suzi always drinks  $\frac{1}{6}$  of the carton.

What fraction of the carton of cat milk is left over?

Show how you figured it out.

$$\frac{216}{216} - \frac{198}{216} = \frac{18}{216}$$

$$\frac{1}{3} \times \frac{12}{12} = \frac{12}{36}$$

$$\frac{1}{36} \times \frac{15}{36} = \frac{21}{36}$$

$$\frac{21}{36} \times \frac{1}{6} = \frac{162}{216}$$

$$\frac{21}{18} \times \frac{3}{12} = \frac{18}{36}$$

$$\frac{1}{18} \times \frac{3}{12} = \frac{18}{36}$$

$$\frac{1}{18} \times \frac{3}{36} = \frac{316}{216}$$

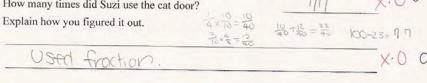
$$\frac{1}{18} \times \frac{3}{36} = \frac{316}{216}$$

4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.

Sammy used it for  $\frac{1}{4}$  of the times and Tommy used it for  $\frac{3}{10}$  of the times.

How many times did Suzi use the cat door?



# **Berta**

1. Cindy feeds them on Cat Crunchies.

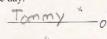
Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of

What fraction of a whole box do the cats eat, in all, each day? Show how you figured this out.



2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

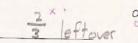


Which of the two cats sleeps for longer?

How much longer does it sleep each day? Show how you figured this out.

3. Cindy's cats often share a carton of cat milk. Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and

Suzi always drinks  $\frac{1}{6}$  of the carton.



What fraction of the carton of cat milk is left over? Show how you figured it out.



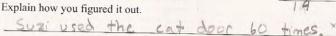
4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.

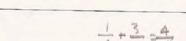


Sammy used it for  $\frac{1}{4}$  of the times and Tommy used it for  $\frac{3}{10}$  of the times.

How many times did Suzi use the cat door?

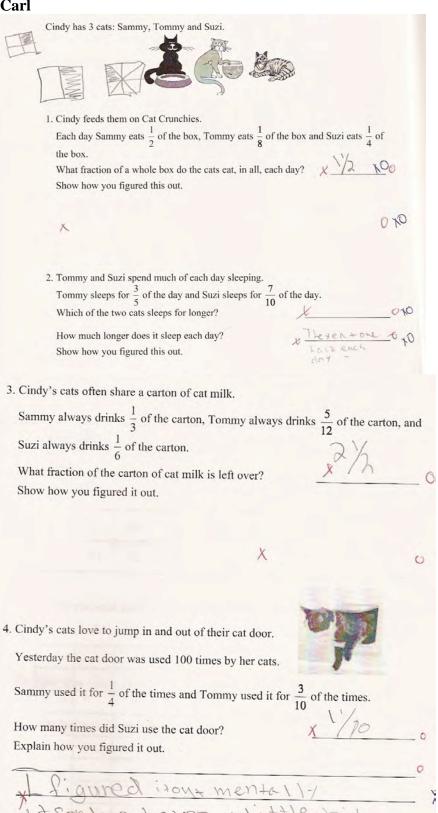








## Carl



# D

Ocrothy	
1. Cindy feeds them on Cat Crunchies.  Each day Sammy eats $\frac{1}{2}$ of the box, Tommy eats $\frac{1}{8}$ of the box the box.  What fraction of a whole box do the cats eat, in all, each day? Show how you figured this out.	and Suzi eats $\frac{1}{4}$ of $\frac{3}{4}$ $\frac{1}{4}$
Tommy and Suzi spend much of each day sleeping.	
Tommy sleeps for $\frac{3}{5}$ of the day and Suzi sleeps for $\frac{7}{10}$ of the day Which of the two cats sleeps for longer?	Suzin C
How much longer does it sleep each day?	
Show how you figured this out.	
DDD Tommy  DDD Tommy	× 100
3. Cindy's cats often share a carton of cat milk.	
Sammy always drinks $\frac{1}{3}$ of the carton, Tommy always drinks $\frac{5}{12}$ of the	e carton, and
Suzi always drinks $\frac{1}{6}$ of the carton.	5 × 0
What fraction of the carton of cat milk is left over?	12 / 20
Show how you figured it out.	20
X X	100
4. Cindy's cats love to jump in and out of their cat door.	2
Yesterday the cat door was used 100 times by her cats.	Amiliani.
Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times	mes.
How many times did Suzi use the cat door?	14 10
Explain how you figured it out.  1 add 1+3=4 then 10+4=14 and	that

XO

# Ernie

rnie	
1. Cindy feeds them on Cat Crunchies.	
Each day Sammy eats $\frac{1}{2}$ of the box, Tommy eats $\frac{1}{8}$ of the box and Suzi eats $\frac{1}{4}$ of	
the box. $\frac{2}{8}$	
What fraction of a whole box do the cats eat, in all, each day?	M
Show how you figured this out.	U
-SUZI X GY	1
Sammy	
2. Tommy and Suzi spend much of each day sleeping.	
Tommy sleeps for $\frac{3}{5}$ of the day and Suzi sleeps for $\frac{7}{10}$ of the day.	
Which of the two cats sleeps for longer?	
longer	0
How much longer does it sleep each day?  Show how you figured this out	0
Show how you figured this out. Towns of South Should be south the south of the sout	
5 10 5 × 0 10	
Day chart	
Copyright © 2007 by Mathematics Assessment Resource Service All richter records  Page 4  Cindu's Cats Toet 5	
3. Cindy's cats often share a carton of cat milk.	
Sammy always drinks $\frac{1}{3}$ of the carton, Tommy always drinks $\frac{5}{12}$ of the carton, and	
Suzi always drinks $\frac{1}{6}$ of the carton.	
What fraction of the carton of cat milk is left over?  \( \frac{1}{2} \) is left \( \times 6 \) \( \times 6 \)	
Show how you figured it out.	
× GYD	
4. Cindy's cats love to jump in and out of their cat door.	
Yesterday the cat door was used 100 times by her cats.	
Sammy used it for $\frac{1}{4}$ of the times and Tommy used it for $\frac{3}{10}$ of the times.	
How many times did Suzi use the cat door?  Explain how you figured it out.	
Explain how you figured it out.	
#+10=14 100-10 XO	

# **Fanny**

Cindy has 3 cats: Sammy, Tommy and Suzi.

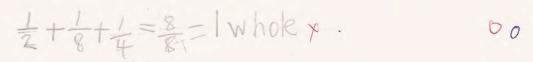


1. Cindy feeds them on Cat Crunchies.

Each day Sammy eats  $\frac{1}{2}$  of the box, Tommy eats  $\frac{1}{8}$  of the box and Suzi eats  $\frac{1}{4}$  of the box.

What fraction of a whole box do the cats eat, in all, each day?

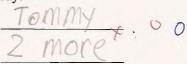
Show how you figured this out.



2. Tommy and Suzi spend much of each day sleeping.

Tommy sleeps for  $\frac{3}{5}$  of the day and Suzi sleeps for  $\frac{7}{10}$  of the day.

Which of the two cats sleeps for longer?



How much longer does it sleep each day? Show how you figured this out.

B + 3 is almost a whole x.00

50 he whouldn't sleep more.

# Fanny, part 2

3. Cindy's cats often share a carton of cat milk.

Sammy always drinks  $\frac{1}{3}$  of the carton, Tommy always drinks  $\frac{5}{12}$  of the carton, and

Suzi always drinks  $\frac{1}{6}$  of the carton.

What fraction of the carton of cat milk is left over? Show how you figured it out.

I added 5 plus the ones to get 7 and 6 goes into 122 times. v.o o

4. Cindy's cats love to jump in and out of their cat door.

Yesterday the cat door was used 100 times by her cats.



Sammy used it for  $\frac{1}{4}$  of the times and Tommy used it for  $\frac{3}{10}$  of the times.

How many times did Suzi use the cat door?



Explain how you figured it out.

#### Performance Assessment Task

## Cindy's Cats Grade 5

This task challenges a student to use knowledge of fractions to solve one- and multi-step problems with fractions. A student must show understanding of operations and fractions to find equivalent numerical representations to add and subtract fractions and compare size of fractions. A student must demonstrate an understanding of operations with fractions to multiply a fraction times a whole number.

#### Common Core State Standards Math - Content Standards

#### **Number and Operations - Fractions**

#### Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12 (in general, a/b +c/d = (ad +bc)/bd).

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result  $2/5 + \frac{1}{2} = 3/7$ , by observing that  $3/7 < \frac{1}{2}$ .

# Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product  $(a/b) \times q$  as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a  $\times q \div b$ . For example, use a visual fraction model to show  $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with  $(2/3) \times (4/5) = 8/15$ . (In general,  $(a/b) \times (c/d) = ac/bd$ .)

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

#### Common Core State Standards Math - Standards of Mathematical Practice

#### MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### **Assessment Results**

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
5	2007	8	4	51.9