Cereal

This problem gives you the chance to: • use ratios in context

Here is some information on the side of two boxes of cereal.

Tasty Oats 12 grams of protein in 100 grams of cereal Cornbits 5 grams of protein in 45 grams of cereal 1. How many grams of Tasty Oats cereal will give you 9 grams of protein? grams Show your work. 2. Which cereal, Tasty Oats or Cornbits, has the higher ratio of protein? Explain how you figured it out. 7

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Cereal Test 7

Cereal Test	7 Ru	bric
The core elements of performance required by this task are: • use ratios in context Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: 75 grams	2	
Shows work such as: 100 x 3/4	1	3
 Gives correct answer: Tasty Oats, supported by some correct explanation such as: 	2	
Ratio of protein in Tasty Oats $=\frac{12}{100}=0.12$		
and		
Ratio of protein in Cornbits $=\frac{5}{45} = 0.111$	2	
or Method involving making either the amount of protein or the amount of	or	
cereal equal.	2	
Partial credit		
Explanation, but does not show calculations.	(1)	4
Total Points		7

Looking at Student Work - Cereal

This question was quite challenging for many students, so looking at types of strategies used by successful students may clarify some of the teaching implications for this problem and how to look at ratio/rate problems.

Student A solves the problem in the most typical fashion, the approach anticipated by the rubric. The student uses a proportion to solve for the number of grams of cereal in part one. The student divides the denominator into the numerator to compare the size of the ratios using decimal values.

Student A



Another way to think about comparing the ratios is to make the amount of cereal the same for both brands. This involves having students pick a common denominator to represent the amount of the cereal. In reality, the decimal comparison is also a way of using a power of ten for the common denominator. Student B takes the approach of finding the amount of protein for 100 grams of both brands of cereal.

Student B

1.	How many grams of Tasty Oats cereal will give you 9 grams of protein?
	Show your work.
	12 grams Plotein - grams Protein
	100 grams total X 12x = 900
2.	12 $12\times -75Which entropy Case as Complete the second seco$
-	which cereat, Tasty Oats of Combits, has the higher ratio of protein?
	$\frac{12}{100} = \frac{5}{45} = \frac{1}{100} + \frac{45}{45} = \frac{500}{45} \times = 100 \text{ M/m}.$ Tosty outs V
	Explain now you figured it out.
	· · · · · · · · · · · · · · · · · · ·
11	is Tasty pats because I converted combits to be
m	grams total instead of 45.1 cross multiplied then
<u>diı</u>	vided by the number multiplied by X. Now both grams
it na	protein from both cereals was out of 100 grams total Tasty
	is had just a ting bit more.
ł	Finding common denominators for 45 and 100, leads to many choices.
	4500ths. What is different about what each student understands about
f	fractions to arrive at these choices of denominators?

Teacher Notes about the mathematics:

Student C 1. How many grams of Tasty Oats cereal will give you 9 grams of protein? \checkmark 7.5 grams Show your work. protein = 9 . R B] 1205025 23 * 9 grans ×3 = 22 225 275 2. Which cereal, Tasty Oats or Combits, has the higher ratio of protein? $\frac{45}{900} \frac{2}{100} \frac{2}{100} \frac{2}{600} \frac{2}{600} \frac{2}{100} \frac{2}{10} \frac{2}{100} \frac{2}{10} \frac{2}{100} \frac{2}{10} \frac{2}$ -Oats Tasty Explain how you figured it out. То figure this $\overline{\omega}t$ Ī cangreit _the denomination 50 thet Herry were equal. 30 T multiply-01 NU 45 ech which equalect ad 20 ъ and -vHipland R and ĸU 102 ul-a Then I compared 44.1 the larger, which nos mai Tusty Outs.

Notice how Student D also uses equivalent fractions to solve for part one instead of using a proportion.

Student D

1. How many grams of Tasty Oats cereal will give you 9 grams of protein? 2 Show your work. <u>3grams Protein</u> ×3 25grams CerealX3 12gr. Protien grams ł 100grams Coreal aram S A 2. Which cereal, Tasty Oats or Combits, has the higher ratio of protein? 12 - 3×9 IDU 257 Explain how you figured it out. Ind TH. ommor 20 00 proceeded 2 NPI 109 ρ ruter. e 7 Student E 2. Which cereal, Tasty Oats or Cornbits, has the higher ratio of protein? 12 795 SAU Jas 100 001 Explain how you figured it out. solved this 750 ar × 100 ân tast oats

Instead of comparing ratios by making the denominator the same, it is possible to make the amount of protein (or numerators) the same. Students F and Student G think about how much cereal is needed to yield 9 grams of protein. Student G has some great thinking, but struggles with how to formalize the mathematical notations and symbols.

Student F

1. How many grams of Tasty Oats cereal will give you 9 grams of protein? Show your work, erains 9×100=900 $\frac{9}{x}$ 900÷12=751 X=75 2. Which cereal, Tasty Oats or Cornbits, has the higher ratio of protein? Explain how you figured it out. giorns of profe 00 r001 e Ce (c Student G 1. How many grams of Tasty Oats cereal will give you 9 grams of protein? 7ť ^v 2 grams Show your work. \$ 3/25 2. Which cereal, Tasty Oats or Cornbits, has the higher ratio of protein? oas Explain how you figured it out. Corrbit tous 44 grams per OSeventh Grade - 2004 pg. 23

Student H

1. How many grams of Tasty Oats cereal will give you 9 grams of protein?	70
Show your work. $\frac{4}{12} = \frac{1}{100}$	grams 2
$\frac{3}{4} = \frac{\times}{100}$	1 8
4x - 30 °	. V 1
x75	-
2. Which cereal, Tasty Oats or Combits, has the higher ratio of protein?	·
Tasty Dats = $\frac{12}{100} = \frac{3}{25}$ Corn bits = $\frac{5}{5} = \frac{1}{9} = \frac{3}{27}$ Explain how you figured it out. $\frac{9}{7} = \frac{3}{27}$	Tasty Dats 207
The ratio of the grams of protein to the grams	ofcerbal
for Tasty pats and complits are is and to respi	actively persues
+= 3, we see that The numerator of both	ratios are the
serve, but the denominanter of the cornhiber ratio) is luce a medilate
its rutio smaller	- ager, making

The most common procedure for students was to divide the numerator into the denominator, but not to understand how that calculation helps to solve the problem. Students I and J understand in some way that this makes a unit ratio, how much cereal is needed to make 1 gram of protein. Student I is articulate about the unit ratio. Student J understands the inverse relationship between the sizes of the number to the amount of protein.

Student I

- 1. How many grams of Tasty Oats cereal will give you 9 grams of protein? <u>75</u> grams \mathcal{V} Show your work. $9 = \frac{3}{4}$ of 12 for $12 + 4 = \frac{1}{4} \times 3 = \frac{3}{4}$ for if you do the some for 100 you get $100 + 4 = 25 \times 3 = 75$
- 2. Which cereal, Tasty Oats or Combits, has the higher ratio of protein?

Toto Explain how you figured it out. 2 5 7 12 -0 $i \alpha$ you (σ) G. . 15 ju ~0 CGraw-Hill LLC. Copyright © 2004 manufact Rescurre Service. All rist Page 2 Test 7 Cereal

Student J

•	
1. How many grams of Tasty Oats cereal will give you 9 grams of error.	V2V
Show your work.	^{n?} <u>75</u> grams 2
$8.33 = \frac{12}{11} g = \frac{100 gc}{-gc}$	998= 25 gc
2. Which cereal, Tasty Oats or Combits, has the higher ratio of protein?	••• ••
Explain how you figured it out.	tacty criting
	V 2.
- the lower number will how higher millio of protein	candy

Student K recognizes that 12 and 100 share a common factor of 4 and uses this information to solve for the amount of cereal in part 1 by making a ratio table.

Student K



Some students do not understand the multiplicative relationship involved in this problem. Student L tries to subtract equal quantities in part 1. Student M tries to add a constant to make things come out evenly.

Student L



While Student N appears to not be using the multiplicative relationship in part one because he is using subtraction, Student N's strategy works because he is really thinking about taking away 1/4ths from different size wholes.

Student N

75 / _______ 1. How many grams of Tasty Oats cereal will give you 9 grams of protein? Show your work. 12-3=9 100-25=75 1 2. Which cereal, Tasty Oats or Cornbits, has the higher ratio of protein? 100y2 Explain how you figured it out. 72 bobivib tett I າລາ Cereal nye 270 ANIDE 45 and 69 Saw that matel of the reveal was protein. This evidence the Tarony Dats cereal contained a higher percentage of protein. I also divide 75 by 5 and set t. I then divided 12 by 4 and got 3. I found the common denominator Page 2 35. I found the common denominator by multiplying thy 33 and 33 by 9. By doing this I found that Trating Oats cereal contained 335 grams of protein and Combits contained 335. This showed that Tasty Oats had the higher ratio as well.

Sometimes students learn procedures, but remember them imprecisely or don't understand the reasoning behind the procedure. Student O uses crossmultiplying to compare the ratios in part 2. While this procedure can work, Student O does not put the results in the correct order so makes an incorrect comparison. What understanding does the student need about common denominators to help her interpret these results?

Student O

<u>، د</u>

2. Which cereal, Tasty Oats or Combits, has the higher ratio of protein? 10 Compity X 00 Explain how you figured it out. Combits has a ligner V71000 ि vatio 00 \times 540 <700 = 741 Test 7 Cereal

Teacher Notes:

Frequency Distribution for each Task – Grade 7 Grade 7– Cereal





Score:	0	1	2	3	4	5	6	7
% < =	38.8%	42.9%	54.8%	70.0%	75.6%	79.7%	83.0%	100.0%
% > =	100.0%	61.2%	57.1%	45.2%	30.0%	24.4%	20.3%	17.0%

The maximum score available for this task is 7 points. The cut score for a level 3 response, meeting standards, is 3 points.

More than half the students (57%) could use the ratio for Tasty Oats to find the amount of cereal needed to get 9 grams of protein. A little less than half the students (45%) could find the amount of cereal for 9 grams of protein and show their calculations. Almost 20% of the students could find the cereal needed to for 9 grams of protein, and compare two ratios to find the one with the rater amount of protein per grams of cereal. Almost 39% of the students scored no points on this task. 87% of the students with this score attempted the task.

Cereal

Points	Understandings	Misunderstandings
0	87% of the students with this score attempted the problem.	Some students do not understand the multiplicative relationship in using proportions. Students tried to subtract or add to convert from different amounts of cereal and protein. The most common wrong answers in part 1 were 97, 11, and 3.
2	Students could correctly give an answer to the amount of Tasty Oats cereal needed to get 9 grams of protein.	Students did not show their work or left out too many steps to understand what they were thinking.
3	Students could find the amount of cereal for 9 grams of protein and document their process.	Students did not know how to compare ratios. Many students made correct calculations, but couldn't correctly interpret their results.
4	Students with this score, generally could compare the ratios of protein in part and explain their ideas, but miscalculated the amount of cereal in part 1. Students who could solve part 2 used a variety of strategies including finding common denominators, using a unit rate, making the amount of protein constant and comparing the denominators.	The most common error in part 4 was to divide the numerator into the denominator. While this could lead to a correct answer, most students could not use the inverse relationship between the size of this answer and the cereal with the most protein. To use this strategy successfully, students needed to see that one gram of protein for Tasty Oats equaled 8.333 grams of cereal and one gram of protein for Cornbits required 9 grams of cereal. This may be a problem of dimensional analysis, failure of being able to apply a label to the results of calculation.
7	Students could adjust portions of cereal to achieve a given amount of protein. Students could successfully compare ratios using decimals or other strategies.	Other common errors in part 4, were to use imprecise calculations like trying to compare 90 grams of cereal to 100 grams of cereal. An important concept for working with ratios is to make either the numerator or denominator equal, so the other quantity can be compared.

Based on teacher observations, this is what seventh graders know and are able to do:

- Turn fractions into percents
- Find units per gram
- Find lowest common denominator

• Turn information into accurate ratios in fraction form

Areas of difficulty for seventh graders, seventh graders struggled with:

- Understanding ratios in a meaningful way
- How to use ratios to make a comparison
- Understanding how division effects labels in ratio problems

Questions for Reflection on Cereal:

- What opportunities have your students had with fractions this year?
- When learning about fractions, do you think they understand the importance of equivalent fractions and how they are useful to solving problems?
- Do your students know how to find common denominators?
- What experiences have your students had with solving ratio/proportion problems? Have they used ratio tables?
- Have they been introduced to the idea of unit rates? How could this idea help them solve this task?
- Do students understand that to make a comparison they need to make part of the ratios constant?

Look at your student work. How many of your students could successfully find the solution to part one by:

Setting up a proportion?	Using a ratio table?	Finding 1/4 of 12 and 100?	Finding a unit rate?	Other?

In part 2, how many of your students were unsuccessful? Of those, how many calculated 8.333 and 9 but couldn't correctly label them to make the correct comparison? How many used imprecise comparisons like 90 grams of cereal to 100 grams? How many did not attempt this part of the task?

In part 2, look at the successful students. How many of them, tried:

Decimals	Dividing	Percents	Finding	Finding	Unit	Other
	numerator		common	common	Rate	
	by		denominators	numerators		
	denominator					

After examining different strategies to achieve the correct solution, what might you want to remember when you work on this unit next year? What are some specific things you would want to emphasize more or add to the unit?

Teacher Notes:

Implications for Instruction:

Students need to more practice with proportions in everyday situations. Students need a deeper understanding of equivalency to help them understand how to set up and compare ratios and proportions. Students need practice determining what is the best buy or most and least in a comparison. Perhaps more work on a unit value would make the comparisons easier or more attention to labels might help them to use dimensional analysis to compare items. Students should be given enough problems with ratios to know and choose the most convenient strategy among several: finding the unit rate, using equivalent fractions, making the numerators the same so denominators can be compared, or making the denominators the same so the numerators can be compared. It is not enough for students at this grade level to learn an algorithm, but there must be sense making about why it works and how it should be applied within a context. To often students only experiences with an idea are in mathematical exercises, devoid of context, where the purpose of the procedure is unclear or lacking from using the process. Students should have classroom discussions about strategies and be able to generalize that to make a comparison one of the two quantities must be made the same so that the other values can be compared. Too often students don't have the opportunity to work with applications for procedures and therefore can't call up the appropriate tool in a problem-solving situation.

Teacher Notes:

Performance Assessment Task				
Cereal				
Grade 7				
This task challenges a student to use knowledge of proportional reasoning and to use equivalent				

ratios. A student must be able to convert between representations of rational numbers to compare and order ratios. A student must be able to interpret quantities and the associated units to interpret values in the context of a real-world problem.

Common Core State Standards Math - Content Standards

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*

7.RP.2 Recognize and represent proportional relationships between quantities.

- a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationships between the total cost and the number of items can be expressed as t=pn.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

<u>The Number System</u>

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of inters (with non-zero divisor) is a rational number. If p and q are integers, then -(p/q) = -/(-q). Interpret quotients of rational numbers by describing real-world contexts.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative

reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2004	7	3	45%