# Candies

This problem gives you the chance to: • work with fractions and ratios

1. This is Amy's box of candies. She has already eaten 6 of them.



What fraction of the candies has Amy eaten?

2. Valerie shares some of the 12 candies from this box. She gives Cindy 1 candy for every 3 candies she eats herself.

How many candies does she give to Cindy? Show how you figured this out.

3. In a packet of mixed candies there are 2 fruit centers for every 3 caramel centers. There are 30 candies in the packet.

How many caramel centers are there? Show how you figured this out.

4. Anthony makes candies.

First, he mixes 1 cup of cream with 2 cups of chocolate. In all, he uses 9 cups of these two ingredients. How many cups of chocolate does he use in this candy recipe?

Explain how you figured this out.



8

Tasl	k 1: Candies	Rubric	
The con • work Based c	points	section points	
1. (	Gives correct answer: 2/3 or 6/9	1	1
2. 0	Gives correct answer: 3	1	
5	Shows work such as: $1 + 3 = 4$ $12 \div 4 =$		
A	Accept diagrams.	1	2
3 (	Gives correct answer: 18	2	
S	Shows work such as: $2 + 3 = 5$ $30 \div 5 = 6$ $6 \ge 3 = 6$		
A	Accept diagrams.	1	3
4. 0	Gives correct answer: 6	1	
	Gives a correct explanation such as: Anthony mixes a ratio of one cup of cream to two cups of chocolate. The ratio stays the same for different amounts. So I wrote the numbers in a chart like this 1 to $2 = a$ total of 3 2 to $4 = a$ total of 6 3 to $6 = a$ total of 9	1	
I	Accept diagrams.		2
	Total Points		8

# Candies

Work the task and look at the rubric. What strategies do you think students might use to help them solve the task?

What are the big mathematical ideas being assessed?

In this task students need to think about fractions: part/whole relationships and ratios: part/part relationships. Look at student work in part 1. How many of your students put:

	1		1		1
2/3 or 6/9	1/3 or 2/6	3/9	3/6 or 1/2	Whole	Other
				number	

What does this show you about student understanding? Are students frequently asked to think about all the ideas in a model? (What fraction is shaded? What fraction is not shaded? What fraction is represented by all the parts?)

Next students are asked to think about a situation of distributing items in a ratio. How many of your students put:

3	4	9	1/4	6	4/12	1	12	Other

What is each student probably thinking? What are they confused about? How can these misconceptions be confronted? What task or problem might you pose for class discussion to help students clarify their thinking?

In part 3, students needed to think about how many candies made up a group, find the number of groups in the whole, and then use that scale factor to find the number of caramel candies. Now look at student work on part 3. Make a note of the types of models or strategies that students used and the answers they came up with. How many of your students put:

18	5	10	5	3	4	90	16/17	Other

What confused them? What models did they use? How could you make use of these models for further instruction? What labels did students use to help them make sense of their calculations? What labels might have helped them to clarify the ideas of the problem?

6	18	3	4	9	2	5	Other

Did it seem that more students were able to solve this than part 3? What about this task made it more accessible to students? How can you help students link their strategies to work on part 3?

How can you use models from good student work for classroom discussion to help other students develop some productive habits of mind and set classroom norms for showing their work or explaining their thinking? What are examples of student work that you valued?

# Looking at Student Work on Candies

Student A uses three different models to think about the ratios. In part 2 the student uses a dealing out strategy; 1,2,3 for Valerie then 1<sup>st</sup> for Cindy, etc. In part 3 the student uses a table, which probably uses groups of caramels and groups of fruit centers but also shows equivalent ratios. In the part 4 the student uses a scale factor of three to solve the problem.



Ratios represent a part/part relationship, which combined represent the whole. Student B, while using a drawing and counting strategy, clearly shows that each ratio is growing in groups of 5 in part 3 and growing in groups of 3 in part 4.

### Student B

3. In a packet of mixed candies there are 2 fruit centers for every 3 caramel centers. There are 30 candies in the packet. How many caramel centers are there? Show how you figured this out. (110-10 () 5 111 4. Anthony makes candies. First, he mixes 1 cup of cream with 2 cups of chocolate. In all, he uses 9 cups of these two ingredients. How many cups of chocolate does he use in this candy recipe? Explain how you figured this out. If you add 1+2 cups T+ 3+3=9 cups dh cups of cream you cups and OF Choc Copyright © 2007 by Mathematics Assessment Page 1 Candies Test 5

Student C very clearly labels the relationship of the whole back to the part. *How does this kind of labeling help students to make sense of the meaning of ratios? How can you help students develop this ability to explicitly describe the relationships in the problem?* 

## Student C

3. In a packet of mixed candies there are 2 fruit centers for every 3 caramel centers. There are 30 candies in the packet.

How many caramel centers are there? Show how you figured this out.

4. Anthony makes candies.

First, he mixes 1 cup of cream with 2 cups of chocolate. In all, he uses 9 cups of these two ingredients.

How many cups of chocolate does he use in this candy recipe? CUPS V

Explain how you figured this out.

ale.

5<sup>th</sup> grade – 2007 Copyright © 2007 by Noyce Foundation Resource Service. All rights reserved. Student D makes good use of the diagram to describe and label what each column represents in terms of fractional quantity and context. Student D makes a table to show how the pattern grows in equal amounts every time. In part 4 the student relates the scale factor to the unit amount (cream). This makes sense because its easier to think about 1 part out of three, than 2 parts of three. The student can then subtract the amount of cream to find the remaining part, chocolate.



5<sup>th</sup> grade – 2007 Copyright © 2007 by Noyce Foundation Resource Service. All rights reserved. Student E also has a nice model to use in solving the problems. Which part of the model relates to the size of the whole and which part relates to the growth factor? What has the student misunderstood in part 1 and 2?

## Student E

1. This is Amy's box of candies. She has already eaten 6 of them. 6 What fraction of the candies has Amy eaten? 2. Valerie shares some of the 12 candies from this box. She gives Cindy 1 candy for every 3 candies she eats herself. How many candies does she give to Cindy? Show how you figured this out. 3. In a packet of mixed candies there are 2 fruit centers for every 3 caramel centers. There are 30 candies in the packet. How many caramel centers are there? Show how you figured this out. 4. Anthony makes candies. First, he mixes 1 cup of cream with 2 cups of chocolate. In all, he uses 9 cups of these two ingredients. How many cups of chocolate does he use in this candy recipe? Explain how you figured this out. is how

Students had difficulty understanding what to do with the scale factor. Student F is able to think about the part/part/whole relationship in 2 and 3 (3+1 makes a group of 4, 3+2 makes a group of 5). The student solves part 2 by counting the number of groups of 4, which is the same as the number of candies. The student then counts how many groups of 6 there are in 30. *Why does this strategy no longer work? What is different mathematically in part 3?* 

#### Student F

2. Valerie shares some of the 12 candies from this box. She gives Cindy 1 candy for every 3 candies she eats herself. How many candies does she give to Cindy? Show how you figured this out. 3H=4. /4+4=8+4=12  $\checkmark$ . 3. In a packet of mixed candies there are 2 fruit centers for every 3 caramel centers. There are 30 candies in the packet. How many caramel centers are there? Show how you figured this out. 3r2:5/St 5:10+5:15+5:20+5=25+5,30. 4. Anthony makes candies. First, he mixes 1 cup of cream with 2 cups of chocolate. In all, he uses 9 cups of these two ingredients. How many cups of chocolate does he use in this candy recipe? Explain how you figured this out. 2 with Page 1 Candies Test 5

Student G is able to multiply by the scale factors in both 3 and 4, but doesn't recognize which number represents the solution in 3. Six is the scale factor for part 3, 18 represents the number of caramels. Notice how the labels in the upper margin help the student think about the relationships for part 4. *Why might part 4 be easier for students to think about than part 3*?

## Student G

Joup 2 pup X.	3
Candies This problem gives you the chance to: • work with fractions and ratios	
1. This is Amy's box of candies. She has already eaten 6 of them.	
What fraction of the candies has Amy eaten?	1.
2. Valerie shares some of the 12 candies from this box. She gives Cindy 1 candy for every 3 candies she eats herself.	
How many candies does she give to Cindy? 3 Kee Show how you figured this out.	ause she bood Ballyundg
3. In a packet of mixed candies there are 2 fruit centers for every 3 caramel There are 30 candies in the packet.	centers.
How many caramel centers are there? $6x2-12$ Show how you figured this out. $6x3+18$	anel 0
<ul> <li>4. Anthony makes candies.</li> <li>First, he mixes 1 cup of cream with 2 cups of chocolate.</li> <li>In all, he uses 9 cups of these two ingredients.</li> <li>How many cups of chocolate does he use in this candy recipe?</li> </ul>	1
Explain how you figured this out. I figured it out by multyplying 1x	3+2X3= 1
	$-(\epsilon)$

5<sup>th</sup> grade – 2007 Copyright © 2007 by Noyce Foundation Resource Service. All rights reserved. Student H is making sense of how the ratio grows in part 3, but finds the scale factor instead of the total number of caramels. In part 4 the student also finds the scale factor. What question might help to push the students thinking. *Would you be willing to use this student's work on the board for class discussion? What part might you show? What questions might you ask? What is the mathematics you want students to make sense of from this model?* 

#### Student H

How many caramel centers are there? 2, 3, 2, 3, 2, 3, 2 Show how you figured this out. 2 = fruit center D 3= caramel center 4. Anthony makes candies. First, he mixes 1 cup of cream with 2 cups of chocolate. In all, he uses 9 cups of these two ingredients. How many cups of chocolate does he use in this candy recipe? Explain how you figured this out. ceam 1+2=3 1+2=3 3+3+3=9

Student I is struggling with the idea of equal groups In part 2 the student is trying to make equal groups of 4, but miscounts. *What has the student overlooked?* In part 4 the student is able to think about how the pattern grows in two different size groups to find the total. But in part 3 the student makes the common mistake about thinking additively instead of multiplicatively. The student notices that the numbers in the ratio differ by one and attempts to maintain that ratio to create the new ratio representing 30 objects. This is different from thinking about how many groups of 5 fit into 30. *What kinds of questions help students to transition from noticing additive to multiplicative thinking?* 

Student I

eats herself. How many candies does she give to Cindy? Show how you figured this out. 000 600 600 600 3. In a packet of mixed candies there are 2 fruit centers for every 3 caramel centers. There are 30 candies in the packet. How many caramel centers are there? Show how you figured this out, 4. Anthony makes candies. First, he mixes 1 cup of cream with 2 cups of chocolate. In all, he uses 9 cups of these two ingredients. How many cups of chocolate does he use in this candy recipe' Explain how you figured this out. eanials 8 5<sup>th</sup> grade – 2007

Copyright © 2007 by Noyce Foundation Resource Service. All rights reserved. Student K is able to draw models to make sense of the ratio in part 2. However in part three the student mistakes the two fruit centers for every 3 caramels as 2 out of every three. Mathematics has a very precise use of language and literacy instruction in mathematics needs to pay to these subtle, but different uses of words. *What kind of discussion might students have if they were asked to model 2: 3 for 30 and 2/3 of 30? What would they draw? Why do you think the student doesn't attempt to make a model for part 4? Why might the student change strategies for this question? How could you design a interview for a student with this type of response?* 

Student K	
	1/1
How many candies does she give to Cindy? 3 Candfes	V. Ibod
3. In a packet of mixed candies there are 2 fruit centers for every 3 caramel centers.	
There are 30 candies in the nacket	4
There are so canales in the packet.	. D
How many caramel centers are there?	elcenter
4. Anthony makes candies.	D
First he mixes 1 cup of cream with 2 cups of chocolate.	
In all he uses 9 curs of these two ingredients	T
How many cups of chocolate does he use in this candy recipe?	0
now many cups of chocolate does ne use in uns candy recipe.	21.1
Explain how you figured this out.	D
= multiplied y x 82 = 181	

Again, look at the work of Student L. The student makes a similar model for part 3, yet has a very good understanding of the relationships in part 4. In part 2 the student seems to think arrays are models for multiplication and doesn't relate it to the context. While all 3 problems are looking at ratios, the student makes 3 different interpretations.

## Student L



# 5<sup>th</sup> Grade Task 1

Student Task	Work with fractions and ratios to reason about candies in a box. Find						
	equivalent ratios.						
Core Idea 1	Understand numbers, ways of representing numbers, relationships						
Number	among numbers and number systems.						
Properties	Recognize and generate equivalent forms of commonly used						
	fractions.						
Core Idea 2	• Reason about and solve problem situations that involve more						
Number	than one operation in multi-step problems.						
Operations	• Develop and use strategies to solve problems involving number						
	operations with fractions relevant to students' experience.						

# Mathematics of this task:

- Ability to use models to understand fractions and to solve ratio problems
- Ability to think about a ratio as part/part model and to think about the pattern growing in equal groups or a unit composed of the sum of the parts
- Ability to find a scale factor and apply it to a ratio

Based on teacher observation, this is what fifth graders know and are able to do:

- Naming a fraction from a model
- Extending a ratio with small numbers
- Use a dealing model to extend a ratio
- Make models to help them visualize the information

# Areas of difficulty for fifth graders:

- Identifying the unit or whole being described in a ratio
- Applying the scale factor to the original ratio
- Making labels to identify what is found with each calculation or to make sense of the quantities in a problem

#### Task 1 - Candies

Mean: 4.50 StdDev: 2.79

Table 25: Frequency Distribution of MARS Test Task 1, Grade 5

Task 1	Student	% at or	% at or
Scores	Count	below	above
0	883	9.5%	100.0%
1	1058	20.9%	90.5%
2	790	29.5%	79.1%
3	1039	40.7%	70.5%
4	661	47.8%	59.3%
5	819	56.7%	52.2%
6	1172	69.3%	43.3%
7	615	75.9%	30.7%
8	2230	100.0%	24.1%





The maximum score available for this task is 8 points. The minimum score needed for a level 3 response, meeting standards, is 4 points.

Most students, 90%, could identify the fraction from a model. Many students, 70%, could identify the fraction of candy eaten in part 1 and find the amount of chocolate needed to extend a recipe from 3 cups to 9 cups. More than half the students,52%, could also find the amount of candies Cindy would receive in part 2 and explain how to figure it out. 24% of the students could meet all the demands of the task, including finding a scale factor and changing a ratio from 2:3 to 12:18, when the total candy would be 30 pieces. Almost 10% of the students scored no points on this task. 93% of the students with this score attempted the task.

# Candies

Points	Understandings	Misunderstandings
0	93% of the students attempted the task.	Students confused the model in part one for what was in the box instead of what was missing. 14.5% put either 1/3 or 3/9 instead of 2/3. 8% made the model into a ratio, giving 3/6 or 1/2.
1	Students could name the fraction represented by the missing part in a model.	Students had trouble finding the amount of chocolate needed for a recipe. 8.5% of the students multiplied 2 x 9 to get 18. 5% confused scale factor, 3, with the amount of chocolate needed. Other common answers were 4,9, 2 and 5.
3	Students could name a fraction and use a ratio to find the amount of chocolate needed to make 9 total cups of a recipe.	Students had trouble thinking about a ratio for dealing out candies in a box of 12. 18% simply divided one of the numbers of the ratio into 12 to get 4. Other common answers were 2,9, 15 and 4/12.
4	Students could name a fraction, extend a ratio out a small distance, and find the number of candies for Cindy in part 2.	Students had difficulty explaining their thinking in part 2.
5		Students missed all of part 3, extending the ratio 2:3 out to 30 total candies. 8.5% could find the scale factor of 6, but didn't know what to do next. 7% thought about a ratio of 2 fruits and 1 caramel to make a total of groups of 3 instead of 2:3 giving a total of groups of 5. Other common answers were 5 and 3.
6		Students with this score missed either all of 2 or all of 4.
8	Students could identify a fraction from a model, apply a ratio to a model to find the number of candies Cindy would receive, find a scale factor to find the increased amount of a given ingredient.	

# **Implications for Instruction**

Students need to work with fractions and ratios in a variety of contexts. They should be comfortable using pictures and diagrams to help them reason about the relationships being expressed. Fractions represent part/whole relationships, while ratios represent part/part relationships. Students need to have language to help them describe these relationships. When learning multiplicative thinking, they need to think the size of the group. In a ratio of 2:3, the two parts add together to make a group or unit of 5 objects. If there are a total of 45 objects the scale factor would be 9, because 9 groups will fit into 45. This scale factor can then be used to find the number of each kind of object. Helping link the language of group size and number of groups to student diagrams can help students to think more clearly about the relationships being expressed.

# Action Research

# Looking at student work:

Often when planning remediation or helping students who are behind, teachers think about the students who are almost there. What are the few steps they need to be successful? But what is it that the students who are at the lowest end of the spectrum need? How are their issues different?

Sit down with colleagues and examine the following pieces of student work. Consider the following questions:

- 1. What are the strengths, if any, that the student has? What are the concepts the students understand about the situation? How might these strengths be used to help build their understanding of the whole situation?
- 2. How did students use representations? Were the representations accurate? Why or why not? What would have helped the student to improve their representation?
- 3. What misunderstandings does the student have? What skills is the student missing? What does this suggest about a specific course of action to help this student?
- 4. How are the needs of each of these students the same or different?

After your have carefully looked at each piece of student work, see if you can devise a plan of experiences/ discussions/ tools that might help these students to make more sense of these situations. While you don't have these exact students in your class, each member of the group will probably have students with similar misunderstandings. Identify students who you think are low and plan different approaches for attacking the problems outlined here. Have each person in the group try out a different course of action and report back on the how the lesson or series of lessons effected the targeted students. See if you can all use some similar starting problems and bring work of the students to share. What types of activities or experiences made the most noticeable improvement in student work?

In thinking about the mathematics of this task:

- What does the student understand about the models? How does the student make use of models giving? What kind of models does the student make for himself/herself to help solve the problem?
- Does the student understand what the whole is in the fraction? In the ratios?
- Does the student find the scale factor for part 3 and 4? Does the student recognize the scale factor?
- Are there issues of mathematical literacy that come into play? Be specific.

# Jonah



#### Manny



Eva



#### Laura



## Performance Assessment Task

Candies

## Grade 5 task aligns in part to CCSSM grade 6

This task challenges a student to use models to understand and describe fractions and to solve ratio problems. Students must be able to distinguish between part/part relationships and part/whole relations to make sense of and solve for the number or portion of candies in a box and find and use scale factor to solve a problem.

## Common Core State Standards Math - Content Standards

## **Ratios and Proportional Relationships**

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.2 Understand the concept of a unit rate a/b associated with a rat a:b with  $b \neq 0$ , and use rate language in the context of a ration relationship.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

## Common Core State Standards Math – Standards of Mathematical Practice

#### MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
Grade 5	2007	8	4	59%