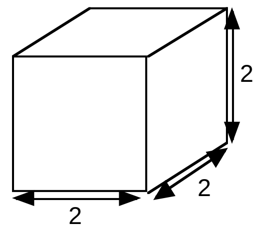
Being Building Blocks

This problem gives you the chance to:
• work with area and volume

Barbara's baby brother, Billy, has a set of building blocks.

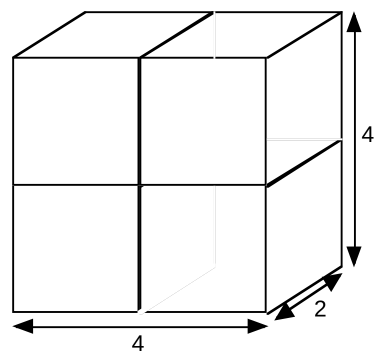
Each block is 2 inches long, 2 inches wide, and 2 inches high.



1. How many faces does the block have? ____________

2. What is the volume of the block? ____________ cubic inches

   Show how you figured this out.



3. Billy has built this shape from his 2 × 2 × 2 blocks.

   a. What is the surface area of the shape?

   _____________ square inches

   Show how you figured this out.

   b. What is the volume of the shape? _____________ cubic inches

   Explain how you figured this out.

   ____

   ____

7

Task 5: Building Blocks — Rubric

The core elements of performance required by this task are: • work with area and volume Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **6**	1	1
2. Gives correct answer: **8** cubic inches	1	
Shows work such as:		
2 x 2 x 2 =	1	
or length x breadth x height		2
3 a. Gives correct answer: **64** square inches	1	
Shows work such as:		
2((4 x 4) + (4 x 2) + (4x 2))	1	
b. Gives correct answer: **32** cubic inches	1	
Gives explanation such as:		
There are 4 cubes and each is 8 cubic inches. So 4 times 8 makes 32	1	4
or 4 x 4 x 2 =		
Total Points		**7**

Building Blocks

Work the task and look at the rubric. What are the key mathematical ideas being assessed?

Look at student work for part 2, finding the volume of the original block. How many of your students put:

8	6	4	24	12	10	Other

Why do you think students might make some of these errors? What are some of their misconceptions?

Now look at student work for part 3, finding surface area. How many of your students put:

64	32	8	16	4	12	10	256	Other

What are the students doing? Can you figure out the logic of their errors?
How many of your students made models or marked up the diagram to help them think through the different pieces of surface area they were calculating?
How many of your students used labels about sides, tops, etc. to help them track what they had calculated?
How many tried to multiply something by six, because there are six sides on the shape?
What types of experiences have your students had with calculating surface area?

Now look at student work for part 4. How many of your students put:

32	48	10	16	8	96	4	Other

Can you figure out where some of these answers are coming from? How are the misconceptions different for different responses?

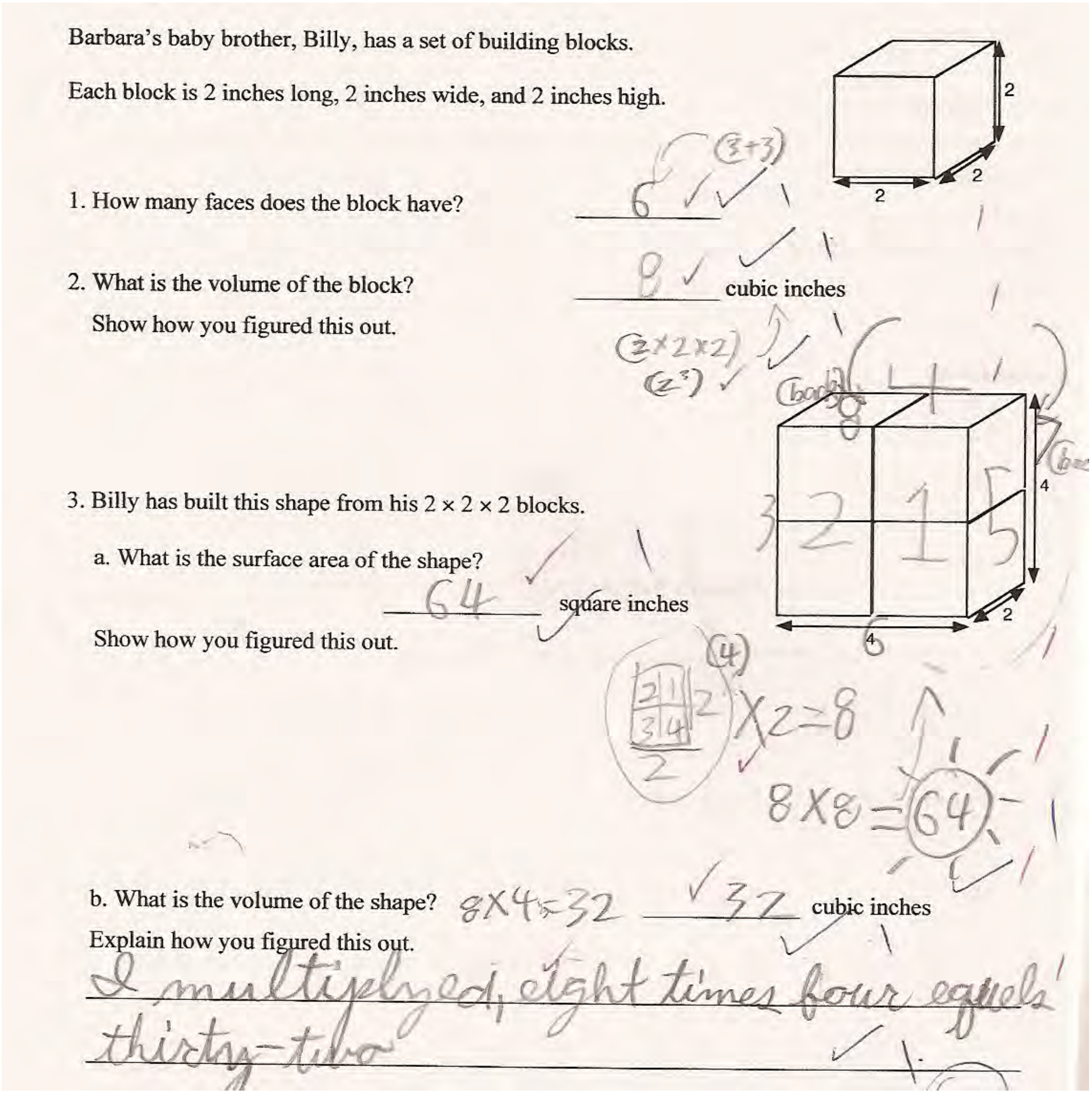
What types of activities and discussions will help students sort through the errors in their logic?

How many of the errors do you think result from students' inability to visualize a shape in all dimensions?
What activities or experiences do your students have to help them develop their spatial visualization? How often do students get opportunities to sketch all the sides of a shape or try to draw 3-dimensional shapes?

Looking at Student Work on Building Blocks

Student A is able to use the diagram to think about all the sides. To find surface area, the student has identified a unit (2 x4) and shown in the diagram how that unit is iterated 8 times around the surface.

Student A



Barbara's baby brother, Billy, has a set of building blocks.

Each block is 2 inches long, 2 inches wide, and 2 inches high.

1. How many faces does the block have? 6 (3+3)

2. What is the volume of the block? 8 cubic inches

   Show how you figured this out.

   (2x2x2)

   (2³)

3. Billy has built this shape from his 2 × 2 × 2 blocks.

   a. What is the surface area of the shape? 64 square inches

   Show how you figured this out.

   (4) X2=8

   8X8=64

   b. What is the volume of the shape? 8X4=32 32 cubic inches

   Explain how you figured this out.

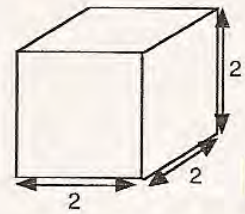
   I multiplyed, eight times four equels thirty-two

Notice how Student B uses shading to help him visualize the different sides of the shape in part 3. Student B shows calculations for each of the six sides for finding the surface area in part 3. The student is able to articulate and use a formula, but makes a calculation error in 3b.

Student B

Barbara's baby brother, Billy, has a set of building blocks.

Each block is 2 inches long, 2 inches wide, and 2 inches high.



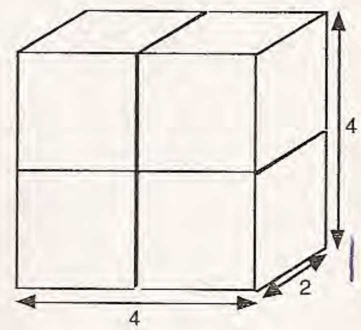
1. How many faces does the block have? ___6___ ✓ ✓

2. What is the volume of the block? ___8___ ✓ cubic inches ✓

   Show how you figured this out.

   L × W × H = Volume ✓
   2 × 2 × 2 = 8 ✓
   (4 × 2 = 8) ✓

3. Billy has built this shape from his 2 × 2 × 2 blocks.



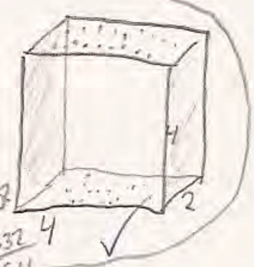
   a. What is the surface area of the shape? ✓ ___64___ square inches ✓

   Show how you figured this out.

   ( ) = 4×2
   ( ) = 2×4
   ( ) = 4×4

   each face appears 2x so...

   4×2 = 8 > 16
   4×2 = 8
   2×4 = 8 > 16 > 32
   2×4 = 8
   4×4 = 16 > 32 → 32 + 32 = 64
   4×4 = 16



   ✓

   b. What is the volume of the shape? ___48___ ✗ cubic inches

   Explain how you figured this out.

   The length of the shape is 4", the width 2in, and the height 4in. The formula for volume is L×W×H=V ✓ L=Length=4in W=Width=2in H=Height=4in (V=volume)
   so 4×2×4= (8×4) = 48in³ ✓
   L×W×H= ✓

   FW

   6 6 7

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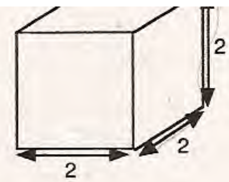
Building Blocks Test 6

Student C does not visualize that the 6 sides are not of the same size in part 3.

Student C

Each block is 2 inches long, 2 inches wide, and 2 inches high.



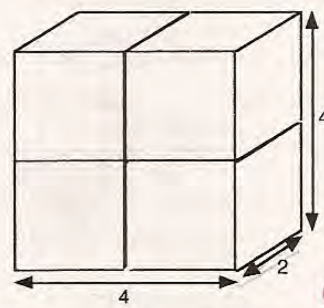
1. How many faces does the block have? 6 ✓ 1

2. What is the volume of the block? 8 ✓ cubic inches 1

   Show how you figured this out.

   L×W×H   2 ×2 = 4 ×2 = 8 ✓ 1

3. Billy has built this shape from his 2 × 2 × 2 blocks.



   a. What is the surface area of the shape? 48 ✗ square inches 0

   Show how you figured this out.

   2 ×4 = 8   8 ×6 = 48   W×H×# of faces 0

   4 ×2 = 8 ×4 = 32

   b. What is the volume of the shape? 32 ✓ cubic inches 1

   Explain how you figured this out.

   I got this by knowing that Length times width times heighth = volume. ✓ 1

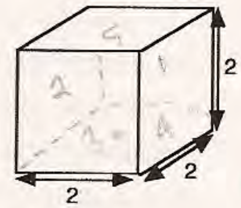
(5)

Student D does not understand that surface area includes all the faces of the shape. The student only finds the surface area for one face of the small block.

Student D

Barbara's baby brother, Billy, has a set of building blocks.

Each block is 2 inches long, 2 inches wide, and 2 inches high.



1. How many faces does the block have? 6 ✓

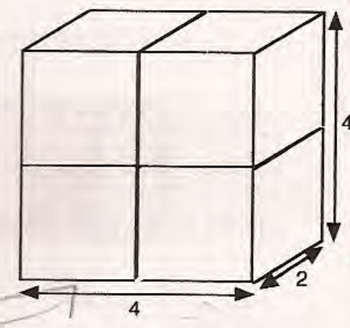
2. What is the volume of the block? 8 ✓ cubic inches

   Show how you figured this out.

   2×2×2=2³ ✓

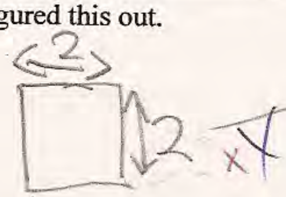
   2×2 ×2 4×2=8

3. Billy has built this shape from his 2 × 2 × 2 blocks.



   a. What is the surface area of the shape? 4 ✗ square inches

   Show how you figured this out.



   2×2=4

   b. What is the volume of the shape? 32 ✓ cubic inches

   Explain how you figured this out.

   I found this out by multipling all the numbers shown on the picture and got 32. ✓

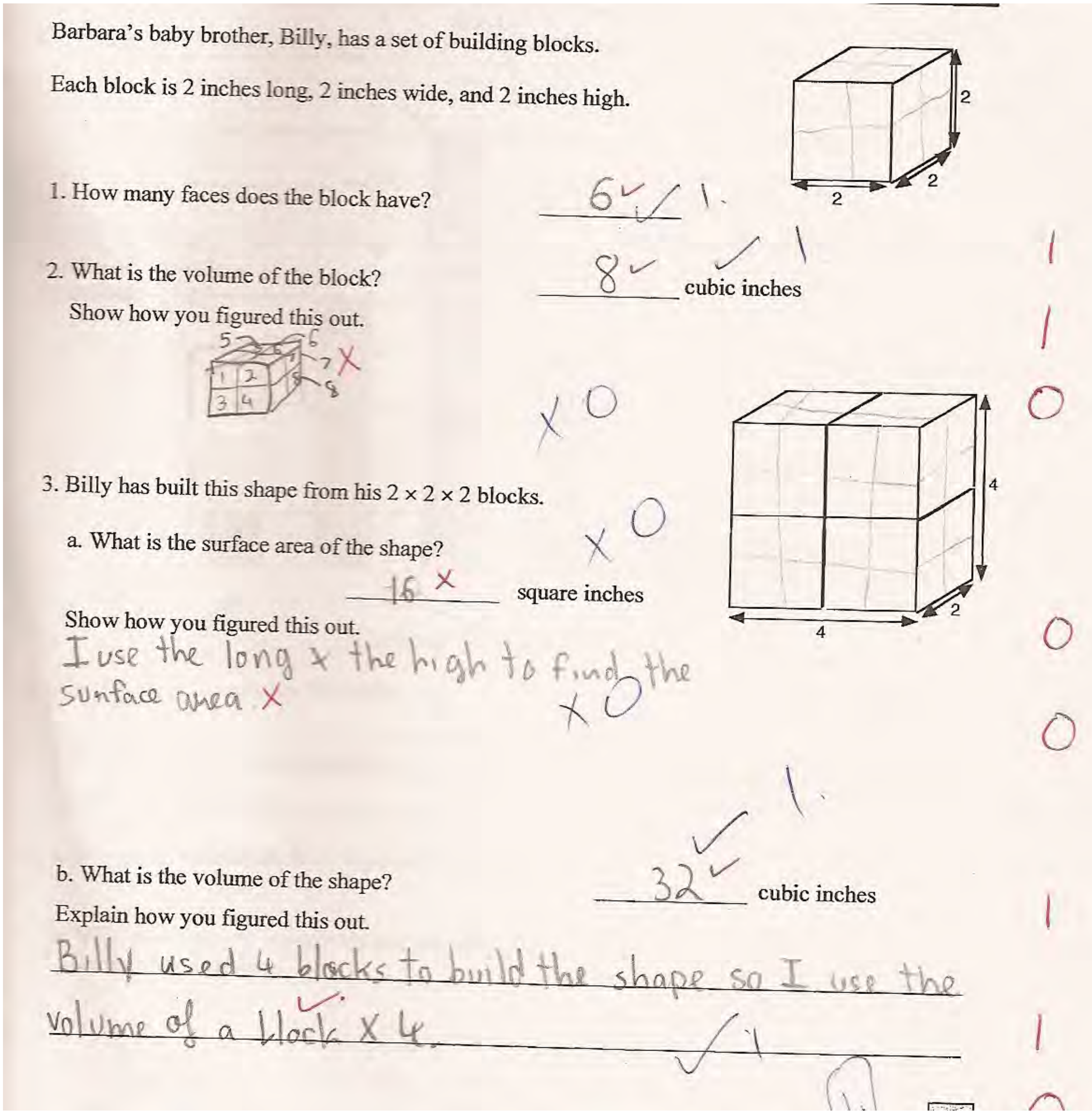
   4×4×2=32

   16 ×2=32

7

Student E divides the small shape into unit blocks and counts the blocks to find volume. This strategy continues in finding the volume in part 3b. The student seems to have found only the surface area of the front of the block in part 3b. *What do you feel comfortable that this student understands? What are you concerned about? What might be a good next task for this student?*

Student E



Barbara's baby brother, Billy, has a set of building blocks.

Each block is 2 inches long, 2 inches wide, and 2 inches high.

1. How many faces does the block have? 6

2. What is the volume of the block? 8 cubic inches

Show how you figured this out.

3. Billy has built this shape from his 2 × 2 × 2 blocks.

a. What is the surface area of the shape? 16 square inches

Show how you figured this out.

I use the long x the high to find the sunface anea.

b. What is the volume of the shape? 32 cubic inches

Explain how you figured this out.

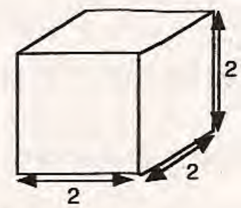
Billy used 4 blocks to build the shape so I use the volume of a block x 4.

In looking at the Student F's work in part 2, she appears to understand the formula for volume. However her thinking breaks down when applying the formula to the larger shape. *Why do you think the student doesn't revert back to the original formula in 3b?Can you see anything in the drawing that may have led to the expanded formula? What do you think the student is thinking about in finding the area in part 2b? Is the student thinking 2 for a small square and there are 8 visible small squares?*

Student F

Barbara's baby brother, Billy, has a set of building blocks.

Each block is 2 inches long, 2 inches wide, and 2 inches high.



1. How many faces does the block have? 6 ✓

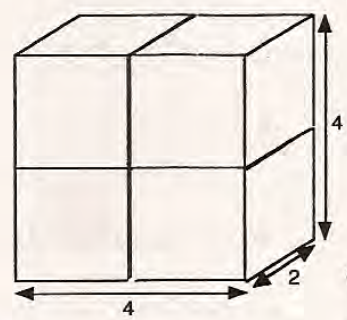
2. What is the volume of the block? 8 ✓ cubic inches

   Show how you figured this out.

   formula: long × wide × high ✓

   $2 \times 2 \times 2 = 2^3$ ✓

3. Billy has built this shape from his 2 × 2 × 2 blocks.



   a. What is the surface area of the shape? 256 ✗ square inches

   Show how you figured this out.

   2^8 2×2×2×2×2×2×2×2 = 256 ✗ (4 8 16 32 64 128)

   Formula: wide × high

   b. What is the volume of the shape? 512 ✗ cubic inches

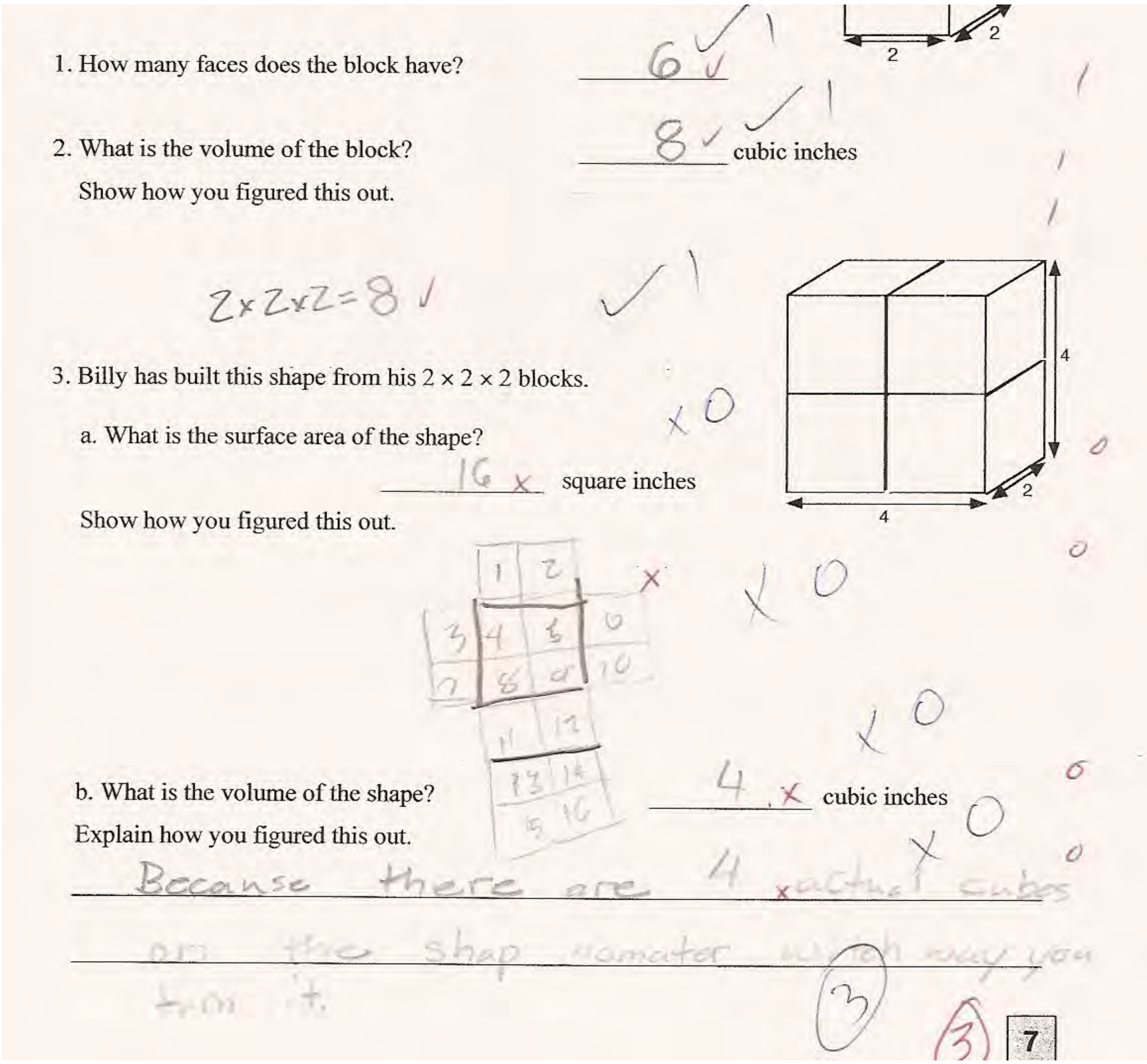
   Explain how you figured this out.

   I just multiplied the length, width, and height with each other

(3)

Understanding scale is difficult for students. In part 3a Student G realizes that there are sixteen units of the same size needed to cover the shape, but the student does not remember or isn't able to think about squares that aren't 1 square unit. *What is the size of the student's identified unit?* The same type of error occurs in 3b. The student understands that four blocks or equal-sized units make up the volume of the cube. But the blocks are not 1 cubic inch, but 8 cubic inches. *How do we help students move from units of one to units other than one?*

Student G



1. How many faces does the block have? 6

2. What is the volume of the block? 8 cubic inches

Show how you figured this out.

2x2x2=8

3. Billy has built this shape from his 2 × 2 × 2 blocks.

a. What is the surface area of the shape? 16 square inches

Show how you figured this out.

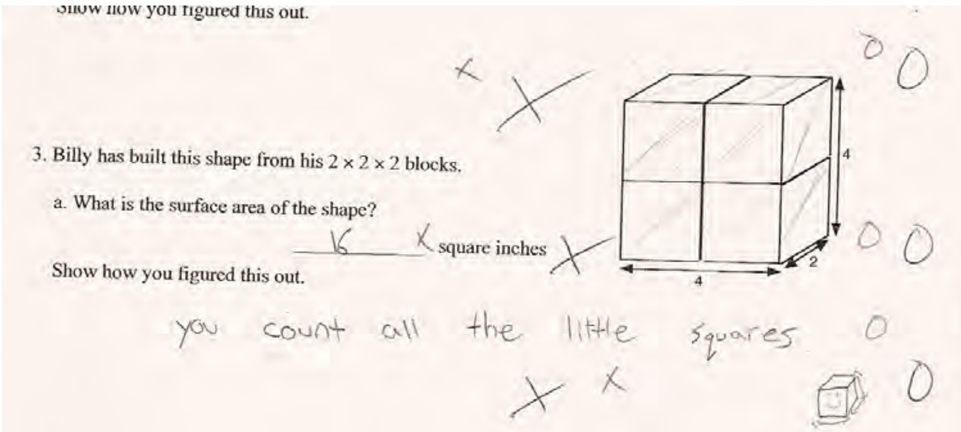
b. What is the volume of the shape? 4 cubic inches

Explain how you figured this out.

Because there are 4 actual cubes on the shap nomater which way you turn it.

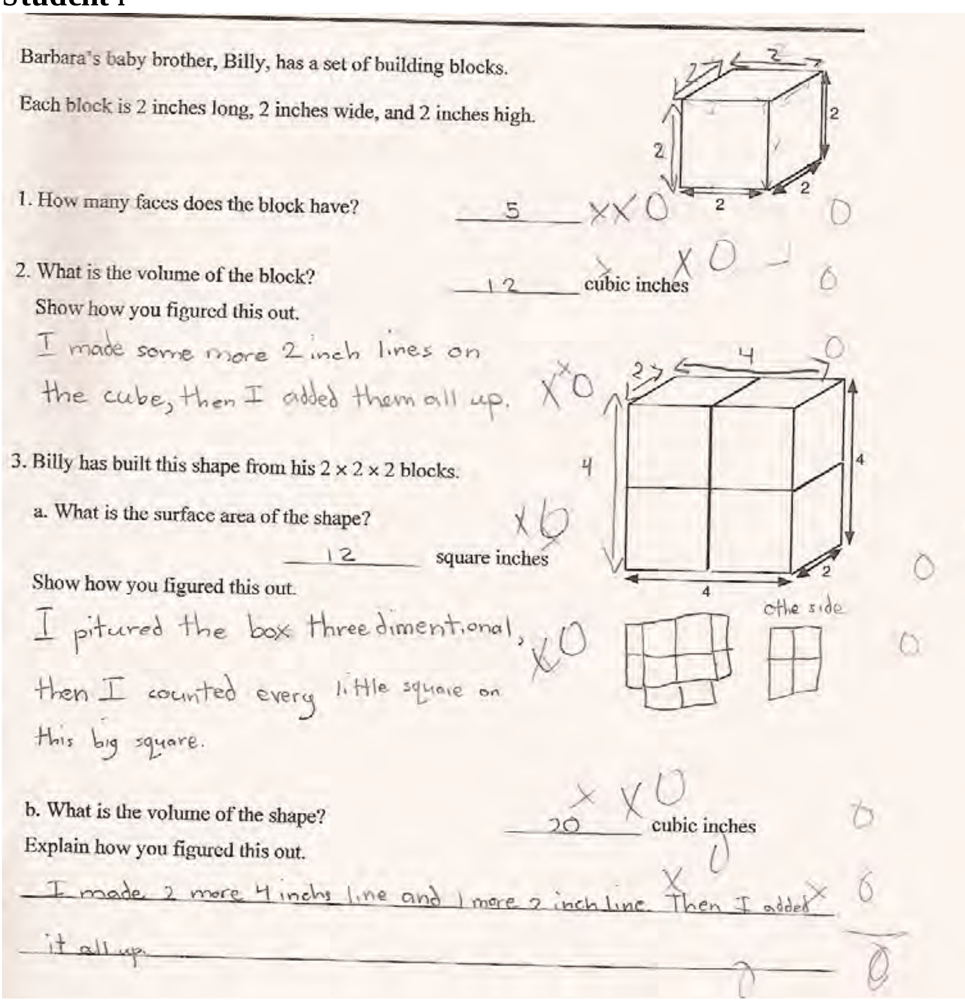
Student H marks the diagram and like G comes up with 16 units instead of 16 units of 4 square inches each.

Student H



Student I is attempting to count the 16 units of 4 square inches, but isn't able to unfold the shape correctly when attempting to draw the net. In part 2 the student seems to be finding a perimeter instead of an area. In part 3b the student is also attempting to add to find a perimeter instead of a volume.

Student I

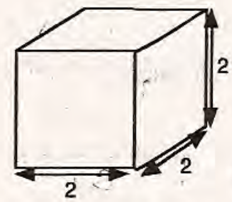


Look at the work of Student J and Student K. *What do you think they understand about 3-dimensional shapes? What do they understand about area and volume?* What *experiences would be good to help these students develop a better understanding of these measurements?*

Student J

Barbara's baby brother, Billy, has a set of building blocks.

Each block is 2 inches long, 2 inches wide, and 2 inches high.



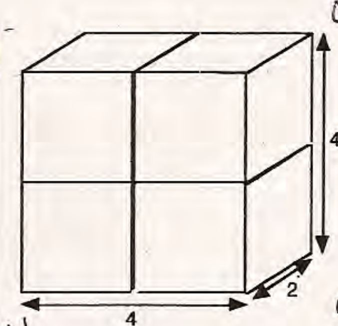
1. How many faces does the block have? 6 ✓

2. What is the volume of the block? 12 X cubic inches

Show how you figured this out.

I figured It out beause I was multiplying each sides So It was 12.

3. Billy has built this shape from his 2 × 2 × 2 blocks.



a. What is the surface area of the shape? 16 X square inches

Show how you figured this out.

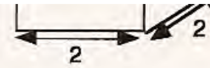
I figured It out because I added all the Sides up and In the cube It kept on going to 4 and then 2.
4X4=16

b. What is the volume of the shape? 10 X cubic inches

Explain how you figured this out.

because you have to add 4+2+4=10.

Student K

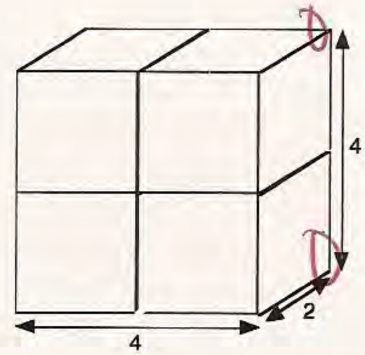


1. How many faces does the block have? 6

2. What is the volume of the block? 6 cubic inches

   Show how you figured this out.

   3 x 2 = 6



3. Billy has built this shape from his 2 × 2 × 2 blocks.

   a. What is the surface area of the shape?

   10 square inches

   Show how you figured this out.

   4 x 2 = 8   2 + 8 = 10

   b. What is the volume of the shape? 16 cubic inches

   Explain how you figured this out.

   I multiplyed some of the inches and added some.

6^{th} Grade Task 5 Building Blocks

Student Task	Work with area and volume using blocks of 2 x 2 x 2.
Core Idea 4 Geometry & Measurement	**Analyze characteristics and properties of three-dimensional geometric shapes and apply appropriate formulas to determine measurements**. Select appropriate type of unit for measuring each attribute (volume may be measured by filling an object.

Based on teacher observation, this is what sixth graders know and are able to do:
- Count the number of faces on a rectangular block
- Find the volume of a cube
- Find the volume of a rectangular prism

Areas of difficulty for sixth graders:
- Finding surface area of a rectangular prism
- Spatial visualization of 3-dimensional shapes
- Applying a scale different than one to a square being stamped or repeated to cover a rectangular prism

MARS Test Task 5 Frequency Distribution and Bar Graph, Grade 6

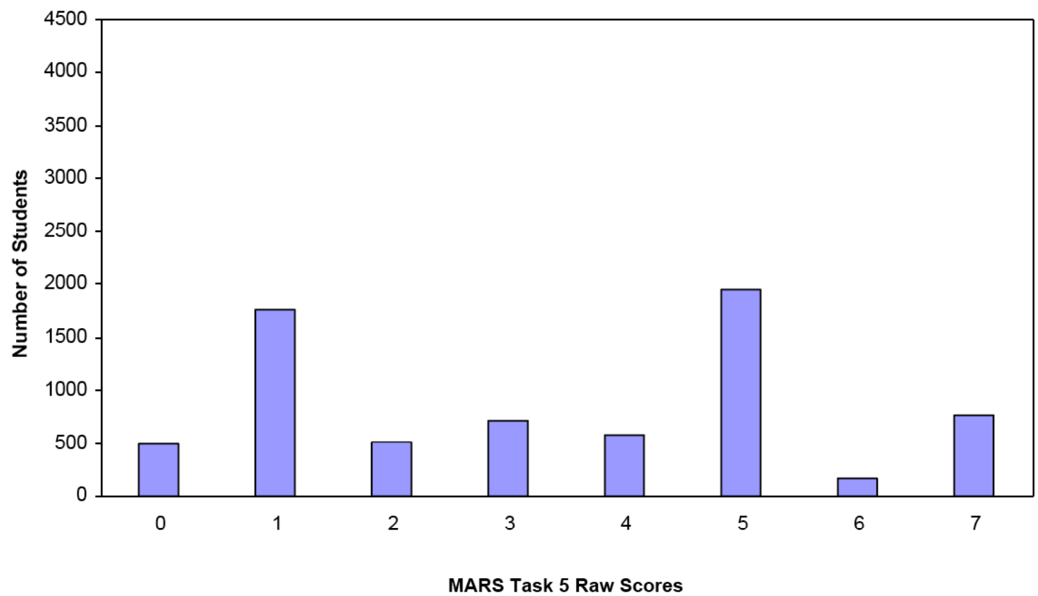
Task 5 - Building Blocks

Mean: 3.37 StdDev: 2.19

Table 34: Frequency Distribution of MARS Test Task 5, Grade 6

Task 5 Scores	Student Count	% at or below	% at or above
0	486	7.0%	100.0%
1	1774	32.5%	93.0%
2	502	39.7%	67.5%
3	714	50.0%	60.3%
4	577	58.3%	50.0%
5	1954	86.4%	41.7%
6	175	89.0%	13.6%
7	767	100.0%	11.0%

Figure 43: Bar Graph of MARS Test Task 5 Raw Scores, Grade 6



The maximum score available on this task is 7 points.
The minimum score for a level 3 response, meeting standards, is 3 points.

Most students, 93%, knew that there are 6 faces on a cube. More than half the students could also find the volume of a cube and show how they figured it out. A little less than half, 42%, could also find the volume of a rectangular prism. 11% of the students could meet all the demands of the task including finding the surface area of a rectangular prism. 7% of the students scored no points on this task. 80% of the students with this score attempted the task.

Building Blocks

Points	Understandings	Misunderstandings
0	80% of the students with this score attempted the task.	Students had difficulty understanding the number of faces on a cube. The most common answers were 3(faces visible in diagram) and 5 (usually forgetting the bottom).
1	Students knew that there were 6 faces on a cube.	Students had difficulty finding volume of a cube. 9% thought the volume was 8. 9% thought the volume was 12. 6% thought the answer was 24.
3	Students knew the faces on a cube and could find the volume of a cube.	Students had difficulty applying their knowledge of volume of a cube to finding volume of a rectangular prism. 7% added the dimensions to get a volume of 10. 4% thought the volume was 48. 4% thought the volume was 16. Answers for volume went as high as 512 and 1096.
5	Students knew the number of faces on a cube and could find the volume of a cube and a rectangular prism.	They struggled with the idea of surface area of a rectangular prism. Almost 18% thought the surface area was 32. 16% thought the surface area was 16. 12% thought the surface area was 8. Other common answers were 4 and 12.
7	Students could reason about 3-dimensional shapes, thinking about the number of faces and finding the area of the faces. Students could also find the volume for rectangular prisms.	

Implications for Instruction

Students should know the formula for the volume of a rectangular prism and be able to substitute numbers into the formula to calculate the volume. Alternately, students should be able to think about how many blocks it takes to build the shape and multiply the volume of one-block times the number of blocks. Students need more opportunities to draw or make representations of three-dimensional shapes as tools for thinking about how the shape is composed or can be decomposed into simpler shapes. Students, who struggle with drawing or representation, are more likely to ignore or "not see" parts of the shape. Triangular dot paper is useful to recording three-dimensional shapes.

Students should also be able to find area of rectangular shapes and be able to organize that information to find surface area. They might not think about the bottom or backside of the shape, but should be able to find a majority of the areas of the surface of a prism. When students develop the habit of mind of using the diagrams as tools for thinking, they are more successful. Simple actions like adding size of the dimensions to more parts of the diagram can help them sort through the parts to be considered.

Ideas for Action Research – Using Student Generated Strategies

Student work on this task brings up some interesting ideas about measurement and iterating units. Having students use these ideas can push their thinking about the shapes in some directions that may help them generalize to other situations or back better connections to understanding where the formulas come from.

Here are some questions you might pose for the class:

In trying to find the surface area, Alice says that she sees 8 equal size shapes that would cover the outside of the blocks. What do you think Alice is looking at? Where are those shapes in the drawing? What are the dimensions of Alice's shape? How could you mark off her shapes in the diagram?

Give students some blank diagrams and consider giving them some blocks or linker cubes to build the shape. (*For your reference look at the work of Student A.*)

Ernie says that when he thinks about unfolding the shapes he counts 16 squares. What do you think Ernie is looking at? Can you draw what Ernie is doing? Does this help him find the surface area?

Here, hopefully students will start to see the difference between the squares in the net and the size of the squares. (*For your reference look at the work of Student E and G.)*

Georgia says that she used a similar method to find the volume of the cubes. First I broke the original cube into smaller cubes, each one by one by one. Can you think what this would look like? How would this help her find the volume? Can you draw what you think she did?

For the next question:

Georgia says that Billy used 4 blocks to build the shape so I use of volume of a block x 4. What is Georgia's number sentence? Does this make sense? Would it work on other shapes made with the blocks? *Hold up some other rectangular prism and ask students to find the volume using Georgia's method. Then have them use the volume formula. Will the answers come out the same?*

(*Look at work of Student E for the last two questions about Georgia.)*

Conner has a different strategy for finding surface area. He said that the area of the side was 2 x 4, which is 8. Then he said that there are 6 sides on the prism, so 6 x 8 is 48. Why does Conner have a different answer from the one we found using Ernie's strategy? Who do you think is right?

(*See work of Student C.)*

How do these types of questions help students develop an understanding of a unit and decomposing shapes? How is this useful mathematically?
How do the questions help students confront and challenge common misconceptions? Do you think this helps students let go of some of their ways of thinking? What evidence did you see of students changing their ideas?

Reflecting on the Results for Sixth Grade as a Whole:

Think about student work through the collection of tasks and the implications for instruction. What are some of the big misconceptions or difficulties that really hit home for you?

____

If you were to describe one or two big ideas to take away and use for planning for next year, what would they be?

____

What are some of the qualities that you saw in good work or strategies used by good students that you would like to help other students develop?

____

Four areas that stand out for the Collaborative as a whole are:

1. Understanding the Operation– Students had difficulty recognizing different types of division actions to help them make sense of a situation. They did not understand about significant digits or level accuracy that would apply to the context of the problem. Many students wrote problems that did not describe a division situation. If students have difficulty understanding the action of division with whole numbers, then they will have problems trying to make sense of operations with fractions and decimals. Many of the issues of division don't come up unless students are working in a context. Working sets of division problems ideas about remainder and rounding do not apply. Students had difficulty matching problem descriptions to calculations in Household Statistics. Often students did not even choose matches that had the correct operations. Being able to move from doing a series of individual calculations to writing the steps into one expressions helps students generalize about problem solutions and sets the stage for later writing procedures down in symbolic notation in algebra.

2. Probability – Some students did not understand the idea of probability as a chance or a prediction. They looked ahead for actual outcomes. Others did not understand the idea of "random" or "equally-likely" events. They tried to find a pattern in the information. Students did not understand how to define sample space and therefore had difficulty quantifying probabilities.
3. Understanding Scale and Frequency – In the data task, Household Statistics, students had trouble distinguishing between categories, number of children in a household, and frequency, how many households had that number of children. This made it difficult for them to find totals. They may have added number of categories or the numbers on the scale instead of the quantities being represented by the bars on the graphs. In Building Blocks students may have identified a unit, how many cubes fit inside a shape or how many squares it takes to cover a shape, but they often forgot to consider the size of the unit. They may have only worked with units of one.
4. Finding Patterns with Number Properties – Students had difficulty finding or describing some of the patterns in Factors. They are still only considering properties like odd and even. They are not comfortable with prime, square, and other more complex types of patterns. As students progress through the grades, the types of patterns or thinking they do needs to be more complex or deeper than work from previous grades. Students are also trying to apply patterns or find patterns in situations where patterns do not apply, like probability.

Performance Assessment Task
Building Blocks Grade 6
The task challenges a student to demonstrate understanding of area, surface area, and volume. A student must be able to analyze characteristics and properties of three-dimensional geometric shapes and apply appropriate formulas to determine measurements. A student must understand the difference between area, surface area, and volume. A student must be able to read given diagrams of three-dimensional shapes. A student must calculate, using formulas, area, surface area, and volume.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Solve real-world mathematical problems involving area, surface area, and volume.** 6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
Common Core State Standards Math – Standards of Mathematical Practice
MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.6 Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
6	2007	7	3	60 %