Aaron's Designs

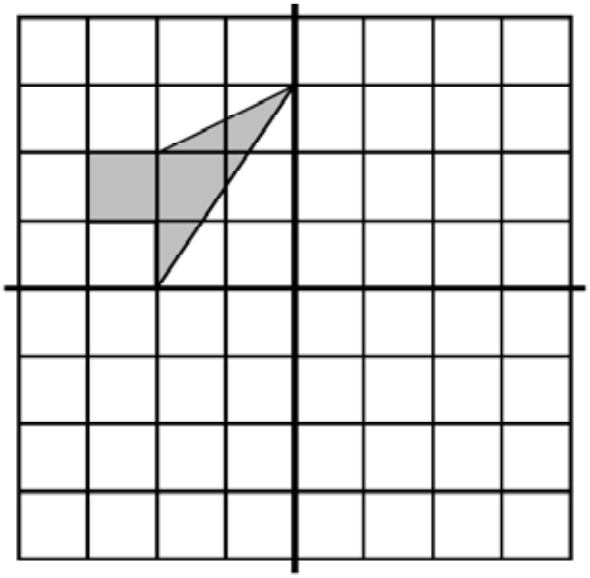
This problem gives you the chance to:
- draw reflections and rotations of a given figure on a grid
- describe transformations needed to make a given pattern

Aaron is drawing some designs for greetings cards.
He divides a grid into 4 quadrants and starts by drawing a shape in one quadrant.
He then reflects, rotates or translates the shape into the other three quadrants.

1. Finish Aaron's first design by reflecting the gray shape over the vertical line.

   Then reflect both of the shapes over the horizontal line.

   This will make a design in all four quadrants.

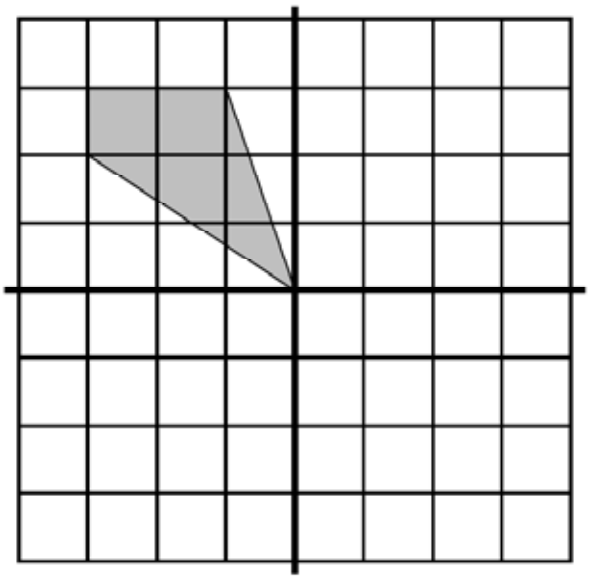


2. To finish drawing Aaron's second design, rotate the gray shape 1/4 of a turn in a clockwise direction about the origin. Then draw the second shape.

   Rotate the second shape 1/4 of a turn in a clockwise direction about the origin. Then draw the third shape.

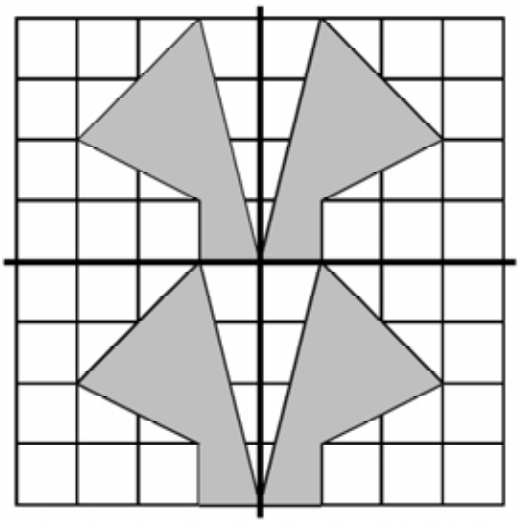
   Rotate the third shape 1/4 of a turn in a clockwise direction about the origin. Then draw the fourth shape.

   This will make a design in all four quadrants.



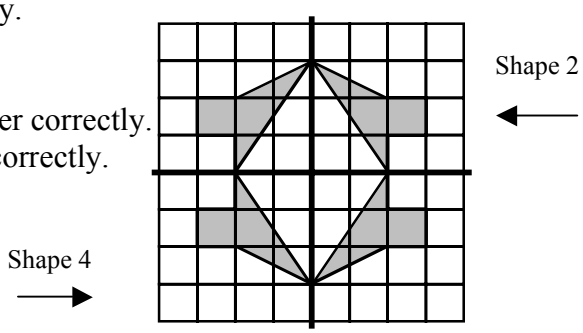
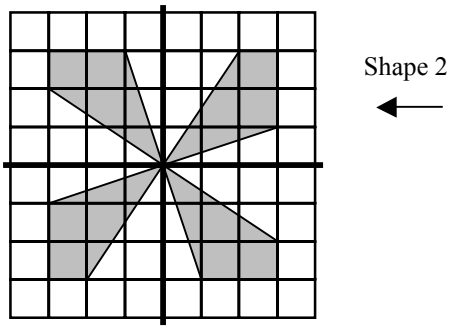
3. This is Aaron's third design.

He started with one gray shape in the top left hand quadrant of the grid and transformed it to make the design.



Describe the transformations that Aaron may have used to draw this design.

8

Aaron's Designs	Rubric	
The core elements of performance required by this task are: • draw reflections and rotations of a given figure on a grid • describe transformations needed to make a given pattern Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Draws all 3 shapes correctly. *Partial credit* Draws shape 2 and one other correctly. Draws shape 2 or shape 4 correctly.  	3 (2) (1)	3
2. Draws all 3 rotations correctly. *Partial credit* Draws shape 2 correctly.  	2 (1)	2
3. Gives a correct description such as: **Reflects** the shape over the vertical line, then **translates/slides** the 2 shapes **down 4 squares**.	1 1 1	3
Total Points		**8**

8th Grade – Task 1: Aaron's Designs

Work the task and examine the rubric.

What do you think are the key mathematics the task is trying to assess?

What opportunities do students in your class have to work with transformations: reflections, rotations, flips, turns, slides?

How are these skills relevant to today's job market?

What kind of work have your students done with symmetry? Do they get practice making their own shapes or drawing in the second part of a shape? How is drawing in a shape different from drawing in a line of symmetry? What mathematics come up in making your own drawing?

Look at student work on part one, using reflections across two lines of symmetry. How many of your students:

Drew all 4 shapes correctly	Slid the top quadrants to the lower quadrants	Slid the shape to right	Put nothing in the lower quadrants	Made a different shape	Didn't draw the shape to scale	Other

Rotating the shape was difficult for students. What opportunities have they had to work with rotations? What strategies might students use to help them solve this task? Try working this part with colleagues. Did you all use the same way? See if you can share some of this variety with students.

Look at part two of the task, rotating a shape. How many of your students:

Made all the rotations correctly	No response	Reflected the shape 4 times	Made a different shape	Other

Many students are using common language rather than mathematical language to describe transformations such as copy and move. What type of work have students done with the transformation?
Look at student work on part three, describing a transformation. How many of your students:

Describe the rotation	Describe the slide down	Use quantity for the slide	Use nonmathematical language	Talked about copying	Inaccurate measurement or scale	Other

As you looked at student work, did you see evidence of students who were operating at a low van Hiele level?

What effort do you make to design and put activities into your curriculum to help students to progress through the van Hiele levels?

What are some implications for instruction that you want to remember after looking at your student work?

Looking at Student Work on Aaron's Designs

Student A is able to do both the reflection and rotations, keeping the shapes in scale as they are moved. Notice in part three the student numbers the quadrants to make the meaning clearer to the reader. The student is able to quantify the distance and direction to move the shapes.

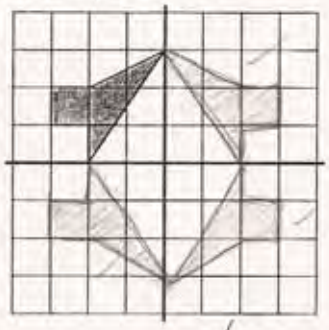
Student A

Aaron is drawing some designs for greetings cards.
He divides a grid into 4 quadrants and starts by drawing a shape in one quadrant.
He then reflects, rotates or translates the shape into the other three quadrants.

1. Finish Aaron's first design by reflecting the gray shape over the vertical line.

   Then reflect both of the shapes over the horizontal line.

   This will make a design in all four quadrants.

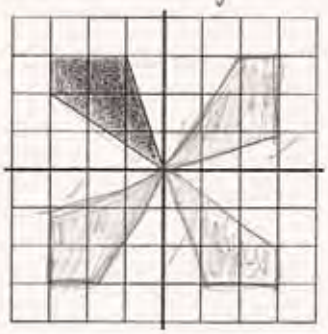


2. To finish drawing Aaron's second design, rotate the gray shape 1/4 of a turn in a clockwise direction about the origin. Then draw the second shape.

   Rotate the second shape 1/4 of a turn in a clockwise direction about the origin. Then draw the third shape.

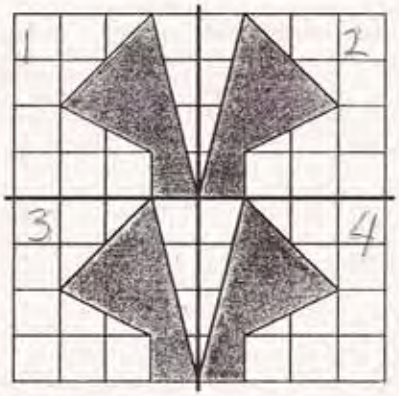
   Rotate the third shape 1/4 of a turn in a clockwise direction about the origin. Then draw the fourth shape.

   This will make a design in all four quadrants.



3. This is Aaron's third design.

   He started with one gray shape in the top left hand quadrant of the grid and transformed it to make the design.

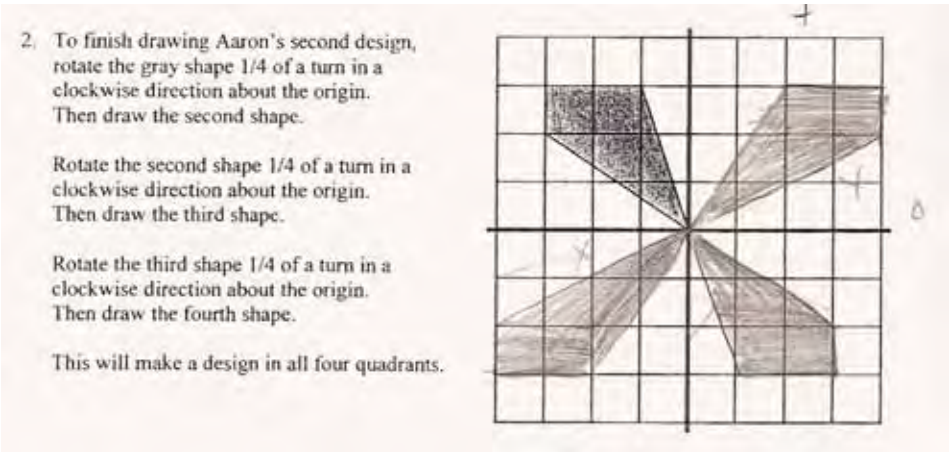


Describe the transformations that Aaron may have used to draw this design.

He probably reflected the first shape over into the second quadrant and then moved both shapes 4 spaces down to create the last two

Student B is able to draw the reflection for part 1 of the task and attempts to do a reflection in part two instead of a rotation. The student distorts the shape, not quite able to track all the measurements of the original design.

Student B



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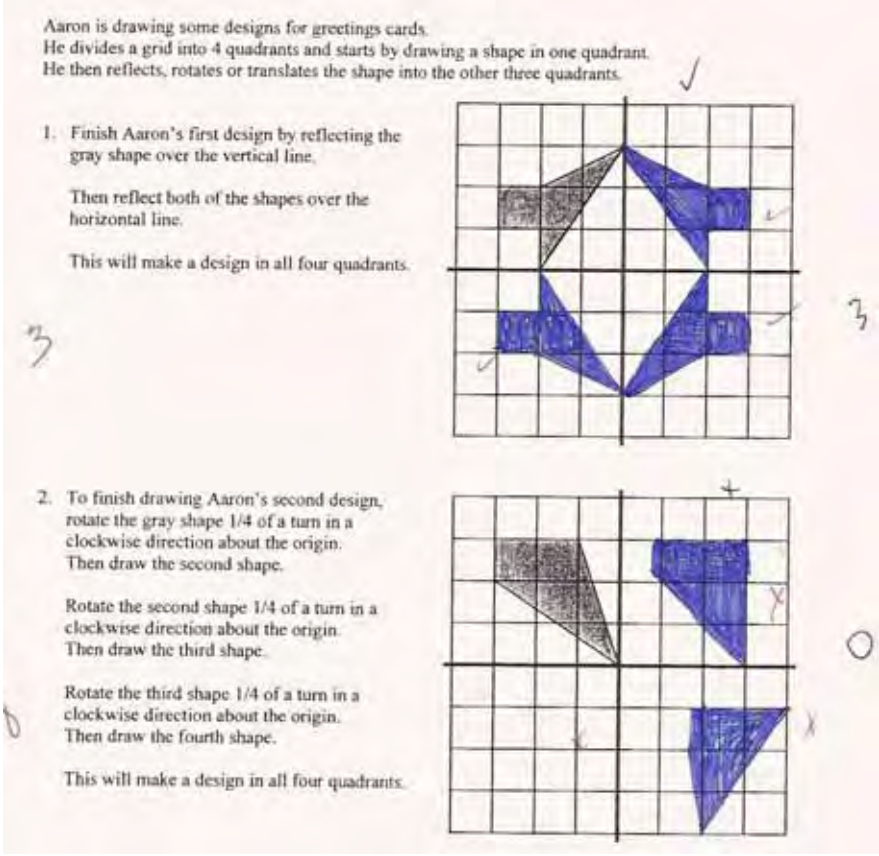
   Rotate the second shape 1/4 of a turn in a clockwise direction about the origin. Then draw the third shape.

   Rotate the third shape 1/4 of a turn in a clockwise direction about the origin. Then draw the fourth shape.

   This will make a design in all four quadrants.

Student C tries to slide the shape in part two, but again can't accurately locate one of the vertices. Notice that not even a distorted version of the shape is maintained when the shape is moved to the 4th quadrant.

Student C



Aaron is drawing some designs for greetings cards.
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1. Finish Aaron's first design by reflecting the gray shape over the vertical line.

   Then reflect both of the shapes over the horizontal line.

   This will make a design in all four quadrants.

2. To finish drawing Aaron's second design, rotate the gray shape 1/4 of a turn in a clockwise direction about the origin. Then draw the second shape.

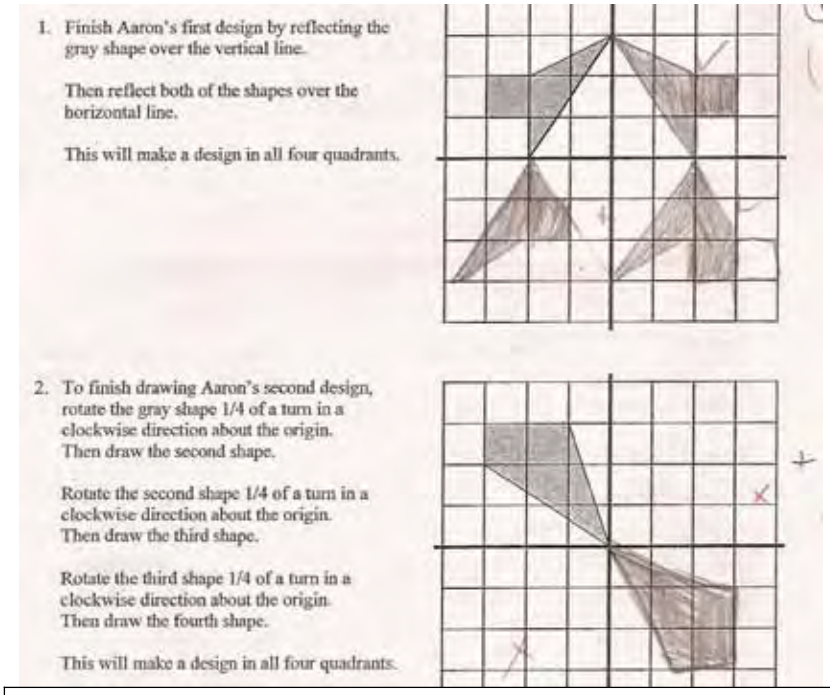
   Rotate the second shape 1/4 of a turn in a clockwise direction about the origin. Then draw the third shape.

   Rotate the third shape 1/4 of a turn in a clockwise direction about the origin. Then draw the fourth shape.

   This will make a design in all four quadrants.

Student D is able to do the reflection across the vertical axis, but changes the shape and size of the design when trying to reflect across the horizontal axis. The student does not understand rotation and the shape also is not scale in part 2.

Student D



1. Finish Aaron's first design by reflecting the gray shape over the vertical line.

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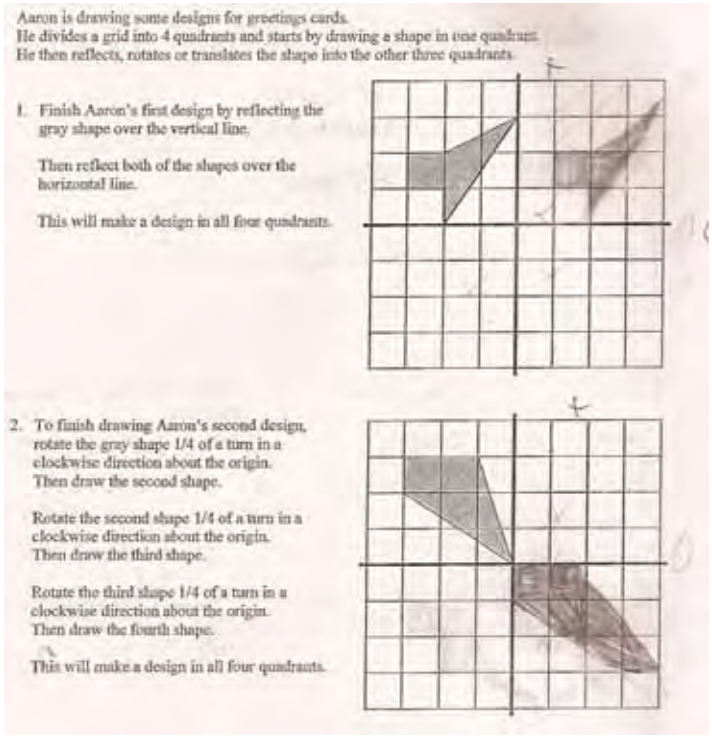
   Rotate the third shape 1/4 of a turn in a clockwise direction about the origin. Then draw the fourth shape.

   This will make a design in all four quadrants.

Student E seems do a horizontal slide in part 1, but is unable to correctly locate two of the vertices. In part two, the student seems to do a diagonal slide, but again distorts the shape by mislocating a vertex.

Student E



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1. Finish Aaron's first design by reflecting the gray shape over the vertical line.

   Then reflect both of the shapes over the horizontal line.

   This will make a design in all four quadrants.

2. To finish drawing Aaron's second design, rotate the gray shape 1/4 of a turn in a clockwise direction about the origin. Then draw the second shape.

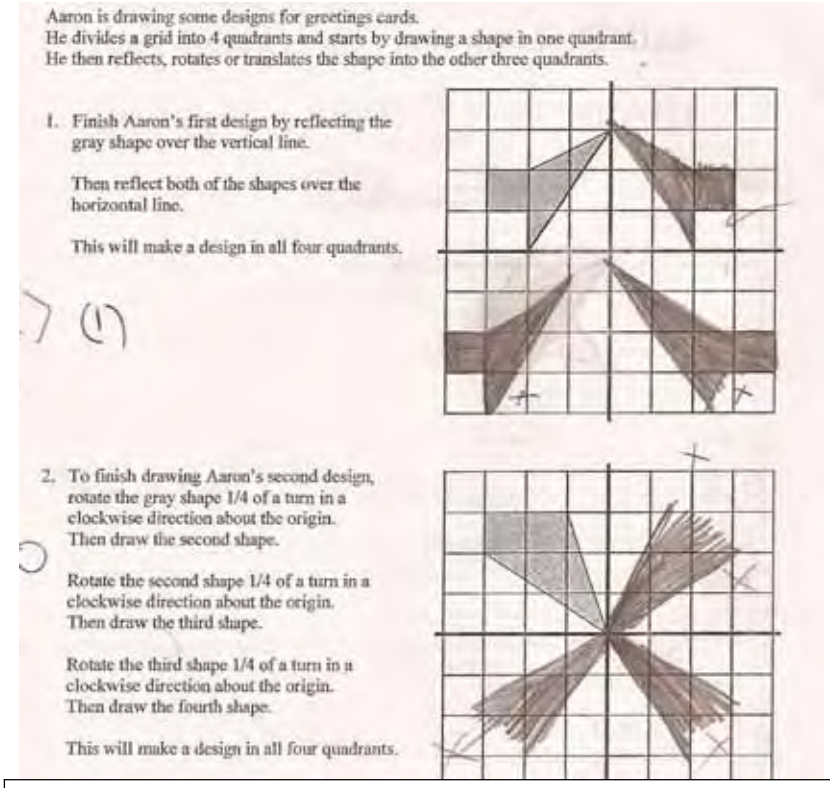
   Rotate the second shape 1/4 of a turn in a clockwise direction about the origin. Then draw the third shape.

   Rotate the third shape 1/4 of a turn in a clockwise direction about the origin. Then draw the fourth shape.

   This will make a design in all four quadrants.

Student F does one correct rotation and then tried to slide the shape down to the lower quadrants, maybe using the drawing in part three as a guide. Notice that the lower right shape is not to scale. In part two the student draws shapes different from the original figure. *Do you think this is caused because the student knows there is some movement but doesn't understand the language enough to finish the end part of each design?*

Student F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1. Finish Aaron's first design by reflecting the gray shape over the vertical line.

   Then reflect both of the shapes over the horizontal line.

   This will make a design in all four quadrants.

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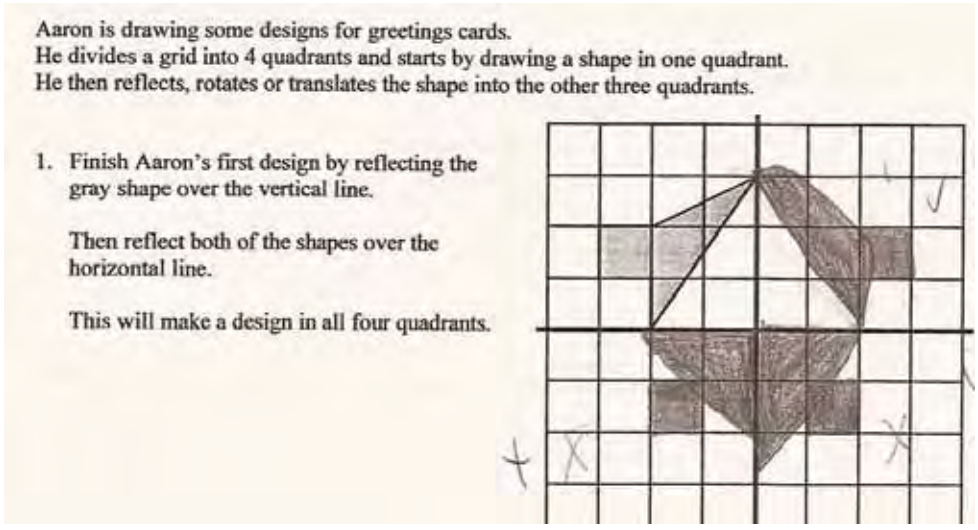
   Rotate the second shape 1/4 of a turn in a clockwise direction about the origin. Then draw the third shape.

   Rotate the third shape 1/4 of a turn in a clockwise direction about the origin. Then draw the fourth shape.

   This will make a design in all four quadrants.

Student G seems to have some vague idea of reflection over the vertical axis, but cannot maintain the original shape with the reflection over the vertical axis.

Student G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1. Finish Aaron's first design by reflecting the gray shape over the vertical line.

   Then reflect both of the shapes over the horizontal line.

   This will make a design in all four quadrants.

Students had trouble with the formal language for transformation. Student H is able to use the language correctly in the first two sentences but then uses reflection incorrectly in the last sentence. The student gives the direction of the slide but not the distance in the second sentence.

Student H

Describe the transformations that Aaron may have used to draw this design.

From the first to the second quadrant he used a reflection. From the second to third quadrant he translates it. From the third to fourth quadrant he reflected image than from the fourth to first quadrant he translated it.

Student B uses the vocabulary of transformation, but uses it incorrectly. There is no mention of direction or distance in the student's directions.

Student B

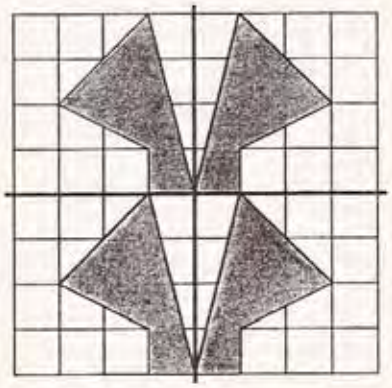
Describe the transformations that Aaron may have used to draw this design.

The object was rotated ½ a turn and reflected on the horizontal line.

Here are some more typical responses to part 3. *What is missing or incorrect in each one?*

3. This is Aaron's third design.

He started with one gray shape in the top left hand quadrant of the grid and transformed it to make the design.



Describe the transformations that Aaron may have used to draw this design.

Aaron first reflected it across the vertical line then repeated the design on the boxes below.

Describe the transformations that Aaron may have used to draw this design.

When he drew the shape in the top left he mirror imaged it on the right then he copied the top shape to the bottom.

He counted the squares and did it.

Describe the transformations that Aaron may have used to draw this design.

He started with simple designs then I think he just start drawing things

Describe the transformations that Aaron may have used to draw this design.

The transformations that Aaron may have used to draw this design is maybe paper.

Eighth Grade

8th Grade Task 1 Aaron's Designs

Student Task	Draw reflections and rotations of a given figure on a grid. Describe transformations needed to make a given pattern.
Core Idea 4 Geometry & Measurement	**Apply transformations and use symmetry to analyze mathematical situations.** • Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling.

Based on teacher observation, this is what eighth graders knew and were able to do:

- Reflect shapes over the vertical axis and slightly less often over the horizontal axis
- Describe a reflection

Areas of difficulty for eighth graders:

- Rotations
- Describing direction and distance for slides or translations
- Maintaining shape and size or scale when making their drawings

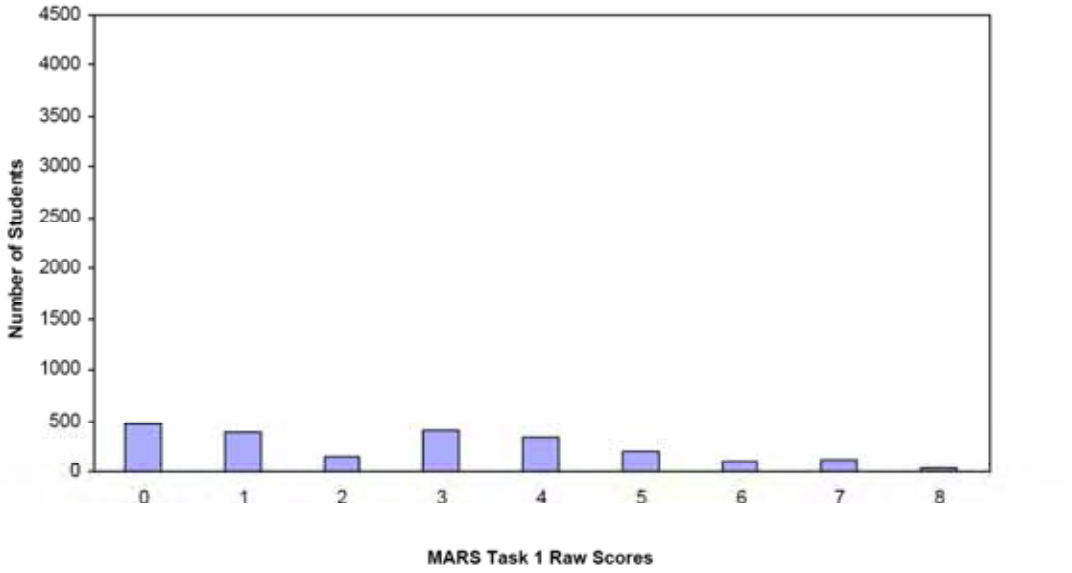
Task 1 - Aaron's Design's

Mean: 2.69 StdDev: 2.18

Table 40: Frequency Distribution of MARS Test Task 1, Grade 8

Task 1 Scores	Student Count	% at or below	% at or above
0	480	21.5%	100.0%
1	388	38.8%	78.5%
2	160	46.0%	61.2%
3	413	64.5%	54.0%
4	337	79.6%	35.5%
5	204	88.7%	20.4%
6	106	93.4%	11.3%
7	111	98.4%	6.6%
8	36	100.0%	1.6%

Figure 49: Bar Graph of MARS Test Task 1 Raw Scores, Grade 8



The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Many students, about 78%, could reflect a design across a vertical axis while maintaining the scale and shape of the original design. More than half the students, 54%, could accurately reflect a design across both a vertical and a horizontal axis. A few students, about 20%, could do reflections and also describe how a design had been reflected as well as give a direction for a slide or translation. Less than 2% of the students could meet all the demands of the task, including reflecting and rotating shapes, describing reflections and translations with direction and quantity for the moves. Almost 22% of the students scored no points on this task. 80% of the students with this score attempted the task.

Aaron's Designs

Points	Understandings	Misunderstandings
0	80% of the students with this score attempted the task.	In attempting the reflections in part one, 12% of the students could not maintain the original shape when making their drawing or did not draw it to scale. 5% of the students slid the shape to the right instead of making a reflection.
1	Students could make a reflection across a vertical axis.	19% of the students slid the design from the top quadrants to the lower quadrants instead of making a reflection. Another 5% put nothing in the lower quadrants.
3	Students could make a reflection across a vertical and a horizontal axis, while maintaining shape and size.	10% of the students did not attempt to describe the translation in part 3 of the task. 19% of the students used nonmathematical language in their descriptions in part 3.
5	Student could make reflections and describe reflections in a transformation.	They had difficulty giving direction and quantity to slides. 18% of the students did not give a reference as to what was being moved or transformed or an axis for making a reflection. 28% of the students gave instructions about copying the design. Students could not do rotations. Almost 50% of them drew reflections for part two instead of rotations. 19% changed the shape completely in their attempt to do a reflection. Almost 10% were unwilling to even attempt a rotation.
6		Students did not give a direction for the slide or "move". They did not quantify the size of the slide or "move".
7		Students did not quantify the size of the move.
8	Students could make reflections and rotations across vertical and horizontal axes, while maintaining shape and size. Students could describe reflections with reference to axis of reflection and slides with reference to direction and size of the slide.	

Implications for Instruction

Students at this grade level need frequent exposure to activities that develop their spatial reasoning and ability to distinguish geometric shapes, properties of shapes, and to develop their reasoning and generalizing skills between properties of shapes. Research suggests that everyone develops through levels of understanding (van Hiele levels) based not on maturity, but on experiences. Research further suggests that the ability to make formal and informal deductions, such as that required in a high school geometry class, without first moving through these lower levels. More than half the students entering a geometry class may still be operating at a level 0 (visualization) or 1(analysis).

Students need opportunities to sort and categorize shapes by their properties. For middle school working with a software program, like Geometer's Sketchpad, can be useful for exploring examples of classes of shapes and can further help students start to build and test conjectures.

Students need more work with drawing rotations, slides, and reflections. Students need to be able to describe the line of reflection or the distance of a slide or other transformation. By making their own drawings and transformations, students learn about the importance of scale and start to see more of the detail in the shapes or designs. Students need more opportunity to work with transformations on a coordinate grid. In a world dominated by special effects in movies, video cell phones, graphic design, missile technology, hdtv, sending images on computers, defense systems, as well as traditional work of engineers, architects, carpenters, it is more important than ever for students to have the visualization skills to function in today's world.

Working with geometry can be very enjoyable for students and give some students a chance to shine, who may have not be so successful in other areas of mathematics. For most students these types of activities and learning experiences are very motivating. Their success can transfer to a positive attitude when they then attempt other activities in the classroom. There are a number of great resources for working with transformations: John Van de Walle – Teaching Student-Centered Mathematics, Connected Mathematics, and Mathematics in Context. Developing spatial skills is directly related to opportunity to learn.

Ideas for Action Research

Clue Cards

Students need to understand the importance of detail, reference to line of reflection, direction, and distance in giving directions. Try making clue cards from some of the student responses to part three in Aarons design and give students grids with only the top left quadrant filled in. Ask students to use the clue cards to try and complete Aaron's third design. Don't give them a lot of definitions or explanations, but tell them that they can look up any words that are unfamiliar to them or use any resources available in the room. Let them take some initiative in their own learning and what they need to know.

When students have completed their designs, have them compare what they made with each other. Did they all draw the same thing for each clue card? Why or why not? Have students discuss which clues were most helpful or most confusing. What would make the clues more helpful?

Now have students make their own simple design that can be made in the 4 quadrants using flips, slides and rotations. Then ask students to write their own clue card for their design. Have them trade with each other or pick three or four to give to the class. See how their thinking has improved.

How did the students do with the idea of being responsible to look up things that they didn't understand? What evidence did you see that they used this opportunity? How is this type of learning different from you "explaining" something to them?

How did the discussion about the clues focus students on the geometric ideas of distance and direction? What other important ideas came up during the discussion?

Performance Assessment Task

Aaron's Designs
Grade 8

This task challenges a student to use transformations, reflections and rotations on a coordinate grid. A student must be able to quantify a transformation with direction and distance. A student must be able to draw a reflection or a rotation given the number of degrees.

Common Core State Standards Math - Content Standards

Geometry

Understand congruence and similarity using physical models, transparencies or geometry software.

8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Common Core State Standards Math – Standards of Mathematical Practice

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
8	2006	8	5	20%