Problem of the Month
Tri-Triangles

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. A POM can also be used school wide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The POM is designed with a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially as the key challenge for grades kindergarten and one. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

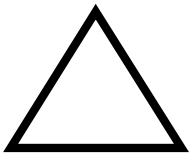
In the Problem of the Month *Tri-Triangles,* students use algebraic thinking to solve problems involving patterns, sequences, generalizations, and linear and non-linear functions. The mathematical topics that underlie this POM are finding and extending patterns, creating generalizations, finding functions, developing inverse processes, exploring non-linear functions, and justifying solutions.

In the first level of the POM, students view a pattern of triangles composed of toothpicks. Their task is to determine the number of toothpicks that make up each pattern. The function is proportional. In Level B, students examine a linear pattern that involves a constant. The task involves a set of triangular tables that are arranged adjacently in a row. The task asks students to determine the relationship between the number of tables and the number of people who can sit around the tables. Students need to extend the pattern. They also find the inverse relationship, i.e., find the pattern number when given the total number of people seated. Level C requires students to determine how a pattern grows. Students need to see that the pattern grows by square numbers. They identify the relationship and then explain a valid process for finding these values. In Level D, students are asked to generalize a rule for finding a value in the triangular number sequence. They are also asked to explain the process for finding an inverse value for the triangular number sequence by finding the term when given the total. In the final Level E, students generate a closed expression for a sequence that grows as the sum of two exponential functions. In addition, the students must justify their findings.

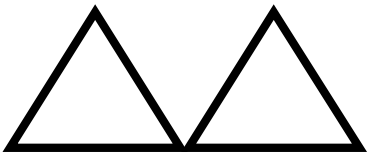
Problem of the Month
Tri - Triangles

Level A

Lisa is making triangle patterns out of toothpicks of all the same length. A triangle is made from three toothpicks. Her first pattern is a single triangle.

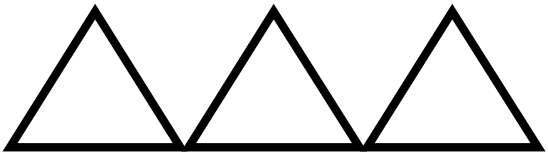


Her second pattern is shown below.



How many toothpicks are needed for her second pattern?

Her third pattern is shown below.



How many toothpicks are needed for her third pattern?

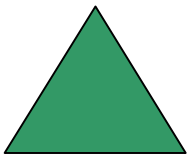
If she continued the same pattern, how many toothpicks are needed for the fifth pattern?

How many toothpicks are needed for the tenth pattern?

If you had 81 toothpicks, what pattern number could you make?
Explain how you know.

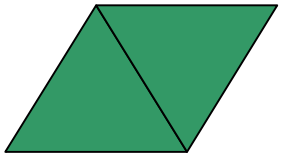
Level B

Your classroom has triangular shaped tables.



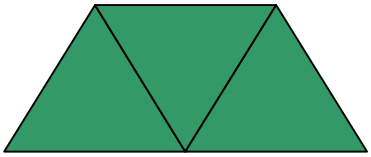
Three students can sit around one table.

Two tables can be pushed together so that two sides are adjacent.



How many students can sit around the tables in this arrangement?

Tables can be added to the arrangement by pushing together tables so that each additional table is adjacent to one side of the row of tables. The arrangement may grow to be a long row of tables.



How many students can sit around three tables in this arrangement?

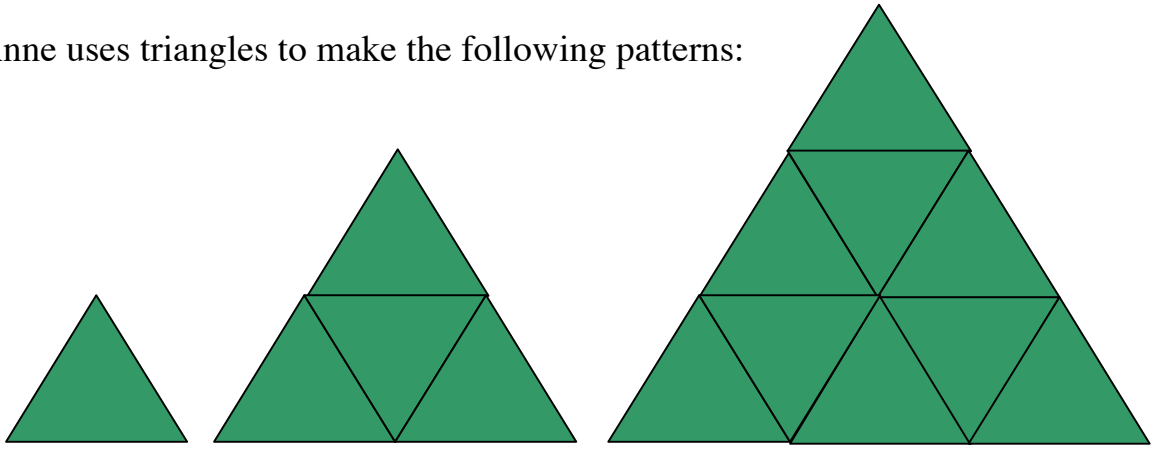
How many students can sit around five tables in a row arrangement?

Without drawing the arrangement, determine how many students can sit around twelve desks in a row. Explain how you figured it out.

How many tables in a row are needed to seat 105 students?
Explain your answer.

Level C

Anne uses triangles to make the following patterns:



The pattern continues in the same geometric design.
Draw Pattern 4. How many triangles are needed?

How many triangles are needed to construct Pattern 7?

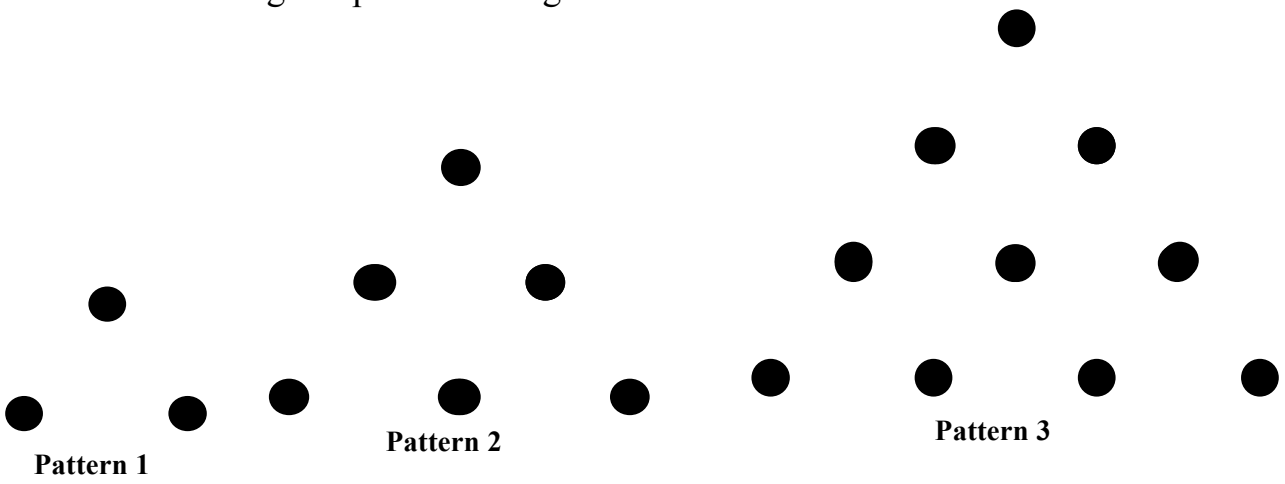
How many triangles are needed to construct Pattern 16?
Explain how you determined your rule.

Write a rule to find the number of triangles needed for the **n**th pattern?
Explain your rule.

Suppose a pattern had 2,025 triangles. What is the pattern number?
Explain.

Level D

Jo constructs triangular patterns using dots.

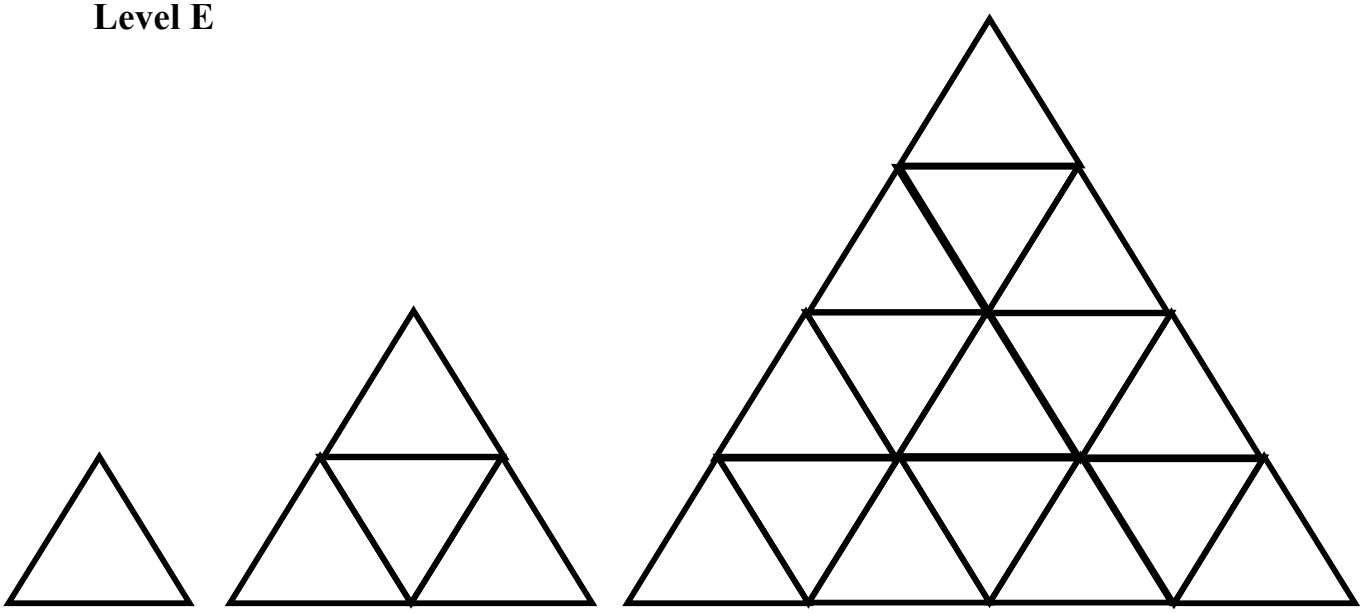


The pattern continues in the same geometric design.
How many dots are needed to make Pattern 5?

How many dots are needed for the **n**th pattern?
Explain your rule.

Jo was born in 1953 and she was wondering if she could make a triangular pattern out of exactly 1,953 dots. If she could, what would the pattern number be? Explain your answer.

Level E



Craig constructs the designs above from equal line segments. The design in Pattern 1 is made up of three line segments. Pattern 2 is made up of nine line segments. Pattern 3 is made up of thirty line segments, and so on.

How many line segments are needed to make Pattern 8?

How many line segments are needed to make Pattern 16?

Determine a function for finding the number of line segments needed to make the pattern for any number **n.** Justify why your function works.

You have 6,294,528 equal line segments. Can you construct a design that belongs in this sequence using just those line segments? If so, what pattern number would that be? If not, how many more line segments might you need to construct a design that fits the sequence?

Problem of the Month

Tri - Triangles

Primary Version Level A

Materials: A picture of the three patterns, paper, a pencil and toothpicks for students to make the different patterns.

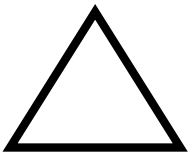
Discussion on the rug: Teacher holds up the pictures of the triangle patterns. **"Here are different patterns. How many toothpicks would it take to make the first pattern?"** Students may build and count. **"How many toothpicks do you need to build the second pattern?"** Students may build and count. Teacher asks the children to think about how the number of toothpicks changed from the first pattern to the second.

In small groups: Each student has access to toothpicks, paper, a pencil and the picture of the first three patterns. The teacher explains that they may either build or draw the pattern to help find the answers. The teacher asks the following questions, only going on to the next question if students have success.

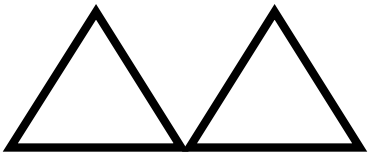
How many toothpicks do you need to build the first pattern?
How many toothpicks do you need to build the second pattern?
How many toothpicks do you need to build the third pattern?
How many toothpicks do you need to build the fourth pattern?
How many toothpicks do you need to build the sixth pattern?

At the end of the investigation have students either discuss or dictate a response to this summary question: **"Tell me how you know."**

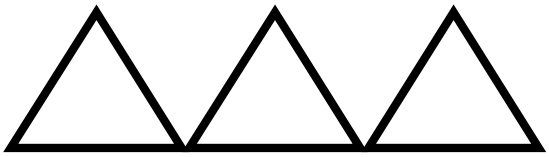
Problem of the Month
Tri - Triangles



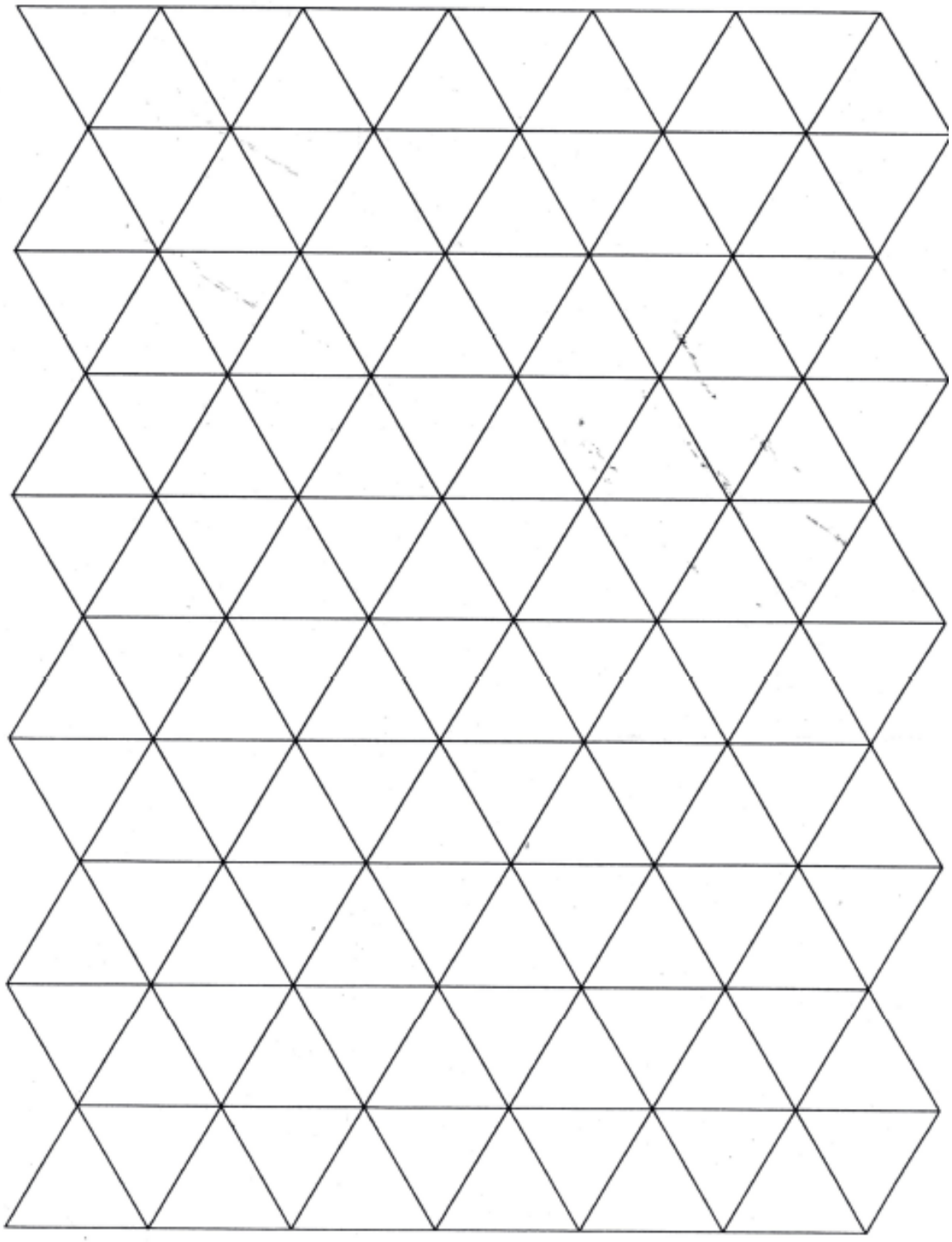
First Pattern



Second Pattern



Third Pattern



Problem of the Month

Tri-Triangles

Task Description – Level A

This task challenges students to view a pattern of triangles composed of toothpicks. Students must determine the number of toothpicks that make up each pattern. This function is proportional.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.1. Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

3.OA.2 Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares,, or as a number of objects in each share when 56 objects are partitioned into equal shares of 8 objects each.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and mathematical quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problems.

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.

3.OA.6 Understand division as an unknown-factor problem.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Ass the reasonableness of answers using mental computation and estimation strategies including rounding.

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Tri-Triangles

Task Description – Level B

This task challenges students to examine a linear pattern that involves a constant. The task involves a set of triangular tables that are arranged adjacently in a row. Students determine the relationship between the number of tables and the number of people who can sit around the tables. Students need to extend the pattern and find the inverse relationship, i.e., find the pattern number when given the total number of people seated.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.

3.OA.6 Understand division as an unknown-factor problem.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Ass the reasonableness of answers using mental computation and estimation strategies including rounding.

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Write and interpret numerical expressions.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluations them.

Analyze patterns and relationships.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane.

Expressions and Equations

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variable using graphs and tables, and relate these to the equations.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Tri-Triangles

Task Description – Level C

This task challenges students to determine values that grow in a square area relationship and then to explain a valid process for finding these values and extending the pattern. They must also find the inverse relationship for specific values.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Write and interpret numerical expressions.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluations them.

Analyze patterns and relationships.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variable using graphs and tables, and relate these to the equations.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Tri-Triangles

Task Description – Level D

This task challenges students to generalize a rule for finding a value in the triangular number sequence. Students also explain the process for finding an inverse value for the triangular number sequence by finding the term, given the total.

Common Core State Standards Math - Content Standards

Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variable using graphs and tables, and relate these to the equations.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Tri-Triangles

Task Description – Level E

This task challenges students to generate a closed expression for a sequence that grows as the sum of two exponential functions. In addition, the students must justify their findings.

Common Core State Standards Math - Content Standards

Expressions and Equations

Expressions and equations work with radicals and integer exponents.

8.EE.1 Know and apply the properties of inter exponents to generate equivalent numerical expressions.

High School – Algebra – Creating Equations

Create equations that describe numbers or relationships.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.

High School – Functions – Building Functions

Build a function that models relationship between two quantities.

F-BF.1 Write a function that describes a relationship between two quantities.

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations and translate between the two forms.

Build new functions from existing functions.

F-BF.4 Find inverse functions.

High School – Functions – Linear, Quadratic, and Exponential Models

Interpret expressions for functions in terms of the situation they model.

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Common Core State Standards Math – Standards of Mathematical Practice

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y -2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month

Tri-Triangles

Task Description – Primary Level

This task challenges students to build a triangle pattern with toothpicks and count the toothpicks. Students also record the findings using drawings and numbers. Students extend the pattern by identifying attributes of the pattern and building.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Count to tell the number of objects.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less that or equal to 20, e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns n all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Work with equal groups of objects to gain foundations for multiplication.

2.OA.4 Write an equation to express the total as a sum of equal addends.

Geometry

Analyze, compare, create, and compose shapes.

K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g. number of sides and vertices/corners and other attributes.

K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.