Problem of the Month
Polly Gone

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used school wide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The POM is designed with a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially as the key challenge for grades kindergarten and one. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *Polly Gone,* students use polygons to solve problems involving area. The mathematical topics that underlie this POM are the attributes of linear measurement, square measurement, two-dimensional geometry, perimeter, area, and geometric justification.

The problem asks students to explore polygons and the relationship of their areas in various problem situations. In the first level of the POM, students are presented with 40 cubes and are asked to make all possible rectangular regions using the cubes as a border. The students are then asked to determine the area of the interior

regions and identify the rectangle with the largest area to make a pen for animals in a zoo. In Level B, students are presented with a triangular shape on grid paper comprised of five smaller polygons. The students are asked to name and determine the area of each polygon. They are also asked to rearrange the shapes to construct as many different parallelograms as possible. In Level C, students are given a rectangle that is subdivided into nine smaller squares. The students are given the area of two of the squares and asked to determine the area of the remaining seven squares. In Level D, the students explore concepts for maximizing area given a fixed perimeter. The students grapple with which polygon will produce the largest area as well as maintain a constant distance from the perimeter to design the playing surface in a sports arena. In Level E, students are asked to construct a geometric figure from a square. An octagon is produced by drawing line segments from each vertex to its opposite midpoints. Students are asked to determine the area of the octagon in relationship to the area of the square. Students are asked to justify their solutions.

Problem of the Month
Polly Gone

Level A

Polly works in a zoo and needs to build pens where animals can live and be safe. The walls of the pens are made out of cubes that are connected together. Polly has 40 cubes and wants to make the largest pen possible, so the animals can move around freely but not get loose. Build the largest area using all 40 cubes. Your walls must:

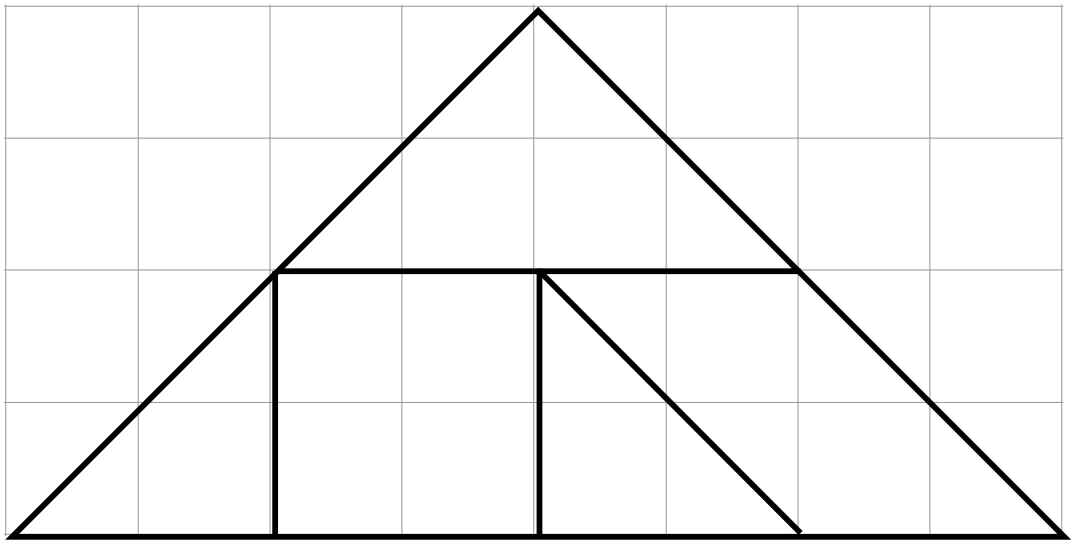
- Be fully enclosed, with no doors or windows so Polly's animals can't get out.
- Have a height of one cube.
- Be joined cube face to cube face.

Help Polly by making pens of several shapes and determine which pen provides the largest area for the animals. You might want to build the pen on the grid paper first, so that it will be easier to determine the area.

Use the grid paper to show the shape of the pen. Explain to Polly why you believe your pen is the largest one that can be made.

Level B

The large triangle is made up of five shapes and is drawn on graph paper.
Name each of the five shapes and determine the area of each.



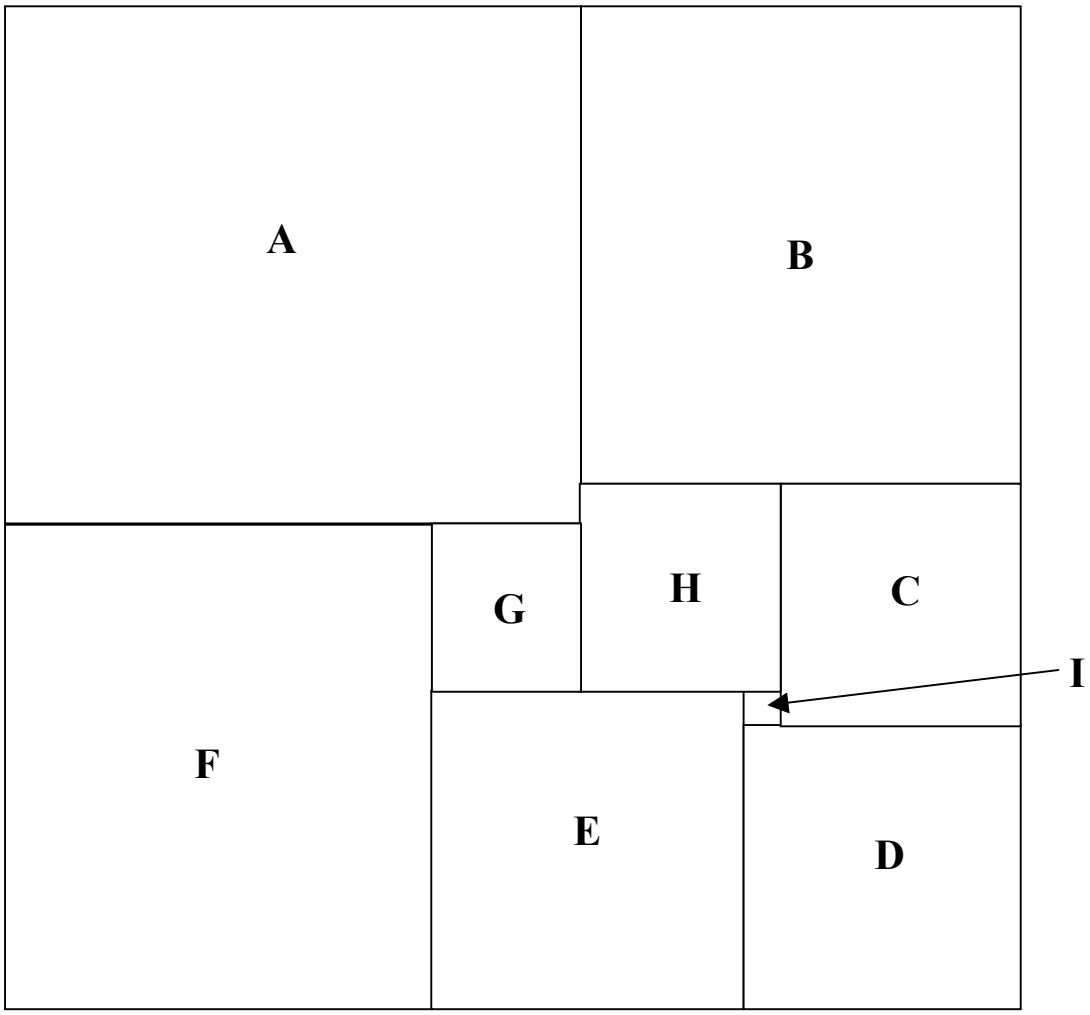
Rearrange the shapes to find all possible parallelograms of any size using any number of them.

Draw a picture of each parallelogram that you have found, and then determine its area.

How did you find all of them? How do you know you found them all?

Level C

In November 1958, the magazine, *Scientific American,* showed this diagram on its cover.



Each of the interior rectangles is a square. If square D is 81 square units and square C is 64 square units, what is the area of the other seven squares? What is the area of the entire figure? What is the perimeter of the entire figure? Explain your solutions.

Level D

A new arena is going to be constructed at a local university. A study is being done to find the best performance or playing area design. Since the arena will be used for many different sports, as well as shows and concerts, the designers want a seating arrangement that allows spectators to be as close as possible to the action. They also want to seat as many front row spectators as possible around the performance area. They have also decided that the boundary of the performance area needs to have straight sides, no curves, due to the building materials they are using. The goal is to have front row seats not more than 20 meters from the center of the performance or playing area.

They want to hire you as a consultant to investigate this matter and explain to them which design would best suit their needs. They need to see several examples of possible performance area designs that will fit their constraints. The final recommendation must explain the advantages of the design in terms of the size of the playing area and the number of people they can seat in the front row.

Level E

Catherine said to Rebecca, "I need to draw an octagon and I want it to be accurate."

Rebecca replied, "I have an easy way to draw an octagon. Start with a large square. Find the midpoint of each side. Now draw a line segment from each midpoint to the two opposite vertices. In the center of the drawing, an octagon will be formed."

"That's a great method, Rebecca, but I want to make my octagon a certain size. How big do I need to make the original square in terms of area to get an octagon of a certain area?" Catherine asked.

Please help Catherine and Rebecca determine these relationships. Fully explain your reasoning.

"After you have drawn your octagon, you will see that it comes out as a beautiful regular octagon," Rebecca exclaimed. "Well, it may be beautiful, but I don't think it is regular," challenged Catherine.

Who is right? Determine your answer using mathematics.

Problem of the Month
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Primary Version Level A

Materials: 20 multi-link cubes per pair, a sheet of 1-inch grid paper to measure and record.

Discussion on the rug: The teacher asks the class, **"Where do the animals in the zoo live?"** The teacher invites students to tell the whole class about cages or pens at the zoo. The teacher says, **"Suppose we want to make a cage or pen where an animal can live. We want to make our pen out of these cubes."** The teacher shows the multi-link cubes to the class. **"We want to make the pen as big as possible so the animal can roam around, but we have only 20 cubes we can use. What shape can we make?"** The teacher asks the class. Students share their ideas. Then the teacher shows two different pens - one with interior dimensions 1 by 7 and a second with interior dimensions 3 by 5 - to illustrate what is meant by different. The teacher says, **"I would like you to go back to your desk and work with your partner and make all the different possible shapes using 20 cubes."**

In pairs: Students have cubes and gird paper available.
Teacher says, **"Look at all the animal pen shapes you made. Which shape has the most room for the animal?"** Students work together to find a solution. After the students are done, the teacher asks students to share their answers and how they know.

At the end of the investigation: Students either discuss or dictate a response to this summary question, **"Show all the shapes you can make with 20 cubes. Explain which shape has the most room for the animal. How do you know?"**

Problem of the Month
Polly Gone
Task Description – Level A
This task challenges students to make all possible rectangles using 40 cubes as a border, to find the area of the rectangles, and to identify the one with the largest area.
Common Core State Standards Math - Content Standards
Measurement and Data **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement 3.MD.6 Measure areas by counting unit squares. 3.MD.7 Relate area to the operations of multiplication and addition. **Geometry** **Solve real-world and mathematical problems involving area, surface area, and volume.** 6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes, apply these techniques in the context of solving real-world and mathematical problems.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month
Polly Gone
Task Description – Level B
This task challenges students to identify shapes drawn inside a triangle and to find their areas. Students are then challenged to cut out shapes and find all possible parallelograms that can be made from some combination of the shapes, and to record the shapes and areas of each. Students must justify how they know they have found all possibilities.
Common Core State Standards Math - Content Standards
Geometry **Classify two-dimensional figures into categories based on their properties.** 5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. **Solve real-world and mathematical problems involving area, surface area, and volume.** 6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes, apply these techniques in the context of solving real-world and mathematical problems. 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Polly Gone

Task Description – Level C

This task challenges students to derive measurements from a collection of different sized squares arranged in a square when the areas of two of the squares are known. Students must use logic and geometric formulas to find the area and perimeter of all the squares including the large composite square.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.7 Relate area to the operations of multiplication and addition.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measurements.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes, apply these techniques in the context of solving real-world and mathematical problems.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Expressions and Equations

Expressions and equations work with radicals and integer exponents.

8.EE.2 Use square root and cube root symbols to represent solutions to equations in the for $x^2 = p$ and $x^3 = p$.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Polly Gone

Task Description – Level D

This task challenges students to explore concepts for maximizing area given a fixed perimeter. The students grapple with which polygon will produce the largest area as well as maintain a constant distance from the perimeter to design the playing surface in a sports arena.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.7 Relate area to the operations of multiplication and addition.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measurements.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes, apply these techniques in the context of solving real-world and mathematical problems.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Common Core State Standards Math – Standards of Mathematical Practice

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MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month
Polly Gone
Task Description – Level E
This task challenges students to draw an octagon from a square and find the relationship between the two areas.
Common Core State Standards Math - Content Standards
Geometry **Solve real-world and mathematical problems involving area, surface area, and volume.** 6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes, apply these techniques in the context of solving real-world and mathematical problems. 7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **Draw construct, and describe geometrical figures and describe the relationships between them.** 7.G.2 Draw geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, notice when the conditions determine a unique triangle, more than one triangle, or no triangle. **High School – Geometry- Congruence** **Make geometric constructions.** G-CO.12 Make formal geometric constructions with a variety of tools and methods. **High School – Geometry – Similarity, Right Triangles, and Trigonometry** **Apply trigonometry to general triangles.** G-SRT.11 Understand and apply the Law of Sine and the Law of Cosines to find unknown measurements in right triangles and non-right triangles.
Common Core State Standards Math – Standards of Mathematical Practice
MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.7 Look for and make use of structure.** Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Problem of the Month
Polly Gone
Task Description – Primary Level
This task challenges students to explore all the rectangles that can be made with 20 multi-link cubes. Students should record their rectangles on graph paper and determine the shape with the maximum area.
Common Core State Standards Math - Content Standards
Measurement and Data **Describe and compare measureable attributes.** K.MD.2 Directly compare two objects with a measureable attribute in common to see which object has "more of" or "less of" the attribute, and describe the difference. **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement 3.MD.6 Measure areas by counting unit squares.
Common Core State Standards Math – Standards of Mathematical Practice
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