

# **Guidelines for Problem of the Month**

## **Why Problem of the Month?**

Problem solving is the cornerstone of doing mathematics. George Polya, a famous mathematician from Stanford, once said, “a problem is not a problem if you can solve it in 24 hours.” His point was that a problem that you can solve in less than a day is usually a problem that is similar to one that you have solved before, or at least is one where you recognize that a certain approach would lead to the solution. But in real life, a problem is a situation that confronts you and you don’t have an idea of where to even start. Mathematics is the toolbox that solves so many problems. Whether it is calculating an estimate measure, modeling a complex situation, determining the probability of a chance event, transforming a graphical image or proving a case using deductive reasoning, mathematics is used. If we want our students to be problem solvers and mathematically powerful, we must model perseverance and challenge students with non-routine problems.

## **How should the Problems of the Month be used?**

The Problems of the Month are designed to be used school-wide to promote a problem-solving theme at your school. Each problem is divided into five levels, Level A through Level E, to allow access and scaffolding for the students into different aspects of the problem and to stretch students to go deeper into mathematical complexity. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important. Administrators, teachers, and parents should facilitate and support students in the process of attacking and reasoning about the problems. Students’ self-analysis of how they went about approaching, exploring, and solving the problems is a critical step in the development of becoming a strong problem solver.

The Problem of the Month is structured to provide reasonable tasks for all students in all grades K-12. The Problems of the Month are designed so all learners start at a level of A (either Primary Level A or Level A). Where a learner will be successful and challenged will depend on his or her maturity as a problem solver. It should not be a race to get through the levels; rather a learner should stay at a level and fully investigate that level (or tangents to the problems at that level) before proceeding to the next level. George Polya states, “It is better to solve a problem five ways than to solve five problems.” A learner should be able to fully explain answers and justify solutions before proceeding to the next level. The Problem of the Month’s levels are related around a big idea. Learners work through the levels to deepen their understanding of the big math idea. Often a learner will benefit from additional exploration of a level, using different assumptions or finding related problems to investigate.

Even though all learners should start at level A, there is a general rule of thumb that may apply – although each Problem of the Month is slightly different in access points and levels of complexity, so the rule may not always hold true. With that caveat, a general rule for learners' success at a given level are: Primary Level A is designed to be accessible to all students and may be challenging for primary students in Kindergarten and 1<sup>st</sup> grade. Level A may be accessible for some students in 1<sup>st</sup> grade, and most 2<sup>nd</sup> and 3<sup>rd</sup> grade students will be successful working on these tasks. Level B may be the limit of where 3<sup>rd</sup> and 4<sup>th</sup> grade students will have success and understanding. Level C may stretch 5<sup>th</sup> or 6<sup>th</sup> grade students. Level D may challenge most middle school students, and Level E should be challenging for most high school students. These grade level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

One caution - the solution is not as important as the process of problem solving. Struggling to get started is a natural part of learning to problem-solve. The educator or parent should not be impatient with the student's struggle. In fact encouraging and supporting the struggle with some frustration is exactly what the student needs. If a method is shown or told then the problem-solving process ends. Asking good, but not guiding, questions that require students to reflect and focus are most helpful. Having the student carefully read or be read the problem and then having the student restate the problem is often valuable. Having the student talk through the approach or the challenge is also effective in having the student rethink a strategy. Encouragement is the important key. In class, a teacher might have different students share their thinking (approach, not solution) with the class. The teacher should be careful not to assign value to the approaches. If students seem to follow an error in their process, the teacher should pose questions that make the class examine the process to uncover the error. A good problem-solver tries, fails, re-evaluates and tries again.

If some students are able to complete all levels with detailed and accurate solutions and justifications, then the students may be challenged to go deeper into a problem or to extend it into another problem that is more complex. For example, in the Cutting the Cube problem, you may ask the student to formally prove the number of unique hexominoes. You could also extend the problem for them to explore septominoes (7 square figures) and develop number patterns for unique solutions of any nth-omino.

## **Role of the principal**

The principal should embrace the concept of problem solving and model problem solving leadership. In that instructional leadership role, the principal should demonstrate being a facilitator of non-routine problems. The principal should begin by facilitating a session with the teachers in which they explore the Problem of the Month prior to presenting the problem to the students. The principal should model the same good attributes of instruction in problem solving with students. The same clarifying questions, without guiding, should be posed. In facilitating the non-routine tasks with the teachers, the principal should expect similar struggles and encourage teachers to reflect on the importance of disequilibrium and perseverance in the process. The teachers should follow this model as they present the problems to their students.

Once the problem is presented to the students in their classrooms, the principal should be visible in facilitating the tasks alongside the teachers. Visiting and/or leading a class as students share ideas and approaches with the other students encourages and empowers both teachers and students. The principal can also play a role in examining the student products with their teachers. Holding student write-up review meetings with teachers is an excellent way to focus on student understandings and their misconceptions. Another method would be for a principal to attend a class session where students are presenting their findings and solutions to the Problem of the Month. Facilitating a presentation session is a great way to model how students can communicate and question each other's thinking. Encouraging deeper understanding and the justification of solutions are important mathematics outcomes of the Problem of the Month process.

## Summary Process

Often teachers use a summary or write-up format for the students to follow as they prepare to share their solutions and analysis of their problem solving process. Below is a sample format, which is appropriate for upper elementary grade students. Simple versions are used for primary students.

### Problem of Month Write-up

- **Problem Statement**  
In your own words, state the general overall problem clearly enough that someone unfamiliar with the problem could pick up your paper and understand what you are asked to do.
- **Process**  
Describe in detail how you attempted to solve this problem. You may want to consider some of the following questions. You should also include things that didn't work.  
How did you get started?  
What approaches did you try?  
Where did you get stuck?  
Did you talk to anyone about the problem?  
Did talking to someone help or hinder you?  
What drawing, chart, graph, or model did you use?
- **Solution**  
State your solutions as clearly as you can. Include any charts, graphs, lists, and so on that you used to help you. If you were able to generalize the solution, include your results. Defend why you believe your solution is correct or the best possible answer. Your explanations should be written in a way that will be convincing to someone else.
- **Learning**  
Reflect on the problem. What did you learn? What mathematics did you use?

### Including buy-in from the stakeholders

The students, parents and teachers are the chief stakeholders in the Problem of the Month program. Obtaining commitment and buy-in by all is essential for a healthy and productive program. Assisting teachers with knowledge and skills is necessary to facilitate a strong learning experience. Informing parents of the program and supporting them with how they can help their students will make the program even more successful. Motivating students to want to problem-solve and encouraging their perseverance will be

the ultimate reward. The ultimate goal of math power for all students will never be realized if students aren't developed into strong problem solvers.

Below are sample letters to parents and teachers.

Dear Parents,

Problem solving is a fundamental goal of any strong mathematics program. Our school is committed to making each of our students a problem solver. It is never too early or too late to develop the real life learning skills of problem solving. Therefore, our school is embarking on a Problem of the Month program to help achieve this important goal.

Each month we will present our students with a non-routine problem for them to attack and solve. The problem will have several levels so that all students at our school will be able to work on a part of the problem appropriate to their learning development. All students should start with level A and work through the different tasks. It is understood that some students will not get too far into the problem. The process of attacking and struggling on a non-routine problem is important to learn. When your child has reached the maximum level of his/her understanding, please celebrate their progress.

Trials, errors, and retries are key attributes of good problem solvers. We ask you to encourage your students to persevere. Many students might want to initially give up. The best support for your student is encouragement through good questions. Some good questions are: *What have you tried? Why do you think it doesn't work? Have you tried to make the problem simpler? What do you need to know to be able to solve the problem?* There are many other good questions; however, leading or guiding questions are not helpful. The process of finding and understanding a solution outweighs the benefit of having a correct answer. Doing the problem for the student actually hurts the problem-solving process. Many students will receive the hidden message that they can't solve problems by themselves, and will learn to stop and wait for someone else to answer.

You will play an important role in supporting your child's work on these problems. Once students have reached their level of understanding, they are asked to complete a write-up of their findings. Students should communicate how they went about solving the problem as well as the solution they found. This write-up helps students understand how they think and approach new problems. We look forward to a partnership with you around problem solving. Thank you for supporting your child.

Sincerely,

Dear Teacher,

As you know, we are committed to improving our math program by emphasizing the importance of problem solving. We have made it one of our school-wide foci. The Problem of the Month is a school-wide program that we can all participate in to encourage and teach our students to be strong problem solvers.

Our first Problem of the Month is titled “Cutting a Cube.” It has five levels of complexity. Although all of our students should start at level A and work through the problem, some of our primary students will not be able to go much beyond the first level. That is okay; what is most important is the process and that students are stretched to go as far as their understanding and skills take them. At the same time we must encourage the students to struggle and persevere to develop their problem-solving skills.

As a facilitator of Problems of the Month, you must be careful not to lead or guide, but rather to pose clarifying questions and questions that require the students to reflect on their work. A good method is to have students from time to time share various processes they have tried. Be careful not to emphasize one solution method over another as students share their ideas. Don’t have students share complete solutions until the conclusion and the summary presentations.

Many students might benefit from hands-on experience in exploring the attributes of the cube or the process of actually cutting a cube apart. Paper cubes can be constructed with masking tape along the edges, and students can use plastic knives to cut cubes into nets. When students are trying to determine which nets work and whether they have them all, a class might share a few examples of nets they have found, and the teacher might pose a question like: *How might we classify the nets we found? How might we know when we have them all?*

You are encouraged to have your students follow the Problem of the Month write-up. This provides a common format for examining student work with your colleagues. Processing the solutions and methods with your students is important in developing their skills.

Thank you for supporting your students in their development as math problem solvers.

Sincerely,

# Scoring Guidelines For Problems of the Month

## Rubrics

The criteria for scoring problems or tasks are called “rubrics.” The rubrics are designed for use in a professional collaborative setting: teachers scoring together around a table with discussion. This type of scoring has proved to be a powerful tool when used by teachers and students in self-assessment. The rubrics are designed to be easily adaptable to classrooms.

The Problem of the Month rubric focuses on the performance rather than on the performer. Scorers are directed by the rubric to the evidence in the response. To help make distinctions, scorers are asked to consider what feedback to the student would be appropriate based on the evidence in the response. The formulation of scoring decisions based, in part, on the feedback idea, has proven helpful to teachers and scorers who seek reliable scores based on sound classroom practice.

A rubric is based on the core elements of performance of the problem or task. The core elements of performance include both the essential mathematics assessed and the process at which the students solve and communicate their results. The core elements of performance are stated in the rubric.

## Generic Holistic Rubric

### 5: Accomplishes the Task with Distinction

The response completely accomplishes the core performance of the task and goes beyond the task requirements. A distinguished performance is exciting – a gem. It excels and merits nomination for distinction by meeting the standards for a “4” and demonstrating special insights or powerful generalizations or eloquence or other exceptional qualities.

### 4: Accomplishes the Task

The response accomplishes the prompted purpose. The student’s strategy and execution are at a level consistent with the core elements of performance including math standards and qualitative demands of the task. Communication is judged by its effectiveness, not by grammatical correctness or length. Although a “4” need not be perfect, any defects must be minor and very likely to be repaired by the student’s own editing, without benefit of a note from a reader.

### 3: Ready for Revision

Evidence in the response convinces you that the student can revise the work to a “4” with help of written feedback. The student does not need a dialog or additional teaching. Any overlooked issues, misleading assumptions, or errors in execution – to be addressed in the



revision – do not subvert the scorer’s confidence that the student’s mathematical power is ample to accomplish the task.

**2: Partial Success with More Instruction Needed**

Part of the core elements of performance is accomplished, but there is a lack of evidence – or evidence of lack – in some areas needed to accomplish the whole task. It is not clear that the student is ready to revise the work without a conversation or more teaching.

**1: Engaged Task with Little Success**

The response may have fragments of appropriate material from the core elements of performance and may show effort to accomplish the task, but with little or no success. The task may be misconceived, or the approach may be incoherent, or the response might lack any correct results. Nonetheless, it is evident that the respondent tackled the task and put some math knowledge and tools to work.

**0: No Response or Off Task**

There is no evidence that the task was engaged. The response is blank or there are marks, words, or drawings unrelated to the task.