STACI CARIGNAN: Gabby, what did you get? Four -- four girls had short blond hair?

STUDENT: Yeah, because, well, I put down four people because eight had long hair and four had short hair, and they said that, "How many people will have short, red hair?" So, the short blond hair girls are the ones -- all they have to do is dye their hair red ...

STACI CARIGNAN: [laughs] Do you think anyone's gonna dye their hair red?

STUDENT: ... so it would be short.

STACI CARIGNAN: Is that what the problem's asking us, Gabby? Are they gonna be dyeing their hair? No. I want you guys to keep talking as I walk away. And as I walk away, I want you to think about -- how did you get that four girls? Because you guys all agreed that four girls had short, or had red hair, right? Is there some kind of diagram you can draw to show that? Or is there some kind of visual representation you can use to show that? Is there?

STUDENT: I guess.

STACI CARIGNAN: You guess? You want to try it out? That's the spirit. All right, keep going. I'll be back.

So, what is, what is the problem asking us? What does it want us to find out?

STUDENTS: How many people have red hair.

STACI CARIGNAN: How many people have red hair. Okay, so boys or girls have red hair. Do any of the boys have red hair?

STUDENT: No.

STACI CARIGNAN: No, so we know that ...

STUDENT: [inaudible] girls.

STACI CARIGNAN: ... we're looking for girls here. Okay. So, how many boys and girls are at the party?

STUDENT: 16. 16 boys and girls.

STACI CARIGNAN: How do you know that 16 boys and girls are at the party?

STUDENT: There's 2 divided by 30 -- 2 divided by 32 is 16.

STACI CARIGNAN: 2 divided by 32, or 32 divided by 2?

STUDENT: 32 divided by 2.

STACI CARIGNAN: So there's 16 boys and there's 16 girls. Okay. So, let's go to the next part of the problem. Let's go back to the first part. "At Leslie's party, one-fourth of the people had long hair." So, what's -- how many people are at the party?

STUDENTS: 32.

STACI CARIGNAN: 32. So what would be a fourth of 32?

STUDENT: Eight.

STUDENT: Eight.

STUDENT: No, four. Eight, yeah.

STACI CARIGNAN: Eight. Agree? So, then how many people had long hair?

STUDENT: Eight.

STUDENT: Eight.

STACI CARIGNAN: Eight. Do any of the boys have long hair?

STUDENT: No.

STACI CARIGNAN: So, it's only the girls who have long hair. Did you somehow note that on your paper somewhere, that if you have 16 girls, we know that eight of them have long hair?

STUDENT: I put eight short hair, eight long hair.

STACI CARIGNAN: Okay. So if eight of them have long hair, how many girls does that leave?

STUDENT: It leaves [inaudible] eight more girls.

STACI CARIGNAN: We still have eight more girls, okay? Let's go to the next part of the problem. One-half of the people at the party were boys, and one-fourth of the girls had short, blond hair. One-fourth of the girls ... Well, how many girls are at the party?

STUDENT: 16.

STACI CARIGNAN: So if one-fourth of the girls had short, blond hair, how many girls actually had short, blond hair? So what --

STUDENT: Eight.

STUDENT: Four.

STUDENT: Four.

STACI CARIGNAN: So one-fourth -- you're saying one-fourth of 16 is four, Nyema?

STUDENT: Hmm?

STACI CARIGNAN: You're saying one-fourth of 16 is four? So how many people had short, blond hair, then?

STUDENT: Four.

STACI CARIGNAN: Okay, so if we had four people with short, blond hair, and we had eight girls with long hair, how many girls is that so far? What's the four girls with the short hair, and the eight girls with the long hair? How many do we have?

STUDENT: 12.

STACI CARIGNAN: We have 12. But how many girls came to the party?

STUDENT: 16.

STUDENT: 16.

STUDENT: 16.

STACI CARIGNAN: So, how may girls do we have unaccounted for right now? How many girls we don't know about their hair?

STUDENT: Three.

STACI CARIGNAN: Three?

STUDENT: Three.

STUDENT: Four.

STACI CARIGNAN: Why don't you guys talk about that? And really take a look at the girls, and -- because we know there's 16 girls, and we've split them up a couple different ways, so you may want to go back and talk about, right now, how many girls have what kind of hair. Okay? I'm going to put a clue up on the board in about two minutes, so be checking for that, okay? Nice job. Keep going. I like the way you guys are proving to one another. I like that everyone's voice is being heard now. Thank you all.

Whew!

STUDENT: ... people that have long hair. And we have one-fourth of boys, so that's eight. Then we still have this four, and those four are the ones with red hair. So all together we have four groups, so that's how I got my answer before. Because we have red-haired group, boys, girls, and long hair, so ...

STUDENT: But blond hair.

STUDENT: Yeah, blond hair. Oh, so the girls have blond hair.

STUDENT: You need blond hair, girls, and boys, and red hair.

STUDENT: Yeah. So ... Um, well ... Nyema, do you get it now?

STUDENT: Yeah.

STUDENT: How -- we got our answers? Do you change your mind?

STUDENT: [inaudible] understand?

STUDENT: I could [inaudible].

STUDENT: So we should write down how we got our answer, so we can remember when we come back.

STUDENT: 16.

STACI CARIGNAN: So -- quick question -- how did you know that there were 16 boys and 16 girls at the party. Juan?

STUDENT: Um, because it -- 16 plus 16 is 32.

STACI CARIGNAN: Okay.

STUDENT: And, um, I put 16 girls and 16 boys.

STACI CARIGNAN: 16 girls and 16 boys, okay. Agree? Okay. So let's -- getting back to our problem. "One-fourth of the people at the party had long hair."

STUDENT: One-fourth of, um ...

STACI CARIGNAN: So how many people had long hair?

STUDENT: One-fourth of ei -- of 32 is eight.

STUDENT: Eight.

STUDENT: So ...

STACI CARIGNAN: Why did you decide one-fourth of 32, rather than one-fourth of 16?

STUDENT: Because, um ... eight girls had long hair?

STACI CARIGNAN: Say that again?

STUDENT: Because four -- eight girls had long hair?

STACI CARIGNAN: So -- and how do we know that eight girls had long hair? Salia?

STUDENT: Because one-fourth of ...

STACI CARIGNAN: Because one-fourth of what?

STUDENT: Uh ... 32?

STACI CARIGNAN: Of 32. Okay. So, if I have 16 boys and 16 girls, you're saying that if these are my ladies right here, you're saying that this would be the amount that represents what?

STUDENT: Girl [inaudible].

STACI CARIGNAN: Girls with long hair, because this is one-fourth of 32, right?

STUDENT: Mm-hmm.

STACI CARIGNAN: Okay. So then, how did you get these two?

STUDENT: Because -- because, um, it's a, um, because it says, "What's" -- because to get -- because half of -- it said half of the kids at the party were boys.

STACI CARIGNAN: Mm-hmm.

STUDENT: So that makes this 16. But how would you split these to make 16, to add them -- to add 16 plus 16 equals 32?

STACI CARIGNAN: Okay. So, if these are my girls with long hair, then that means we've got eight with long hair, eight with some other kind of hair, right?

STUDENT: Short.

STUDENT: Short, red, and blond.

STACI CARIGNAN: Short, red, or blond hair. So, what my real question is, is how did you know that four girls had blond hair, leaving only four girls with red hair?

STUDENT: Because the last ...

STACI CARIGNAN: Hold a second, I'm gonna let -- he's really excited to talk. Go ahead.

STUDENT: Because one-fourth -- one-fourth's supposed to be blond and one-fourth's supposed to be red.

STACI CARIGNAN: One-fourth of what? 16 or 32?

STUDENT: 16.

STUDENT: 16.

STACI CARIGNAN: Why 16, Salia?

STUDENT: Because you already have 16 [inaudible].

STACI CARIGNAN: So that means that 16 -- we have 16 girls. Half of them have this, and the other fourth have this. Is there -- it looks like you've already drawn pictures of that. So, what I want you to think about as I walk away is -- how do you know you're correct? How could you prove it?

STUDENT: You could --

STACI CARIGNAN: Talk -- talk about it [inaudible].

16 boys, 16 girls. Agree, Carolina? How do you know that?

STUDENT: Because there's 16 [inaudible].

STACI CARIGNAN: Okay, so then since it said that one half are boys, one half are girls, we have 16 boys, 16 girls. Jayden, agree? Agree, okay. Now what?

STUDENT: So one-fourth of the girls had blonde, short blonde hair. That's one-fourth of the girls.

STACI CARIGNAN: One-fourth of the girls. So how many girls were there?

STUDENT: 16.

STACI CARIGNAN: 16 girls?

STUDENT: That should have 8, that's 4, 5, 6, 7, 8.

STACI CARIGNAN: Okay.

STUDENT: Eight. And then the other 16 boys. And these are the four girls that are left.

STACI CARIGNAN: Okay. So then what does that represent?

STUDENT: The girls with short red hair.

STACI CARIGNAN: So how do we know, Shamari, how do we know that four of them could possibly have short red hair? How do we know that?

STUDENT: Because 4 times 9 equals 32?

STACI CARIGNAN: So here's what I want you to do. I'm going to walk away and I want you guys to agree, talk with each other, about that. So you guys have some really interesting ideas here.

Really good thinking going on. So write two-fourths, what you told me.

STUDENT: [inaudible]

STACI CARIGNAN: Sure, yes. Maybe we can help her keep track of her thinking.

STUDENT: 16 people.

STACI CARIGNAN: Now ...

STUDENT: That's half, that's half, um, of 32 and half of them [crosstalk].

STACI CARIGNAN: Right. So now what we have to do is figure out what fourth of the people had long hair. Did any of the boys have long hair?

STUDENTS: No.

STUDENT: None of the boys had long hair.

STACI CARIGNAN: So what does that tell us? All of the girls had to have ...?

STUDENT: Long hair.

STUDENT: Two-fourths, would be ...

STACI CARIGNAN: Do what you're thinking, play with the math.

STUDENT: [inaudible]

STACI CARIGNAN: So how are we going to figure that out?

STUDENT: [inaudible]

STACI CARIGNAN: Okay, what are you going to divide?

STUDENT: Um, 16 divided by 2.

STUDENT: How'd you get 16 divided by 2?

STACI CARIGNAN: No, your thinking is good. What's the sixteenths stand for?

STUDENT: Stands for how many people I divided from two-fourths divided by 3.

STACI CARIGNAN: So there's 16 girls. So why don't you label that? Put 16 girls.

STUDENT: 16 girls had short blond hair.

STACI CARIGNAN: Put a G for girls right here -- can you sit down? So this is going to help you keep track of your thinking. So you know where we got the 16 girls?

STUDENT: 16 boys and 16 girls.

STACI CARIGNAN: Now you divided by 2, and why did you divide by 2? You were thinking of the two-fourths. Because first we were looking at two-fourths of 32, and now we're looking at two-fourths of --

STUDENT: Yeah, before what I thought was --

STACI CARIGNAN: 16, right?

STUDENT: What I had is maybe I could divide it by 2 until I can't divide it by 2 anymore. And then I can multiply it by 2 to get [inaudible].

STACI CARIGNAN: Okay.

STUDENT: So maybe --

STACI CARIGNAN: So what does this 8 stand for?

STUDENT: But I thought --

STACI CARIGNAN: You took the two-fourths of 16 --

STUDENT: How many girls.

STACI CARIGNAN: How many girls what?

STUDENT: How many girls have short blond hair.

STACI CARIGNAN: Write it down. Or wait, how many girls had long hair.

STUDENT: Okay.

STACI CARIGNAN: Because none of the boys had ...

STUDENTS: Long hair.

STACI CARIGNAN: Long hair. So write down your thinking, okay. So what do we know for sure based on your math? How many had long hair?

STUDENT: Eight people -- eight girls.

STACI CARIGNAN: Eight girls had long hair. Out of how many?

STUDENT: Out of 32.

STACI CARIGNAN: Out of how many girls?

STUDENT: Oh, 16.

STACI CARIGNAN: Okay, so we know that two-fourths, or half of them, half of the girls had long hair.

STUDENT: So maybe eight of the other girls had short red hair?

STACI CARIGNAN: So how many are left, how many girls are left? If eight of them have long hair, how many girls do we have left?

STUDENT: Eight.

STACI CARIGNAN: Eight. So we have eight left. We know for sure that eight had long hair. Now how can we figure out how many had blonde hair, short blonde hair?

STUDENT: So we have to divide 8 with one-fourth. Because it says one-fourth of the girls had short blonde hair.

STACI CARIGNAN: Can you put another 8 down here for me? I'm sorry, 8 circles.

STUDENT: Oh, okay. 1, 2, 3, 4, 5, 6, 7, 8.

STACI CARIGNAN: Okay, so which one of these -- so half of them had, 16 of them had the long hair, so can you mark off which ones had the long hair? Your brain's on fire and you've got good mathematical thinking. One of the things that mathematicians have to do is we have to keep track of our thinking so we don't get lost. So writing it down what we're thinking, like you've got your drawings here, is really helpful. Okay.

STUDENT: There's too much outlook.

STACI CARIGNAN: Okay, how many have white, blonde, I mean how many have --

STUDENT: Short blonde hair.

STACI CARIGNAN: Short blonde hair?

STUDENT: 1, 2, 3, 4, 5, 6, 7, 8.

STACI CARIGNAN: And how many have -- well, we have blonde hair and red hair.

STUDENT: No, short blonde -- red hair. Eight of them have short red hair.

STACI CARIGNAN: Well if eight of them have short red hair, how many are going to have long blonde hair?

STUDENT: Eight.

STACI CARIGNAN: So you have to figure out how many of the girls that are left have the blonde hair and how many of the girls that are left have red hair.

STUDENT: No, when it says blonde, it means red-blonde hair.

STACI CARIGNAN: So how many of the girls left have red hair, short blonde hair --

STUDENT: Short blond hair.

STACI CARIGNAN: Why don't you underline short blonde hair and how many have red hair.

I'm seeing people snap as they're looking at this, and I've seen quite a few people looking at this. When you see this, what are you thinking about when you see this? What are you thinking about? Bemnet, what do you think about when you see this?

STUDENT: Division?

STACI CARIGNAN: You're thinking division? In what kind of way?

STUDENT: Um, if you have 16 boys that don't have any short hair and you have 8 girls that has long hair, then if you do 16 plus 8, 24, then 32 subtracted by 24 is, yeah, 32 subtracted by 24, then you know how much are left over and you might get how many have short blond hair.

STACI CARIGNAN: What are you thinking about when you see the chart?

STUDENT: A big one, like a big rectangle that's a 16 and then there's two small squares. One says 8, and those two small squares are like 16, divided by 2 equals 8.

STACI CARIGNAN: So you're saying that this square plus this square would be equal to 16?

STUDENT: No, like, I mean those two squares they like represent like the division and like if you do 16 divided by those two squares, it will equal to 8.

STACI CARIGNAN: Okay. Thank you very much. Ladies and gentlemen, I'm going to give you a couple more minutes to continue to work with your partners. By the time we get to 11:15 we're going to stop and we're going to do a little bit of writing about this, and whether you're done or not -- and that's completely okay if you're still thinking about the problem. But I want you to spend the next couple minutes thinking about where you are in the problem, and what this diagram might have to do with the problem. Okay? You guys have a couple more minutes. Continue please, thank you.