Problem of the Month: Digging Dinosaurs

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *Digging Dinosaurs*, students use algebraic thinking to solve problems involving multiple answers, optimum solutions, variables, and functions. The mathematical topics that underlie this POM are equations, optimization, variables, functions, and linear and non-linear patterns.

In the first levels of the POM, students view a drawing of eight dinosaur feet under the water. Their task is to determine multiple solutions for answering the question of how many dinosaurs are standing in the water. As one continues through the levels, students analyze different rate plans in order to visit a museum. The purpose is to determine the least expensive plan for a particular group given the frequency of their visits. In the final levels of the POM students examine a table of data that is linear and then later non-linear. Students are asked to define a rule for

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both situations. In the context of these situations, students focus on algebra and functions.

Mathematical Concepts

Algebra is the cornerstone of secondary mathematics. Algebraic thinking is taught in primary grades with the foundations of algebra taught usually by the end of middle school. Even though the term algebraic thinking is routinely used, it cannot be simply defined. The underpinnings of algebra involve abstractions and language. Identifying the conceptual understandings students must learn about algebra is paramount. One resource is Fostering Algebraic Thinking.

A facility with algebraic thinking includes being able to think about functions and how they work and to think about the impact that a system's structures has on calculations. These two aspects of algebraic thinking are facilitated by certain habits of mind.

Doing – Undoing. Effective algebraic thinking sometimes involves reversibility (i.e., being able to undo mathematical processes as well as do them). In effect, it is the capacity not only to use a process to get to a goal, but also to understand the process well enough to work backward from the answer to the starting point....

Building Rules to Represent Functions. Critical to algebraic thinking is the capacity to recognize patterns and organize data to represent situations in which input is related to output by well-defined rules...

Abstracting from Computation. This is the capacity to think about computations independently of particular numbers that are used. One of the most evident characteristics of algebra has always been its abstractness. But, just what is being abstracted? To answer this, a good case can be made that thinking algebraically involves being able to think about computations freed from the particular numbers they are tied to in arithmetic – that is, abstracting system regularities from computation....

(Driscoll, 1999)



You are swimming under water in a lake and you see dinosaur feet in the water. You don't want to go to the surface in case they are not friendly dinosaurs. Below is a picture of what you see.



How many dinosaurs are standing in the lake?

Explain how you know. Use words and mathematical language to explain your solution.

Level B:

You want to go with your sister to the museum to see the dinosaur exhibit. The museum has three different plans to pay for going to see the dinosaurs.

Museum Rate Plans

Plan A: Pay \$ 3.00 per person to visit the Museum.

Plan B: Monthly membership is \$8.00 for each person, but you can go as many times as you like during the month.

Plan C: A family membership for a month is \$17.00. Everyone in your family can go as often as they like for a month.

You and your sister want to go see the dinosaur exhibit three times this month. Which plan should you buy to save money?

Explain your reasoning.

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Level C:

It is summer vacation and you can go to the museum more often. The rates change for a summer special.

Museum Summer Rate Plans
Plan A: Pay \$ 2.75 per person to visit the museum.
Plan B: Monthly membership is \$7.50 for each person, but you can go as many times as you like during the month.
Plan C: A family membership for a month is \$15.25. Everyone in your family can go as often as they like for a month.

If you and your brother want to go to the museum eight times during the three months of summer, which one plan should you use and when should you go to save the most money?

What if you can't go as you originally planned? What other plans might you use? State when you would attend and the best plan(s) to use. Explain your thinking.

Level D:

You are going on an expedition to dig for dinosaur fossils. In digging you come across a small trunk left by another archeologist. In the trunk is a paper that lists how deep you need to dig to reach dinosaur fossils. Unfortunately, something spilled on part of the paper and it is damaged.

Years ago	Feet to Dig
10,000	14
20,000	22
30,000	30
40,000	
70,	

Using your knowledge of mathematics, find the missing numbers in the chart. Write a generalization or rule to determine any number of feet you must dig given the number of years.

If the Jurassic period was 200 million years ago, how deep would you need to dig at this location to uncover fossils of that era? Explain why your answer is or is not reasonable.

Level E:

Another archeologist looks closely at the paper that was excavated. She says, "Wait, that isn't right. The first depth is not 14 feet. It is actually 4 feet. That is a wrinkle in the paper, not the digit 1." You say, "Well that makes more sense because the functional relationship should not be linear. I think the difference in the depth diminishes as one digs deeper."

Determine a functional relationship that approximates the sequence that begins with 4, 22, 30 and continues to increase, but by an ever decreasing rate of change.

Explain the function and generalize it.

Use your function to determine the depth you would need to dig at that location to uncover a fossil from the Jurassic era. How deep would you need to dig?

Explain if your answer seems reasonable.

Digging Dinosaurs

Primary Version Level A

Materials: A picture of the lake with 8 dinosaur feet for each student.

Discussion on the rug: Teacher holds up the picture. "Tell me about this picture? What do you see?"

Teacher solicits answers from students. "How many dinosaur feet do you see in this picture?"

Teacher solicits answers from students and then states that the class will investigate this question. "We are going to try to figure out how many dinosaurs are standing in the lake."

In small groups: Each student has a picture and the teacher asks the following questions:

"How many legs does a dinosaur have? Which dinosaurs have 2 legs? Which dinosaurs have 4 legs? Are there any other dinosaurs with a different number of legs? How many dinosaurs are standing in the lake? Explain why you think it would be that number. Can you draw a complete drawing of the dinosaurs standing in the lake? Can you show how you got your answer?"

At the end of the investigation have students either discuss or dictate a response to the summary questions.

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Digging Dinosaurs

Task Description - Level A

This task challenges a student to use words and number sentences to describe the number of dinosaurs in a lake given the number of visible dinosaur feet. A student is to determine multiple solutions for answering the question of how many dinosaurs are standing in the water; this depends upon the type of dinosaur. This task challenges a student to use words and number sentences to explain his or her solution.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and under- stand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **Add and subtract within 20.**

1.0A.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

1.0A.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Common Core State Standards Math – Standards of Mathematical Practice

MP. 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Digging Dinosaurs

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Digging Dinosaurs

Task Description – Level B

The problem of the month asks the student to determine the best rate plan for three visits to a museum given three different rate plans. The purpose is to determine the least expensive plan for their three visits to the museum.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **Add and subtract within 20.**

1.0A.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

1.0A.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with equal groups of objects to gain foundations for multiplication.

2.0A.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **Represent and solve problems involving multiplication and division.**

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Common Core State Standards Math – Standards of Mathematical Practice

MP. 2 Reason abstractly and quantitatively.

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MP.4 Model with mathematics.

Digging Dinosaurs

Task Description – Level C

The problem of the month asks the student to determine the best rate plan to visit a museum when given a number of rate plans which have changed to increase variability of both cost and time. The student is to determine and explain which plan to use to save the most money, and to consider which other plan might be used if the original constraints for their visit have changed.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Work with equal groups of objects to gain foundations for multiplication.

2.0A.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **Represent and solve problems involving multiplication and division.**

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Write and interpret numerical expressions.

5.0A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.0A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.2 b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

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Problem of the Month

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Digging Dinosaurs

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Digging Dinosaurs

Task Description – Level D

The problem of the month asks the student to solve problems involving a linear relationship based upon some data retrieved by an archaeologist. The student is given a table of data that is linear and asked to define a rule for the given situation. This reasoning is extended for the student by asking the student to determine the mathematical reasonableness of data from another similar situation.

Common Core State Standards Math - Content Standards

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.2 b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

Reason about and solve one-variable equations and inequalities.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Functions

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Common Core State Standards Math – Standards of Mathematical Practice

MP. 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4 Model with mathematics.

Digging Dinosaurs

Task Description – Level E

The problem of the month asks the student to solve previous problems with additional twists and the expectation of finding the functional rule as well as generalizing and explaining the function itself within the context of the problem. The student is given a table of data that is non-linear and asked to define a rule for the given situation. The student is asked to explain if his or her answer seems reasonable.

Common Core State Standards Math - Content Standards

<u>Functions</u>

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

<u>High School – Functions - Interpreting Functions</u>

Analyze functions using different representations

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.8.,b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

High School - Functions - Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems. F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

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MP.4 Model with mathematics.

Digging Dinosaurs

Task Description – Primary Level

This task challenges a student to use words and number sentences to describe the number of dinosaurs in a lake given the number of visible dinosaur feet. A student is asked about the characteristics of different dinosaurs: How many have 2 feet? How many have 4 feet? Are there any with a different number of feet? How can one determine the number of dinosaurs standing in the lake? A student is asked to show how they determined their answer and to have a discussion with other students, with the whole group, or to dictate a response to the summary questions.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects.)

Count to tell the number of objects.

K.CC.5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and under- stand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

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MP.4 Model with mathematics.