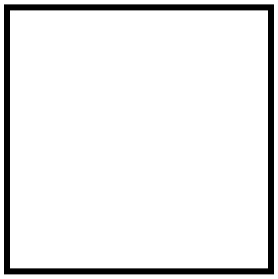
What's Your Angle

Level A

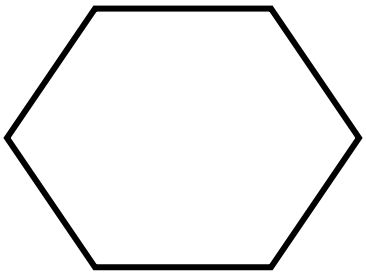
Maggie Graham likes to draw diagonals in different figures. She draws a square.



She draws in all the possible diagonals within the square. How many does she draw?

She draws another figure with a different number of sides. Then she draws in all the diagonals. She counts the diagonals and comes up with a different number than when she counted the diagonals in a square.

Maggie asked herself, "I wonder how many diagonals are in a hexagon?"



How do you know when you have found all the diagonal lines that you can draw in a figure?

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What's Your Angle

Level B

Maggie made a drawing of an octagon (eight-sided figure) and drew all the diagonals. How many diagonals did she draw?

Maggie made a table showing the number of diagonals you can draw compared to the number of sides of the figure. What did Maggie's table look like?

Explain what patterns you see.

Lex, Maggie's friend, says that he drew a figure and there were 16 diagonals, but Lex is wrong. Explain why he is wrong.

Using Maggie's method, can you predict how many diagonals there are in a dodecagon (twelve-sided) shape? Explain how you know.

Write an expression in terms of *n*, the number of sides of a polygon, that gives the number of diagonals possible.

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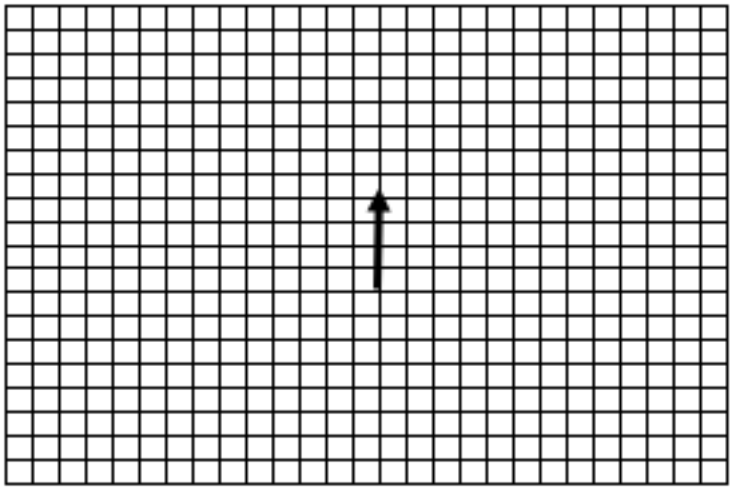
What's Your Angle

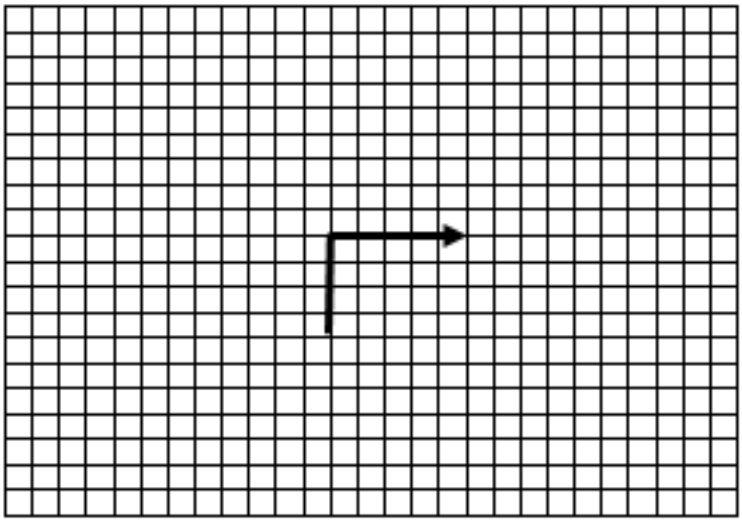
Level C

A spirograph is a geometric representation of a sequence of numbers. A spirograph with the numbers 4, 5, 6 is called an order-three spirograph because it has 3 numbers in its sequence. You create a spirograph using graph paper. Pick a point near the middle of the graph paper and call it home. Follow the steps to create a spirograph.

1. Take the first number in the sequence and draw a line "up" the paper with that distance.



2. Turn right 90˚ and draw a line the distance of the second number in the sequence.



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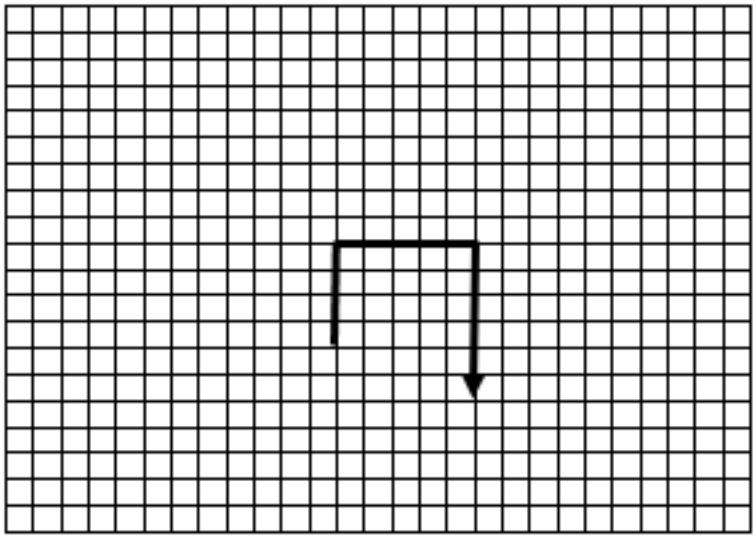
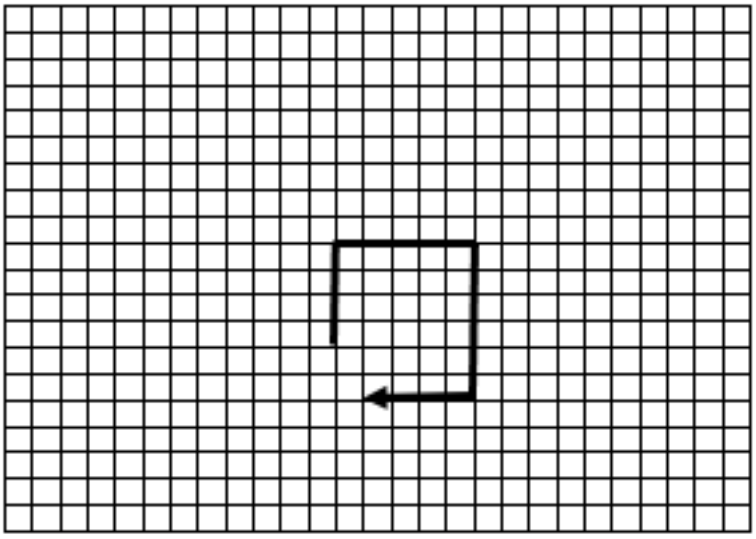
3. Turn right again 90˚ (now you are pointed down) and draw a line the distance of the third number in the sequence.



4. Again turn right 90˚ (now you are pointed left) and draw a line the distance of the next number in the sequence. If—as in an order-three sequence—you have run out of numbers, start again with the first number in the sequence.



5. Continue with the process, turning right and drawing a line segment the distance of the next number in the sequence, until you get back to home. Home is the place you started, and after turning right, you will just continue to repeat over the same path.

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Now that you know how to draw spirographs, experiment with their designs and, after some exploration, answer the following questions.

What patterns did you find in spirographs?

How are the designs impacted by the:

- Size of the numbers in the sequence?
- The number of numbers in the sequence (order size)?
- The arrangement of the numbers in the sequence?

Describe the relationship between the order number and the number of loops in the design.

Do all spirographs eventually return to home? Explain.

What is the relationship between the order of a spirograph and the number of cycles of sequence numbers used to return home?

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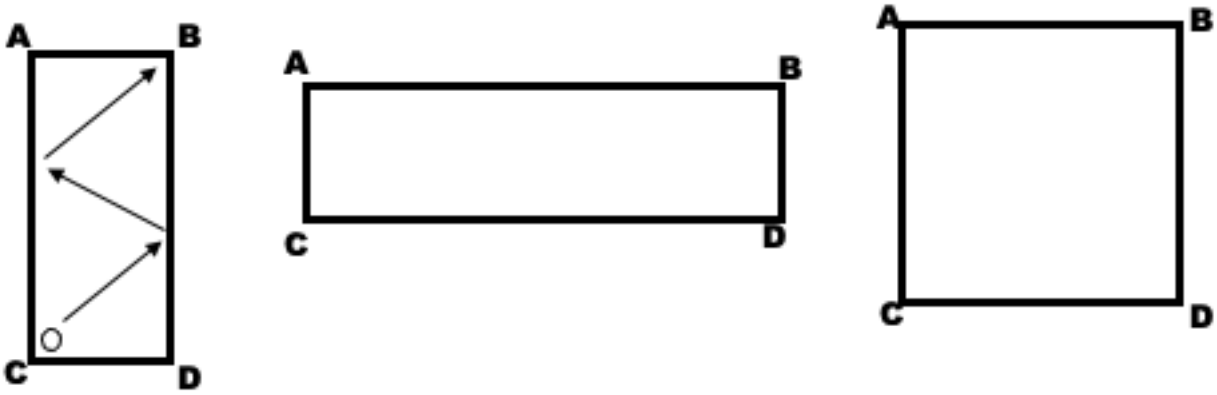
What's Your Angle

Level D

Perplexing Paula Pocket loves to play pool. Besides being a great pool player, she makes pool tables. The pool tables are indeed perplexing. They are all rectangles of different sizes with only four pockets. The pockets are at the four corners of the tables, labeled in the orientation shown below.



What makes these tables most interesting is the different games Paula plays on them. She picks a table and lays the cue ball right in front of Pocket C. She challenges her opponent to guess the pocket where the cue ball will drop. Then she always strikes the ball at a 45-degree angle to the side of the table. The cue banks off different sides of the table until it finally drops in one pocket. If you just guess, you have a 1 in 4 chance of guessing the right pocket. But Paula also asks you to predict the number of times the ball will bank off a side before dropping in a pocket.

Remember, she has almost an unlimited number of tables with different dimensions. Since Paula doesn't like fractions, all her tables have whole-number dimensions in feet.

You want to be able to beat Paula at her own game. Determine a strategy that you can use to play with Paula. A successful strategy would enable you to accurately predict the number of times a cue ball would bank off a wall and exactly which pocket it would fall into, once Paula has picked a table and given you the dimensions.

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What's Your Angle

Level E

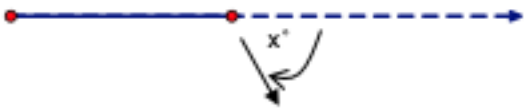
If we can't all be stars, maybe we can draw them.

Drawing Process

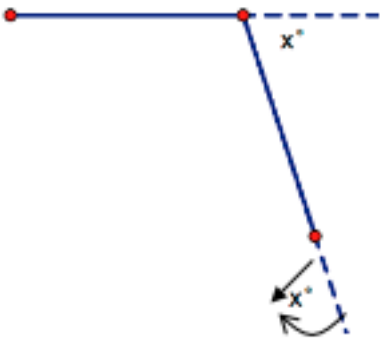
Step 1: Record your beginning point and orientation (the relative position and direction that you will draw).

Step 2: Draw a line segment 5 centimeters in length in the direction you are oriented.

Step 3: At the end of your line segment, turn your orientation **x** degrees clockwise.



Step 4: If you reached your starting point, then stop. Otherwise, continue by going back to Step 2.



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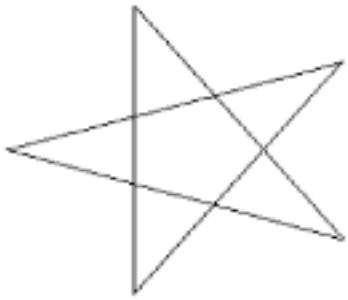
1. Try out this drawing process, inserting 90 degrees for **x**. What figure was drawn? How would you have known without having to actually draw the figure?

2. Use the drawing process with **x** = 72˚. What figure is drawn? Explain how you know without drawing.

3. Use the drawing process with x = 80^0. What figure is drawn? Explain how you know without drawing.

4. For the figure below, determine the interior angle of each point of the star and the exterior angle needed to make the turn in the drawing procedure in order to draw the figure.



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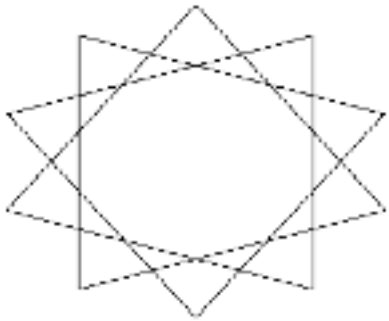
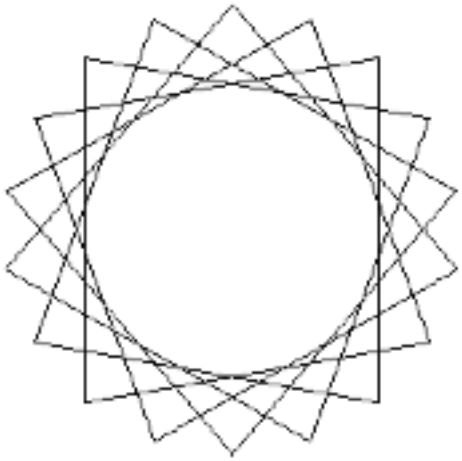
5. For the figure below, determine the interior angle of each point of the star and the exterior angle needed to make the turn in the drawing procedure in order to draw the figure.



6. For the figure below, determine the interior angle of each point of the star and the exterior angle needed to make the turn in the drawing procedure in order to draw the figure.



Determine a generalized procedure for finding the interior and exterior angles of a star-shaped figure. Explain the mathematics behind your process. How do you know it always works?

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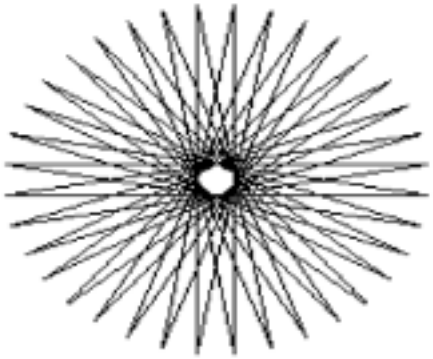
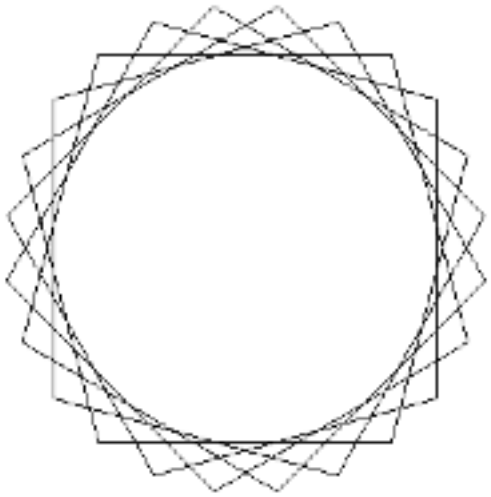
7. Test your procedure on the following two-star figures.

Explain how you might predict how many points a star will have, given the turning angle of ***x*** degrees.





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