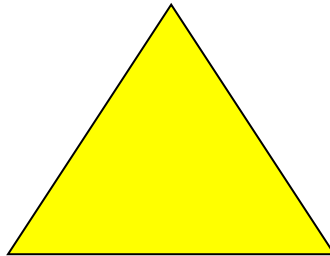


Surrounded and Covered

Level E

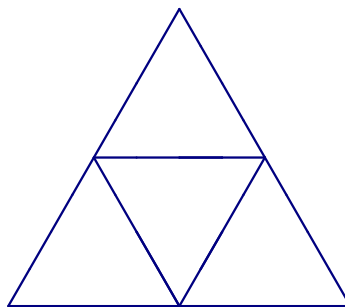
A fractal is a geometric figure that has self-similarity, which is created using a recursive process, and which is infinite in structure. A fractal is an object whose detail is not lost as it is magnified. In fact, the structure looks the same as the original. In contrast, a circle appears to become straight as a portion of it is magnified.

Sierpinski's triangle is a fractal. This fractal is generated through a recursive process. The fractal can be viewed and analyzed at each stage of development. In its original stage, the fractal is an equilateral triangle. For our purposes, we will call this initial shape Stage 0. For measurement purposes, we will say it has a perimeter of three linear units and one area unit. We will define the area as the yellow triangular region. We will define the perimeter as the linear units bordering the yellow area.



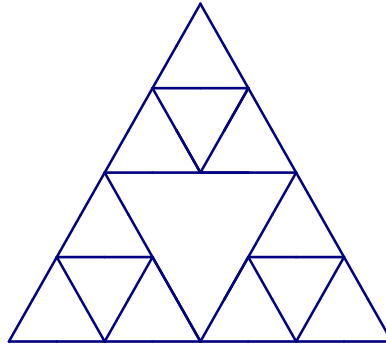
During the first iteration of the recursive process, a triangle is cut out of the center of the original equilateral triangle. The smaller cutout triangle is similar to the original but with a length of $\frac{1}{2}$ the Stage 0 length.

Therefore, the perimeter, defined as the distance bordering the yellow area, has increased by $\frac{3}{2}$ linear units, making the perimeter $\frac{9}{2}$ units in length. The yellow area has been decreased by $\frac{1}{4}$, making the new area $\frac{3}{4}$ units in size. Below is Sierpinski's triangle at Stage 1.



— Inside Problem Solving: Surrounded and Covered —

The process is repeated in stage 2, with three smaller triangle cutouts in the remaining three sections of the original triangle. What is the total perimeter at Stage 2? What is the total area at Stage 2? Explain your calculations.



Sierpinski's triangle is generated following this recursive process. Draw a picture of Stage 3. Determine the area and perimeter of the triangle at this stage. Explain the mathematics.

Sierpinski's triangle is created using the recursive process indefinitely. The fractal has an infinite structure. Find the area and perimeter of Sierpinski's triangle for any Stage n .

Justify the mathematics.

Find the actual area and perimeter of Sierpinski's triangle as n approached infinity. Justify your solution.