Level A

On television, there is a new game show called *Take Two*. The game is played with two players. There are nine coins lined up in a row.

Each player takes turns. On a turn, a player must take two coins away. The game starts with the first player taking away two coins. Then it is the other player’s turn. The second player takes two coins away. The game continues with alternate turns. The player who can’t pick up exactly two coins loses. If there is only one coin left when it is a player’s turn, then the player whose turn it is loses and the other player wins.

Which player do you want to be, the first player or the second player?

Explain your choice.

Is this a fair game? Why or why not?
Many viewers complained about the game show called *Take Two*, so the producers changed the show’s name and rules. The new show is called *Pick Up Coins*.

They also changed the number of coins from 9 to 22 and changed the rules so on a turn a player could choose to pick up either one coin or two coins.

The new rules are: Start with 22 coins. The players take turns. On a turn, a player must take either one or two coins away. Then it is the other player’s turn. The second player takes either one or two coins away. The game continues with alternating turns. The player who picks up the last coin loses. The other player wins.

Determine a strategy for playing this game.

Explain a strategy that may ensure you win the game.

Is this a fair game? Explain your reasoning.
The game show *Pick Up Coins* is getting top ratings, so the rival network has begun a new show called *Pick Up Sticks*. The game starts with an arrangement of 31 sticks in a 3-by-4 rectangular-shaped grid as shown below:

The game is played by two players who alternate taking turns. A turn consists of picking up sticks along a path. During any one turn, a player may start wherever she or he likes, picking up as many sticks as he or she wishes as long as the sticks follow a path. A path is a sequence of adjacent sticks, starting at one end of the stick, leading to a stick at the opposite end, and so on. For example, Path A is legal and Path B is illegal because the path has been retraced.

The winner is the player who picks up the last stick!

Determine a strategy for playing this game. Should you start first or second? Explain a method for ensuring you win.
Level D

It’s time to play *Let’s Make It Real*, hosted by Mighty Fall.

There are three doors. Behind one door is a fabulous prize. Behind the other two doors are bogus prizes. When you pick the first door, another door will open and show you a bogus prize, leaving the door you picked and the third door closed. You once again pick one of the two closed doors. You will either get the fabulous prize or the bogus prize.

Determine the best strategy for winning the fabulous prize.

Using mathematics, explain why your strategy is the best.
“Hi, I’m Pat Saywhat, and this is Banana. Welcome to

**WHEEL OF TORTURE!**

Today we have several contestants who will try their luck and risk their earnings to become today’s Big Wheel winner. (Crowd roars.)

The rules of the game are quite simple. We have several contestants backstage in soundproof booths. One by one, we will invite a contestant to come out and try his or her luck at the Big Wheel. As you can see, our Big Wheel has six equal sectors (pie-shaped regions). In each of the sectors is a dollar amount, except for one that is labeled ‘LOSE ALL YOUR $$.’”

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$2,000
$1,000
$4,000
$5,000
LOSE ALL YOUR $$
$3,000
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“Contestants may spin the wheel as many times as they would like until either they decide to stop and keep the money or the big wheel lands on LOSE ALL YOUR $$$. If that happens, the contestant loses the game, but we have some very nice consolation prizes. The outcome of each player’s turn is unknown to the other contestants, so they must use the best strategy, since they are not aware of what has transpired prior to their turn. After the last contestant has completed a turn at the Big Wheel, the player with the most money wins the game and keeps all the money. (Crowd roars.) Banana, I sure hope these contestants have come up with a sound mathematical strategy!”

Use your knowledge of mathematics to determine the optimum strategy for playing the game. Describe your strategy. Please describe the steps to follow in playing the game and at what point to stop and keep the cash. Explain the reasoning behind your game strategy. What mathematics did you use to arrive at your game strategy? Why do you believe that it is the best possible strategy?