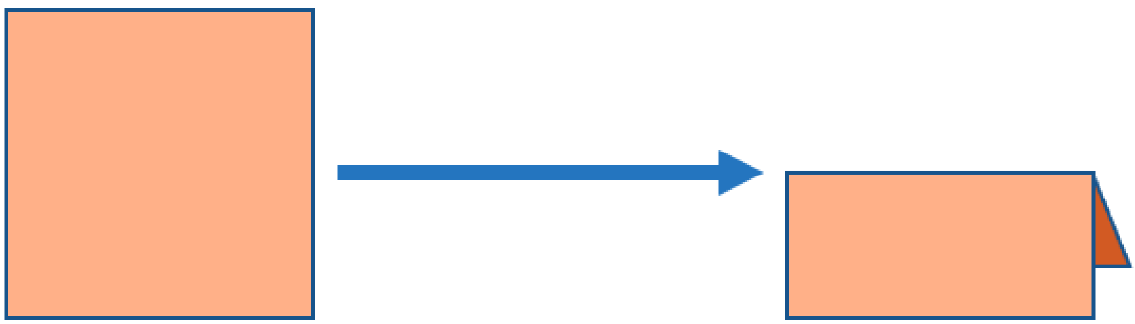
inside + x = ÷
mathematics

Inside Problem Solving

Cut It Out

Level A

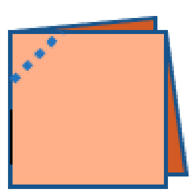
Start with a square piece of paper that is 8 inches in length. Fold the piece of paper in half, bending the top edge down to meet the bottom edge of the paper.



Now fold the sheet in half again by bending the left side over to meet the right side.

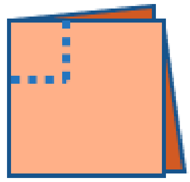
How do the shape and size of the folded paper compare to the original sheet of paper? Describe its dimensions and area.

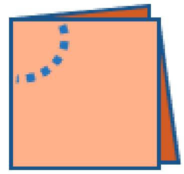
Now make the cut (dotted line) at the top left-hand corner of the folded paper.



Without unfolding the paper, draw and describe what the paper will look like when unfolded. Explain how you know.

Repeat the folding process with new sheets of paper. Make the following cuts (dotted lines).



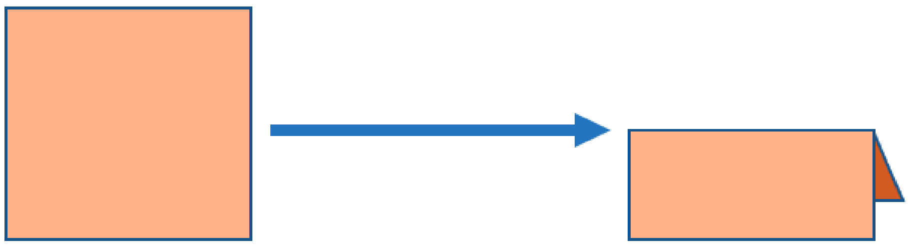


Before unfolding the paper, predict what the original paper will look like after the cut. Draw an illustration and explain your reasoning.

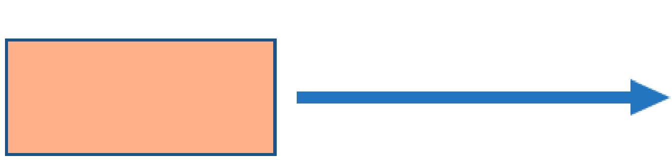
Cut It Out

Level B

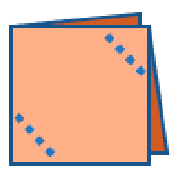
Once again, start with a square piece of paper that is 8 inches in length. Fold the piece of paper in half, bending the top edge down to meet the bottom edge of the paper.



Now fold the sheet in half again by bending the left side over to meet the right side.

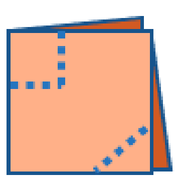
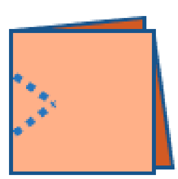


Now make the cuts (dotted lines) at the top right-hand corner and bottom left-hand corner of the folded paper.



Without unfolding the paper, draw and describe what the paper will look like when unfolded. Explain how you know.

Repeat the folding process with new sheets of paper. Make the following cuts (dotted lines).

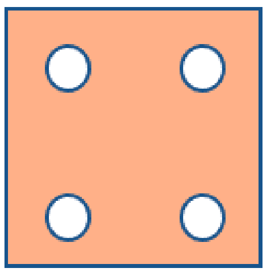
Before unfolding the paper, predict what the original paper will look like after the cut. Draw an illustration and explain your reasoning.

Cut It Out

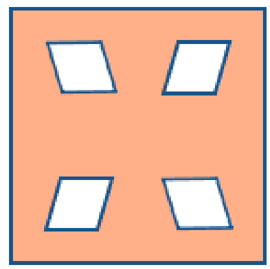
Level C

Examine each of the following images. Each image is a blank sheet of square paper that contains a set of holes. The paper was folded a **number of times** and then **one continuous cut** was made to produce the image. Determine how the paper was folded and how the cut was made to produce each of the six images below.

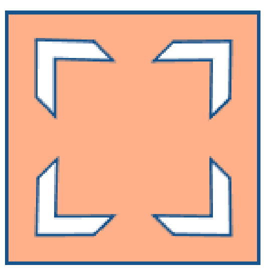
A.



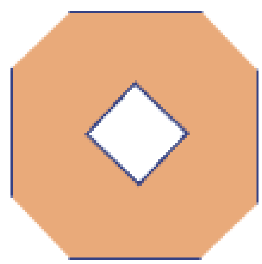
B.



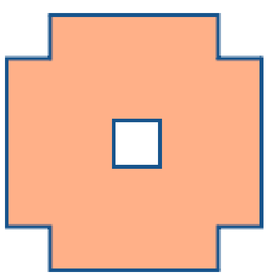
C.



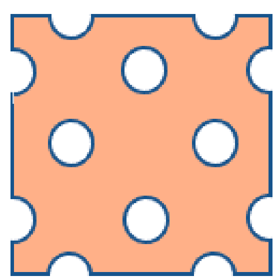
D.



E.



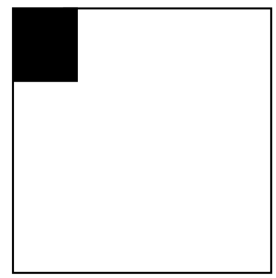
F.



Cut It Out

Level D

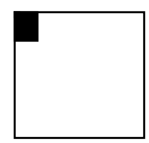
A *fractal* is an image that has self-similarity. In this activity, create a fractal. Start with a square sheet of tissue paper 8 inches on each side. List the area and perimeter of the paper.



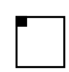
1. Fold the paper by bringing the left side over to the right. Then bring the top down to the bottom. If you were to open the paper up, it would be divided into 4 congruent squares. Now imagine cutting a square measuring 1 inch on a side out of the most folded corner (top left corner) of the folded paper.

   What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result.

   Determine the new area of the paper (excluding the hole). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance bordering each hole). How do the area and perimeter compare to the original paper?



2. Take the folded sheet and fold it again by bringing the left side over to the right and the top down to the bottom. If you were to open the paper up, it would be divided into 16 squares. Now imagine cutting out a square measuring 1/4 inch on a side from the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the holes). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance around each hole). How do the area and perimeter compare to the original paper?



3. Take the folded sheet and fold it again by bringing the left side over to the right and the top down to the bottom. If you were to open the paper up, it would be divided into 64 squares. Now imagine cutting a square measuring 1/16 inch on a side out of the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the holes). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance around each hole).

4. Imagine taking the folded sheet and folding it again for the fourth time using the same process. How many sub-squares would the folded paper contain? Now imagine cutting a square measuring 1/64 inch on a side out of the upper left corner of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Determine the new area and perimeter of the paper. How do the area and perimeter compare to the original paper?

Examine the process you followed in the previous steps. A ***fractal*** contains an infinite number of iterations (steps).

Explain what the fractal would ultimately look like.

Draw a diagram of the fractal.

Explain the size of the fractal at the first five iterations (steps).

Determine the actual size of the fractal in terms of area and perimeter.

Discuss how you found your answers and explain your mathematical reasoning.

Cut It Out

Level E

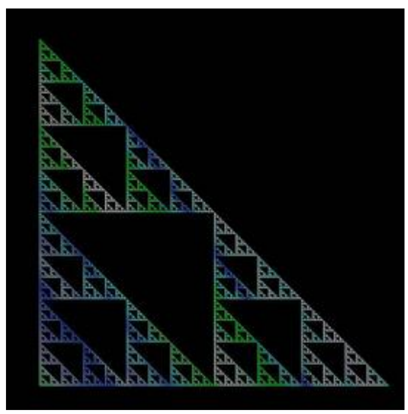
A *fractal* is a geometric figure that has self-similarity, that is created using a recursive process, and that is infinite in structure.

There are two categories of fractals—geometric and random.

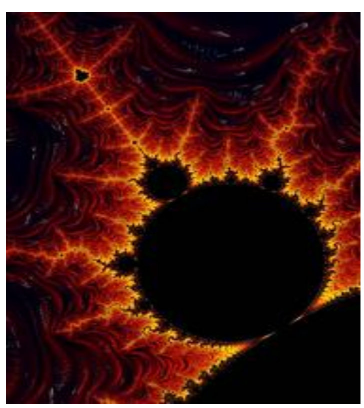
A *geometric* fractal is an endlessly generating pattern of self-similarity. The pattern continually replicates itself in smaller versions. Thus, when a small portion of a geometric fractal is magnified, it looks exactly like the original version.

A *random* fractal also contains self-similar images of itself, only in a disorderly, non-predictable pattern.

Beautiful computer-generated images such as the Mandelbrot Set are examples of these fractals.



Geometric Fractal: Sierpinski Triangle



Random Fractal: Mandelbrot Set

Image credits: Geometric Fractal: Sierpinski Triangle, from istock.com/sakkmesterke, Stock photo ID: 478553781|| Random Fractal: Mandelbrot Set, istock.com/raydex Stock photo ID:187032851

Design a poster/object that contains a fractal.

- The fractal could be a self-similar collage, a series of pictures inside a picture, a self-similar geometric design, or another self-similar unique creation. It must be an original drawing or design.
- Your poster may contain photographs, pictures from periodicals, enlargements and reductions from copiers, and/or computer-generated designs.
- The fractal may be created using a random (chaos) technique or a self-similar drawing.
- You may produce a 3-dimensional model of a fractal.
- Your design must contain at least four iterations of a process that produces some self-similar shapes.

Write a report that describes the fractal and the process that you used to create the design.

- Be sure to describe the relationship between similar objects in your design.
- Identify the self-similar shapes or pictures that you used in the fractal.
- Demonstrate a procedure for finding the size (length, area, volume, angular distance, etc.) of the self-similar objects at any given level of the fractal.