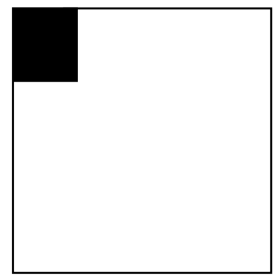
Cut It Out

Level D

A *fractal* is an image that has self-similarity. In this activity, create a fractal. Start with a square sheet of tissue paper 8 inches on each side. List the area and perimeter of the paper.

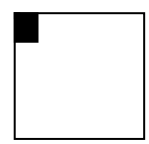
1. Fold the paper by bringing the left side over to the right. Then bring the top down to the bottom. If you were to open the paper up, it would be divided into 4 congruent squares. Now imagine cutting a square measuring 1 inch on a side out of the most folded corner (top left corner) of the folded paper.

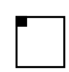
   What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result.

   Determine the new area of the paper (excluding the hole). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance bordering each hole). How do the area and perimeter compare to the original paper?

2. Take the folded sheet and fold it again by bringing the left side over to the right and the top down to the bottom. If you were to open the paper up, it would be divided into 16 squares. Now imagine cutting out a square measuring 1/4 inch on a side from the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the holes). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance around each hole). How do the area and perimeter compare to the original paper?

3. Take the folded sheet and fold it again by bringing the left side over to the right and the top down to the bottom. If you were to open the paper up, it would be divided into 64 squares. Now imagine cutting a square measuring 1/16 inch on a side out of the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the holes). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance around each hole).

4. Imagine taking the folded sheet and folding it again for the fourth time using the same process. How many sub-squares would the folded paper contain? Now imagine cutting a square measuring 1/64 inch on a side out of the upper left corner of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Determine the new area and perimeter of the paper. How do the area and perimeter compare to the original paper?

Examine the process you followed in the previous steps. A ***fractal*** contains an infinite number of iterations (steps).

Explain what the fractal would ultimately look like.

Draw a diagram of the fractal.

Explain the size of the fractal at the first five iterations (steps).

Determine the actual size of the fractal in terms of area and perimeter.

Discuss how you found your answers and explain your mathematical reasoning.