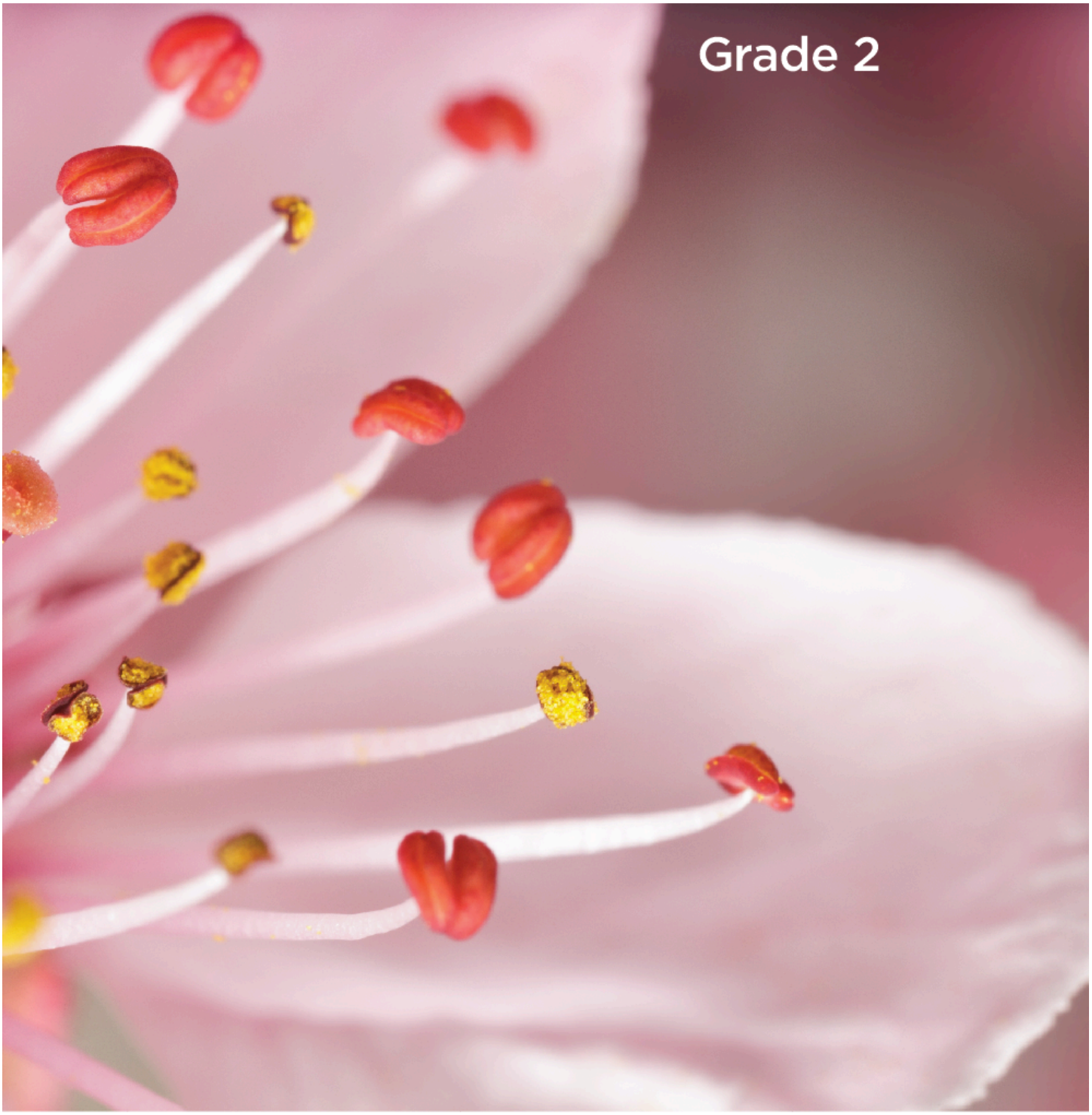
Early**Mathematics**

a resource for teaching young children



a publication of

The Charles A. Dana Center at The University of Texas at Austin

Early Mathematics

A Resource for Teaching Young Children

Grade 2

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The Charles A. Dana Center at
The University of Texas at Austin

2012

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As always, we welcome your comments and suggestions for improvements. Please contact us at **dana-txshop@utlists.utexas.edu** or at the mailing address above.

About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center strengthens our nation's education systems to provide a reliable path to upward mobility for all students. Our work focuses on mathematics and science education, with an emphasis on strategies for improving student engagement, motivation, and persistence. We are dedicated to nurturing students' intellectual passions and ensuring that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy.

We advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards. We help our partners adapt promising research to meet their local needs.

We develop innovative curricula, tools, protocols, instructional supports, and professional development systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance to school and district leadership teams, advise community colleges and states, and collaborate with national partners on work such as our Urban District Leadership Networks, Academic Youth Development project, and Advanced Mathematical Decision Making course.

We have significant experience and expertise in the following:

- Standards development and implementation, systemic reform, and district capacity building
- Education leadership, instructional coaching, and teaching
- K–14 course design and development, learning networks, and programs for bridging critical transitions
- Research, content development, and publishing

The Center was founded in 1991 at The University of Texas at Austin. Our staff of nearly 80 researchers and education professionals has worked with dozens of school systems in nearly 20 states and with 90 percent of Texas's more than 1,000 school districts. We are committed to ensuring that the accident of where a child attends school does not limit the academic opportunities he or she can pursue. For more information about our programs and resources, see our homepage at **www.utdanacenter.org**.

About the Common Core State Standards for Mathematics

This resource is aligned to the Common Core State Standards for Mathematics.

The CCSS for Mathematics and for English Language Arts are copyrighted by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) and are available at **www.corestandards.org/the-standards**; these CCSS are being used under the NGA Center and CCSSO Public License, available at **www.corestandards.org/public-license.**

About the development of this resource

This new revised and expanded edition of *Early mathematics: A resource for teaching young children* consists of materials for 20 sessions for each of four grades—prekindergarten, kindergarten, grade 1, and grade 2—for a total of 80 sessions. We were able to develop these materials because of a generous December 2010 grant from the **Noyce Foundation**.

First edition (2011)

Twenty of these sessions (10 for prekindergarten and 10 for second grade) were initially developed in spring and summer 2011 by early mathematics education experts Brian Mowry (prekindergarten) and Carolyn Moore (second grade), and reviewed in summer and early fall 2011 by ACE: A Community for Education leaders Chetan Makan and Mary Ellen Isaacs, both of whom are experts in designing and implementing early childhood tutoring programs that can be implemented at scale. The materials were also reviewed by Patti Bridwell, who has expertise in professional supports for teachers and tutors.

This first edition was released in fall 2011 as a proof-of-concept resource titled *Early mathematics: Resources for tutoring young children.* These initial 20 sessions were field-tested in fall 2011 by tutors from the Dana Center's ACE: A Community for Education (www.utdanacenter.org/ace) program in Austin, Texas (prekindergarten sessions), and by tutors from Experience Corps (www.experiencecorps.org) in Philadelphia, Pennsylvania (second-grade sessions).

Second edition (2012)

A key finding from the fall 2011 proof-of-concept field testing was that the material as written was probably too complex for paraprofessionals (e.g., tutors) to deliver, but that it could be very effective if delivered by classroom teachers. Based on this feedback, we have substantially revised the initial 20 sessions for this new edition, changing the intended users of this resource from paraprofessionals to classroom teachers.

All 80 sessions are built on recommendations in the 2009 National Research Council report *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Committee on Early Childhood Mathematics; Christopher T. Cross, Taniesha A. Woods, and Heidi Schweingruber, editors). Center for Education, Division of Behavioral and Social Sciences and Education. Washington, D.C: The National Academies Press.

In particular, these session materials speak to the recommendation that:

> Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with *more mathematics instruction time devoted to number* than to other topics.

Accordingly, our materials focus primarily on number.

About the Noyce Foundation

The Noyce Foundation[1] aims to help young people become curious, thoughtful, and engaged learners. The Noyce Foundation focuses on a few key areas:

- improving the teaching of math and science in public schools;
- developing leadership to support student achievement;
- supporting education policy and research; and
- expanding opportunities for students to experience hands-on science in out-of-school settings.

The Noyce family created the Noyce Foundation in 1990 to honor the memory and legacy of Dr. Robert N. Noyce, cofounder of Intel and inventor of the integrated circuit—which fueled the personal computer revolution and gave Silicon Valley its name.

Although he was an individual of daunting talents and intellect who was honored by two presidents as well as by his academic and industry peers around the world, Bob Noyce also remained a humble and approachable man who believed fervently in democracy. In everything the Noyce Foundation undertakes, it remains committed to promoting the qualities that Bob Noyce embodied: optimism, creativity, risk taking, and determination.

In recognition of Bob's concern about the narrowing pipeline of students interested in—and committed to—science-related careers, the Noyce Foundation has focused on mathematics, science, and associated work in research and policy. Much of the Foundation's focus has been on improving instruction in mathematics, science, and early literacy in public schools.

As schools began to intensify their focus on math and literacy in response to No Child Left Behind—leaving science behind in the process—the Noyce Foundation emphasized support for out-of-school science programs that show promise of sustaining and engaging student interest through middle school, a time when students tend to make critical decisions about what subjects they want to pursue in the future. The Noyce Foundation informal science initiative includes support for leadership development in science centers.

For more information about the Noyce Foundation, visit its website at **www.noycefdn.org.** For more information about the Silicon Valley Mathematics Initiative, see **www.svmimac.org.**

[1] This description of the Noyce Foundation's mission and history was adapted from content retrieved from its homepage (www.noycefdn.org) and its About Us page (www.noycefdn.org/aboutus.php) on October 9, 2012.

Acknowledgments

With special thanks . . .

The Dana Center thanks the Noyce Foundation for its generous support of this project. The Noyce family created the Noyce Foundation in 1990 to honor the memory and legacy of Dr. Robert N. Noyce, cofounder of Intel and inventor of the integrated circuit—which fueled the personal computer revolution and gave Silicon Valley its name. For more information about the Noyce Foundation, visit its website at **www.noycefdn.org.**

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Introduction

Background

Early Mathematics—A Resource for Teaching Young Children provides a series of instructional tasks, aligned with the Common Core State Standards for Mathematics, that teachers can use to instruct children in prekindergarten, kindergarten, grade 1, and grade 2. The complete resource includes content for 20 sessions for each of these four grade levels.

The tasks were developed for whole-class instruction with some small-group work, but they are also easily adaptable for tutoring sessions. The estimated timeframe for each session is as follows:

Grade level	Estimated time per session
Prekindergarten	30 minutes
Kindergarten	45 minutes
Grade 1	45 minutes
Grade 2	45 minutes

Most sessions have a literature focus to draw children into the content and/or to keep them connected to a context.

These session materials do not provide everything a child needs to know about a given topic, such as *number*. Rather, each session provides a series of instructional tasks to help you teach selected content and practices described in the Common Core State Standards for Mathematics. You should feel free to modify the sessions as appropriate to meet the individual needs of children in your classroom.

Alignment

We have embedded key Common Core State Standards for Mathematical Practice in each session to help bring out crucial ideas. In most sessions, though, additional Standards for Mathematical Practice beyond those selected may also be relevant.

We chose the content for these sessions based on what content we believe will have the most significant effect on student learning. The language below is drawn from the National Council of Teachers of Mathematics 2006 publication, *Curriculum Focal Points for Prekindergarten Through Grade Eight Mathematics: A Quest for Coherence* (prekindergarten) and the Common Core State Standards for Mathematics (kindergarten onward).

Prekindergarten

(1) developing an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison.

Kindergarten

(1) representing, relating, and operating on whole numbers, initially with sets of objects;

Grade 1

(1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;

(2) developing understanding of whole number relationships and place value, including grouping in tens and ones;

(3) developing understanding of linear measurement and measuring lengths as iterating length units.

Grade 2

(1) extending understanding of base-ten notation;

(2) building fluency with addition and subtraction.

Structure

Each session is divided into three instructional formats—**Activate**, **Engage**, and **Develop**.

The **activate** portion introduces the content in the session and objectives that will be developed in the forthcoming session. In prekindergarten and kindergarten, this section can often occur as a part of the morning circle routine (e.g., calendar, morning message), or it can serve as a transition activity that incorporates songs, movement, and other instructional activities developed to capture the interest and attention of younger students with emerging attention spans.

Then children will **engage** in the content through an activity centered on the content and practices in the standard(s) being addressed. In prekindergarten and kindergarten, this time is spent mostly in whole (or large) group so that the teacher can model the mathematical thinking that children will apply in the Develop section. For younger children, keep in mind that whole-group sessions are designed to last no longer than 20 minutes.

Each session ends with **develop**, which provides children an opportunity to share and analyze their understandings and/or methods. In prekindergarten and kindergarten, the activities in this section can take place during centers, small group, or math station time. Throughout, the role of the teacher will primarily be to ask probing questions to help children make sense of the content in the session.

Early Mathematics
A Resource for Teaching Young Children
Grade 2

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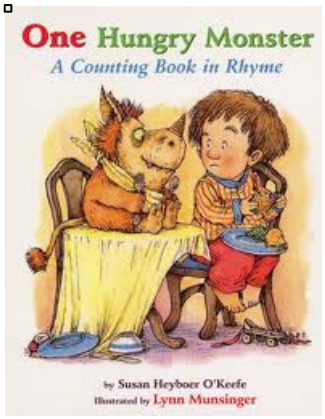
Session 1: Fluently Adding Within 20 (Part I)

At A Glance

In this session, the focus is on helping children acquire fluency with addition strategies. This assumes that children have already learned at least some of the strategies indicated in first grade (1.OA.6). These strategies are addressed in the first grade materials, Sessions 7 through 14. Children use mental strategies to solve addition situations that make sums within 20. The first is an add-to situation where the result is unknown (A + B = ☐). The second is a put-together/take-apart situation where both addends are unknown (C = ☐ + ☐). The second situation allows for multiple solutions. Children share the strategies they used to solve. The strategies are then grouped together in categories and displayed for future reference.

Materials

- *One Hungry Monster* by Susan Heyboer O'Keefe
- Chart paper
- Blackline master, "Tens-Frame"
- Linking cubes



Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

2.OA.2 Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

[2] See standard 1.OA.6 for a list of mental strategies.

MP 7 Look for and make use of structure.

Activate

Introduce and discuss the cover of *One Hungry Monster*.

- Ask, ***"What do you think this book might be about?"***

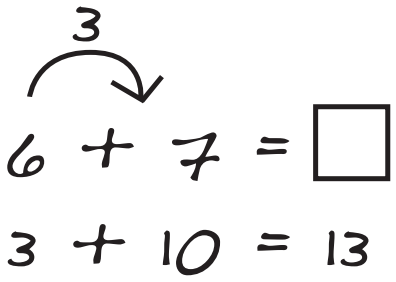
Begin reading, but stop after you read, "Ten hungry monsters, about to fuss and kick, won't get out, they tell me, unless I feed them quick!"

- ***"The little boy needs to find something for the monsters to eat. Let's imagine he runs to the kitchen. He finds 6 bananas on the counter. Then he finds 7 more in the pantry. How many bananas does the little boy have now? Think about how you can solve the problem using mental strategies."***

Have a few children share their strategies. They can also show their thinking using linking cubes or tens-frames. As children do, record their strategies.

You may want to highlight certain ideas if they do not come out naturally by asking questions such as the following:

- How can we use these numbers to make 10? How does that help us solve the problem? (**Make-Ten Extended:** Take 3 from the 6 and put it with the 7 to make 10. Now you have 10 and 3, so it is 13.)
- How can we use doubles to help us think about this problem? (**Doubles Plus One:** 6 and 6 are doubles that have a sum of 12. Now you have 12 and 1, so it is 13.)

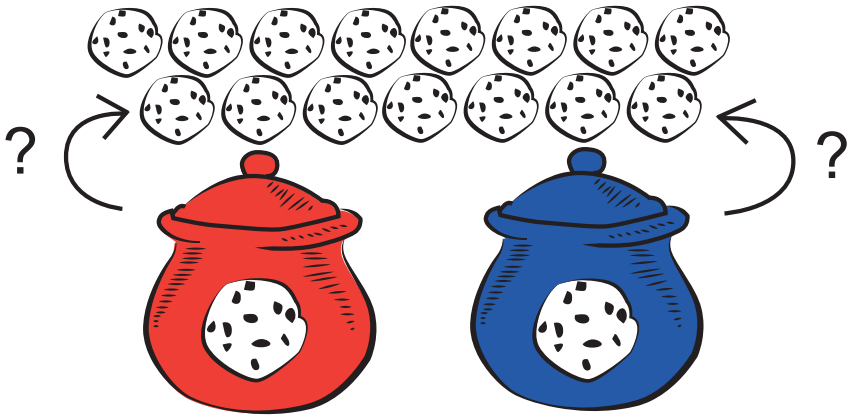


Engage

Introduce the Engage section with the following:

"The little boy now has 15 cookies to feed the monsters with. How many could he have taken from the red cookie jar and how many could he have taken from the blue cookie jar?"

- Give children time to solve in multiple ways for multiple-answer combinations.
- As children share, record equations such as 15 = 8 + 7. Point to one of the equations and ask, ***"How do we know that this expression (fact) equals 15?"***
- Record student thinking when it is helpful to show a strategy. Some children may use subtraction to help them find the missing addends. That is okay because they are relating addition to subtraction, which is an appropriate strategy.
- Ask about several more of the equations you recorded.



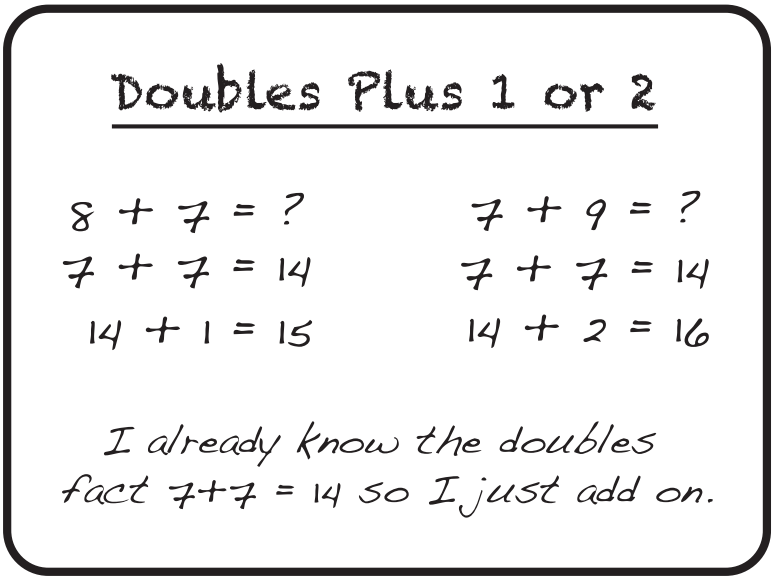
Develop

Introduce the Develop section with the following:

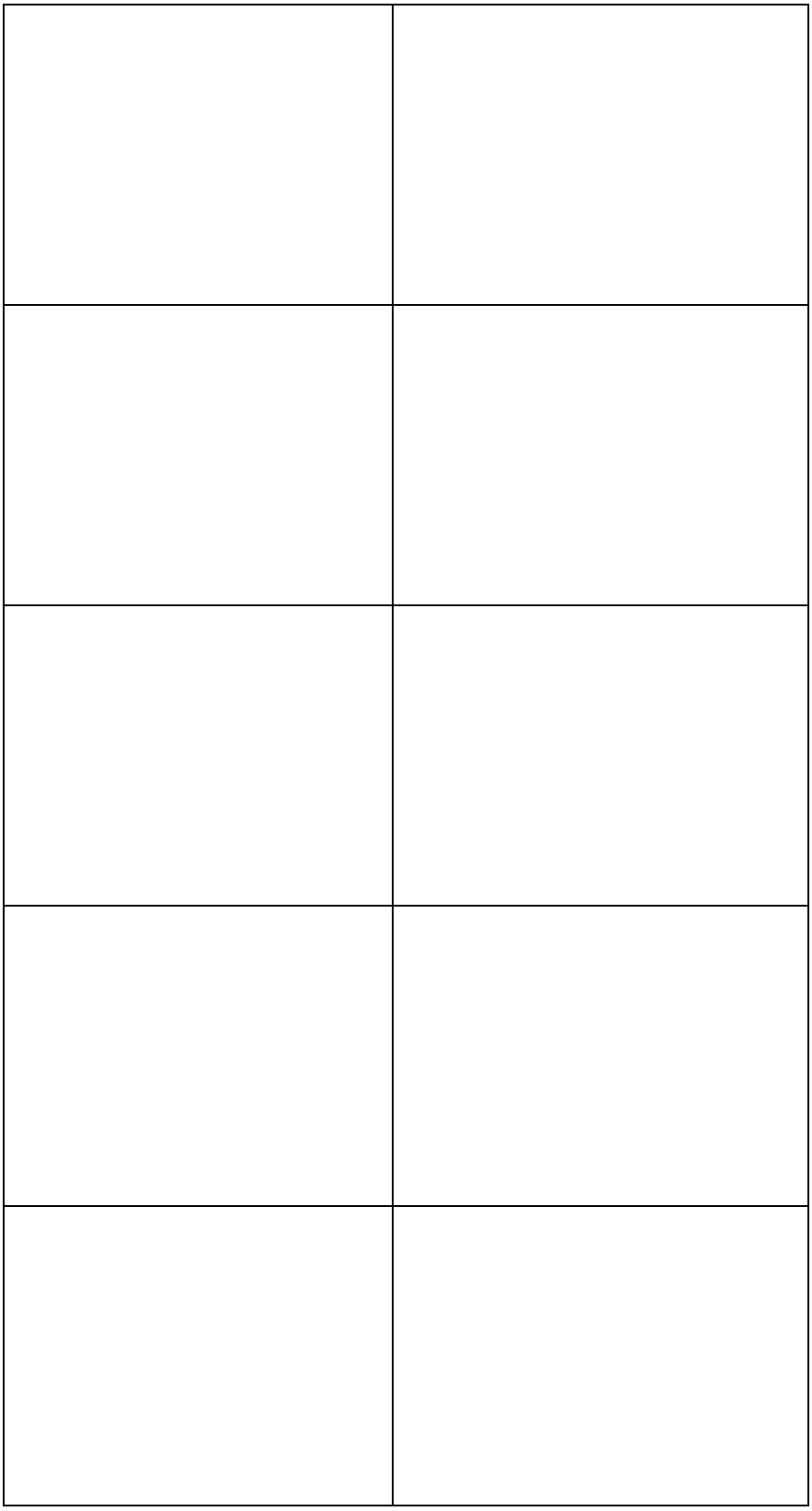
"What do you notice about the strategies we've used today? How could we group some of the strategies together?"

- You may need to do some prompting, but ideally you are looking to create posters that remind children of some of the addition strategies they may use in the future. Possible categories/poster headings include *Make 10 Plus 1 or 2, Doubles Plus 1 or 2, Relate Addition and Subtraction,* and *Count On*.* You do not need a comprehensive list. This is revisited in the next session.
- For each poster, ask children for examples that were not used today in class. They may also use numbers, pictures, and words to describe the action. Record examples and descriptions on the posters and display. Group the posters together under a title of "Addition Strategies" or "Addition Strategies Within 20."

*If the "counting on" strategy is used, try to ensure it is not applied to facts where it would be inefficient (e.g., 7 + 5).



Tens-Frame



Session 2: Fluently Adding Within 20 (Part II)

At A Glance

In this session, children personalize some addition situations so that they can practice solving using mental strategies. The three problem situations are add to, change unknown (A + ☐ = C); put together/take apart, total unknown (A + B = ☐); and add to, start unknown (☐ + B = C). Encourage children that are using inefficient strategies to refer to the posters created in Session 1. Time is available to evaluate strategies and revise or create new posters.

Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

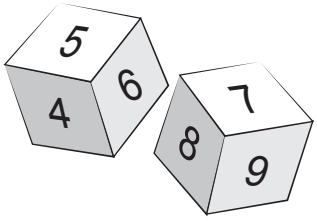
2.OA.2 Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

[2] See standard 1.OA.6 for a list of mental strategies.

MP 7 Look for and make use of structure.

Materials

- *One Hungry Monster* by Susan Heyboer O'Keefe
- Posters made in Session 1
- A six-sided die labeled with the numerals *4, 5, 6, 7, 8,* and *9*
- Blackline master, "Fluently Adding Within 20"



Activate

Read from the beginning of *One Hungry Monster,* but stop after you read, "They paint the peanut butter like lipstick on their mouths, then stamp their feet and boldly say, 'What ELSE is in this house?' "

- Have children brainstorm food that they can gather to feed the monsters. Allow children to be creative. They are feeding monsters after all!
- Record the list for reference.
- Children use the Fluently Adding Within 20 blackline to personalize addition problems.
- It is important that children do not get bogged down in the directions. So, feel free to walk them through that part.

Engage

Have children read and solve their own word problems (without writing on the paper) to be sure they make sense.

Before trading problem sets, ask children not to write on these so they can be reused. You may want to have children sit in a circle so that they can pass the problem sets in the same direction.

- Say, ***"Pass your paper once clockwise. Read No. 1 and solve using mental strategies. You may want to refer to the posters we created (in the last session)."***

You may briefly ask for the strategy that was used or ask them to identify which strategy on the poster that was used. Then ask children to do the same with No. 2. Discuss briefly and then have them solve No. 3.

- ***"Now pass your paper once clockwise again. Read No. 1 of the new problem set and solve using mental strategies."***

Again, you may briefly ask for and record a strategy or two that is used. Continue this process as time permits.

Develop

Direct children's attention to the posters that were created in the last session.

- ***"Which are you most comfortable with? Why? Which are you least comfortable with? Why?"***

Allow children to share a few examples or to connect the examples you recorded during the Engage section to the categories on the posters.

- ***"Is there anything we should add to the posters? Is there another category that we should make a new poster for?"***
- ***"Even though the little boy has tried to feed the monsters again and again they still stamp their feet and boldly say, 'What ELSE is in this house?' "***

Read *One Hungry Monster* to the end. The final frame shows the little boy eating a muffin he kept safe from the monsters, but there is one monster still lurking in the background.

Fluently Adding Within 20

Roll the die with 4, 5, 6, 7, 8, 9 four times and record the results in the spaces below.

1st roll _____ 2nd roll _____ 3rd roll _____ 4th roll _____

Choose three different food items and write them in the spaces below (for example, bananas).

1st food _____ 2nd food _____ 3rd food _____

Personalize the word problems by using the information you recorded above.

1. _____ _____ were on the kitchen table. More _____
1st roll 1st food 1st food

were found in the refrigerator. Then there were 12 _____.
1st food

How may _____ were found in the refrigerator?
1st food

2. _____ blue _____ and _____ purple _____ are in the
2nd roll 2nd food 3rd roll 2nd food

bathtub. How many _____ are in the bathtub?
2nd food

3. Some _____ were on the pillow. _____ more _____ were
3rd food 4th roll 3rd food

under the bed. Then there were 16 _____. How many _____
3rd food 3rd food

were on the pillow?

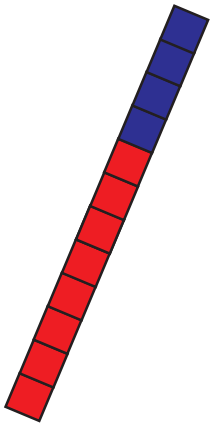
Session 3: Fluently Subtracting Within 20 (Part I)

At A Glance

In this session, the focus is on helping children acquire fluency with subtraction strategies. This assumes that children have already learned at least some of the strategies indicated in first grade (1.OA.6). These strategies are addressed in the first grade materials, Sessions 7 through 14. Children use mental strategies to solve subtraction situations within 20. The first is a take- from situation where the result is unknown (C – B = ☐). The second is a take-from situation where the change is unknown (C – ☐ = A). Children share the strategies they used to solve. Then the strategies are grouped together in categories and displayed for future reference.

Materials

- *One Hungry Monster* by Susan Heyboer O'Keefe
- Tens-frame"
- Linking cubes
- Chart paper



Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

2.OA.2 Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

[2] See standard 1.OA.6 for a list of mental strategies.

MP 7 Look for and make use of structure.

Activate

Introduce the Activate section with the following:

- Ask, ***"What do you think would happen if the monsters in the book* One Hungry Monster *visited our classroom?***
- ***"One of the monsters sees a stack of 12 books on the table. The monster takes 4 books to balance on his head. How many books are on the table now? Think about how you can solve the problem using mental strategies."***

Have a few children share their strategies. They can also show their thinking using linking cubes or tens-frames. As children do, record their strategies.

You may want to highlight certain ideas if they do not come out naturally by asking questions such as the following:

- How can thinking about addition help us solve the problem?
- How can we back down through 10 to solve the problem?

Back down through 10

12 − 4 = ?

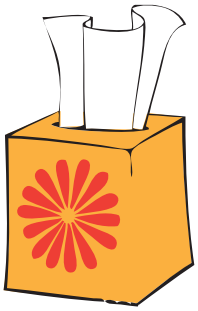
12 − 2 = 10

10 − 2 = 8

Engage

Introduce the Engage section with the following:

- ***"The monsters look in the closet and find some tissue boxes. They start ripping up some of the boxes and blowing the tissues in the air. Now there are 8 boxes of tissue."***
- ***"If we started with 16 tissue boxes, how many were ripped up?"***



You should have opportunities to connect to subtraction strategies such as think addition (doubles), back down through 10, and build up through 10.

- ***"How could we write an equation to represent what happened?"***
- ***"If we started with 12 tissue boxes, how many were ripped up? How do you know? How could we write an equation to represent what happened?"***

You may continue with other numbers from 10 to 20 as needed to gather some strategies for subtraction.

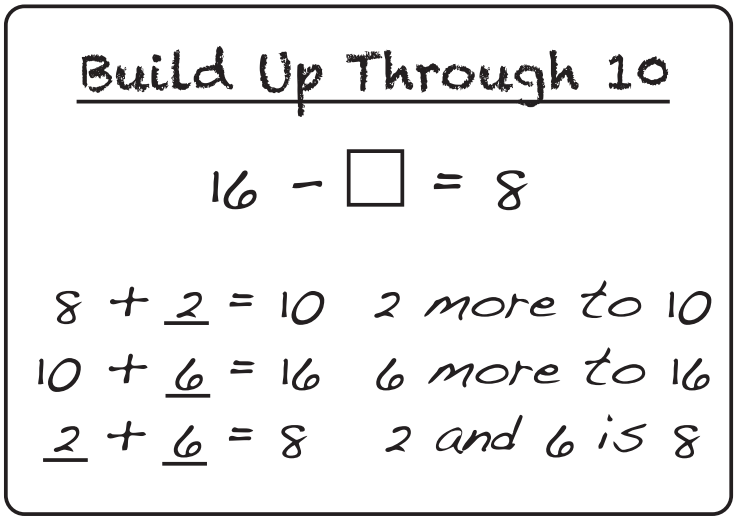
Develop

Introduce the Develop section with the following:

"What do you notice about the strategies we've used today? How could we group some of the strategies together?"

- You may need to do some prompting but ideally you are looking to create posters that remind children of some of the subtraction strategies they may use in the future. Possible categories/poster headings include *Think Addition, Difference is 10,* Build Up Through 10,* and *Back Down Through 10*. You do not need a comprehensive list. This is revisited in the next session.
- For each poster, ask children for examples that were not used today in class. Children may also use numbers, pictures, and words to describe the action. Record examples and descriptions on the posters and display. Group the posters together under a title of "Subtraction Strategies" or "Subtraction Strategies Within 20."

*For example, 14 – 4 or 17 – 7.



Session 4: Fluently Subtracting Within 20 (Part II)

At A Glance

In this session, children personalize some subtraction situations so that they can practice solving using mental strategies. The problem situations are take from, start unknown (☐ – B = A); compare, difference unknown (C – A = ☐); and compare, smaller unknown (C – B = ☐). Encourage children that are using inefficient strategies to refer to the posters created in Session 3. Time is available to evaluate strategies and revise or create new posters.

Common Core State Standards focus

Operations and Algebraic Thinking

Add and subtract within 20.

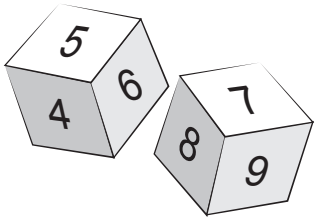
2.OA.2 Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

[2] See standard 1.OA.6 for a list of mental strategies.

MP 7 Look for and make use of structure.

Materials

- *One Hungry Monster* by Susan Heyboer O'Keefe
- Posters made in Session 3
- A six-sided die labeled with the numerals *4, 5, 6, 7, 8*, and *9*
- Blackline master, "Fluently Subtracting Within 20 (Part II)"



Activate

Introduce the Activate section with the following:

Say, ***"After ripping up the tissue boxes the monsters look at you, stamp their feet and boldly say, 'What ELSE is in this classroom?' "***

- Have children brainstorm items that they can gather in the classroom to keep the monsters busy. Record the list for reference.
- Children use the blackline, "Fluently Subtracting Within 20 (Part II)," to personalize subtraction problems.
- It is important that children do not get bogged down in the directions. So, feel free to walk them through that part.

Engage

Have children read and solve (without writing on the paper) the word problems they personalized to be sure they make sense.

Before trading problem sets, ask children not to write on these so they can be reused. You may want to have children sit in a circle so that they can pass the problem sets in the same direction.

- ***"Pass your paper once clockwise. Read No. 1 and solve using mental strategies. You may want to refer to the posters we created (in the last session)."***

You may briefly ask for and record a strategy or two that is used.

- ***"Now pass your paper once clockwise again. Read No. 2 and solve using mental strategies."***

Again, you may briefly ask for and record a strategy or two that is used. Continue this process as time permits.

Develop

Direct children's attention to the posters that were created in the last session.

- ***"Which of these strategies did you use the most? Which are you most comfortable with? Which are you least comfortable with?"***

Allow children to share a few examples or to connect the examples you recorded to the categories on the posters.

- ***"Is there anything we should add to the posters that we have? Is there another category that we should make a new poster for?"***
- ***"The monsters have had a great time thanks to you. They leave one by one only to go into the classroom next door!"***

Fluently Subtracting Within 20 (Part II)

Roll the die with 4, 5, 6, 7, 8, 9 three times and record the results in the spaces below.

1st roll _____ 2nd roll _____ 3rd roll _____

Choose three different classroom items and write them in the spaces below (for example, tissue boxes).

1st item _____ 2nd item _____ 3rd item _____

Personalize the word problems by using the information you recorded above.

1. Some _____ were on the table. The monsters grab _____ _____.
(1st item; 1st roll; 1st item)

Then there were 3 _____ left on the table. How many _____
(1st item; 1st item)

were on the table before the monsters began grabbing?

2. A red monster is juggling _____ _____. A blue monster is juggling
(2nd roll; 2nd item)

14 _____. How many fewer _____ is the red monster juggling
(2nd item; 2nd item)

than the blue monster?

3. A green monster has _____ more _____ than the pink monster.
(3rd roll; 3rd item)

The green monster has 12 _____. How many _____ does the pink
(3rd item; 3rd item)

monster have?

Session 5: Reasoning About Quantities (Number Line)

At A Glance

In this session, children think about where numbers will be placed on a number line based on relationships with other numbers. It is important throughout this session that children are sharing their reasoning for placement. They then think about where 10 more and 10 less than a number would be located. Children count up and down the number line by tens starting at an indicated number. Children explain how they know where to place each number.

Common Core State Standards focus

Measurement and Data

Relate addition and subtraction to length.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

MP 2 Reason abstractly and quantitatively.

Materials

- 10-foot or longer number line on wall or bulletin board with only 0 and 100 on either end of the number line
- Number cards from 1 to 99 (the number cards used in calendar or hundred pocket charts work well)
- Pushpins or some way to attach number cards to the number line

Activate

Present the cards 50, 25, 20, 10, and 40 in that order.

- With each card ask, ***"Where is this number located on the number line? How do you know?"***
- Encourage children to describe where each card would go rather than pointing or showing (e.g., "25 is halfway between 0 and 50.").
- Place each card on the number line based on the comments of the class.
- Discuss relationships of the numbers involved.

Engage

Hand out two cards (one card with number that is a multiple of 5 or 10 and one card with a number that is not a multiple of 5 or 10) to each pair of children.

- Have one pair place one of their cards on the number line and explain why they placed it at that location.
- Then ask the next pair to do the same until all cards are placed on the number line.

Develop

Direct children's attention to a number already on the number line (e.g., 67).

- ***"What is 10 more than this number? Where would it be on the number line? How many times could you count up by 10 before you get to 100?"***

Then count to be sure while pointing to a location on the number line for each number called out. Children help direct you where to place your finger. For example, "77 is a little to the left of 80" or "77 is a little closer to 75 than it is to 80."

- ***"What is 10 less than this number? Where would it be on the number line? How many times could you count down by 10 before you get to 0?"***

Again, count on the number line. Repeat with a few other numbers.

Session 6: Reasoning About Quantities (Odd and Even)

At A Glance

In this session, children think about what it means to be even and odd. They create sets of bugs and describe what makes those sets either even or odd. Children then represent these ideas on a number line and a hundreds chart and look for additional patterns that help define the terms *even* and *odd*.

Common Core State Standards focus

Operations and Algebraic Thinking

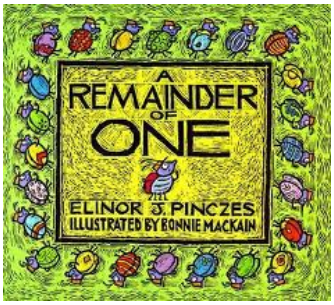
Work with equal groups of objects to gain foundations for multiplication.

2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MP 2 Reason abstractly and quantitatively.

Materials

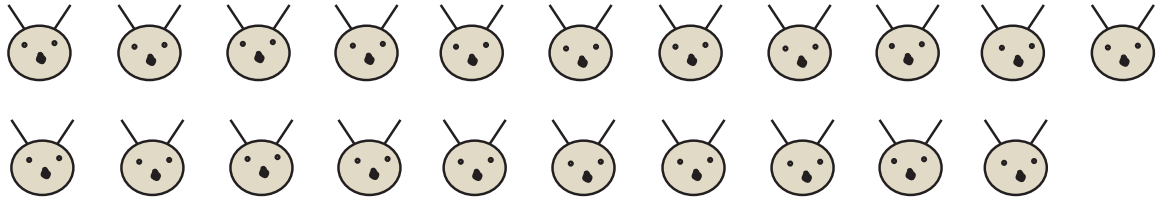
- *A Remainder of One* by Elinor J. Pinczes
- One bag of bugs (manipulatives) per two children (Each bag should have a different number of bugs from 12 to 20, although some repetition of numbers may be necessary.)
- Two different light-colored highlighters, markers, map pencils, or crayons for each child
- Blackline master, "Hundreds Chart"



Activate

Read the first 10 pages of *A Remainder of One*. It begins with the story of Joe, a soldier bug that wants to join in a parade. His squadron lines up by twos, but since there are 25 bugs and Joe hangs off the end, the queen does not like the arrangement. Joe is asked by Sergeant Steven to stand aside so "the troop will be even." Stop reading here.

- Ask, ***"What does it mean for a number to be even?"***
- ***"What could we call it if it wasn't even?"***
- ***"What could Joe do to make the troop even?"*** (Possible answer: Joe could find a partner, and they could both join the troop for the parade.)



Engage

Organize children in pairs, and hand each pair a bag of bugs. Each bag should have a different number of bugs from 12 to 20, although some repetition of numbers may be necessary.

- ***"Take your bugs and make an even troop with two lines that the queen would like. You do not have to use all of your bugs. How many are in your troop? Write an equation to represent your troop such as 6 + 6 = 12. How could you represent the equation on a number line?"***

Have everyone share the number from the troop they made. Record all the numbers so everyone can see them.

- ***"What are some things you notice about these numbers?"***

Have students share the equations they wrote.

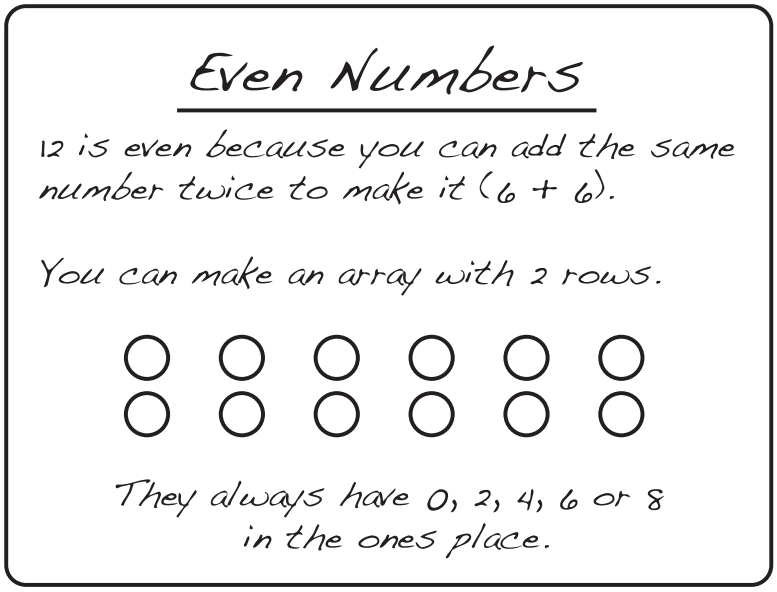
- ***"What do you notice about the equations we made?"***
- ***"Now take your bugs and make a nonexample, an odd troop with two lines that the queen would not like. How many are in your troop? Write an equation to represent your troop such as 6 + 7 = 13 or 6 + 6 + 1 = 13."***

Have everyone share the number from the troop they made. Record all the numbers so everyone can see them.

- ***"What are some things you notice about these numbers?"***

- ***"Look at both sets of numbers and think about the arrangements you made. How are even numbers different from odd numbers? How are the equations different from the ones you made for even numbers?"***

Make a poster about even numbers with some examples in number and picture based on children's observations. Do the same for odd numbers.



Develop

Each student takes a hundreds chart. Have extra copies for mistakes.

Children use two different light-colored highlighters, markers, map pencils, or crayons. They color in the even numbers the class found on the chart in one color and the odd numbers the class found in the other color.

- ***"What do you notice? Is 21 even or odd? How do you know?"***
- ***"Consider the posters you made to be sure. Now color 21 in on your chart."***

Ask the same questions about a few more numbers that are not already on the chart.

- ***"If we count by fives, will we say even numbers or odd numbers?"***

Add any discoveries made from using the hundreds chart to the even and odd posters.

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

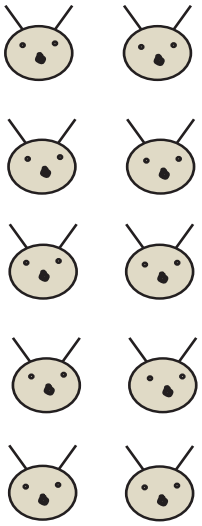
Session 7: Reasoning About Quantities (Arrays)

At A Glance

In this session, children analyze the final array made by the bugs in *A Remainder of One*. They connect it to the number 25 and think about why this is the only arrangement that worked for 25. Children also think about the addition sentence that the arrangement implies. They then use some bugs to create their own arrays.

Materials

- *A Remainder of One* by Elinor J. Pinczes
- A bag of 25 bugs for each pair of children



Common Core State Standards focus

Operations and Algebraic Thinking

Work with equal groups of objects to gain foundations for multiplication.

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

MP 2 Reason abstractly and quantitatively.

Activate

Start from the beginning of *A Remainder of One,* but this time read to the end.

Have children recreate the final arrangement that the queen liked using their own bugs.

- Ask, ***"With 25 soldier bugs, why didn't rows of two, three, or four work?"***

Children should at least know from the last session that rows of two would not work because 25 is an odd number.

- ***"What can we call the arrangement that the queen liked?"*** (Possible responses: Array, rectangular array, rectangle, square, 5 by 5)

If children do not know the term *array* or *rectangular array*, introduce it here.

- ***"How can we show this array in an equation (number sentence)?"***

$$5 + 5 + 5 + 5 + 5 = 25$$

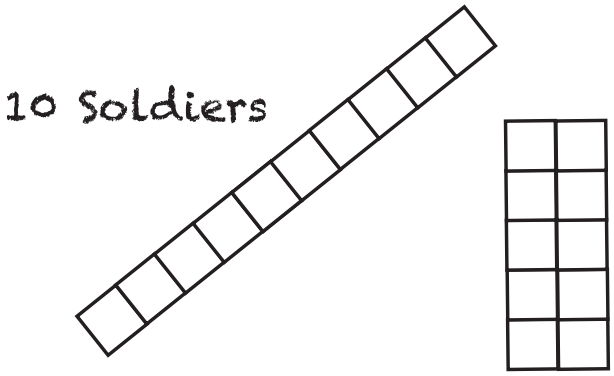
Engage

Introduce the Engage section with the following:

- ***"Let's say your bug army has only 10 soldiers. What rectangular arrays can you make using all 10 bugs? Write an addition number sentence for each array you find."***

Have children do the same thing with 9 soldiers, then 8 soldiers, and then 7 soldiers.

- ***"What do you notice about a bug army with 7 soldiers?"***



Develop

Introduce the Develop section with the following:

- ***"In* A Remainder of One*, Joe's squadron of 25 soldiers made an array that is a square. What other numbers less than 25 can also make a square array?"***
- ***"Use your bugs and write an equation to show each array that makes a square."***
- ***"How do you know you have found them all?"***

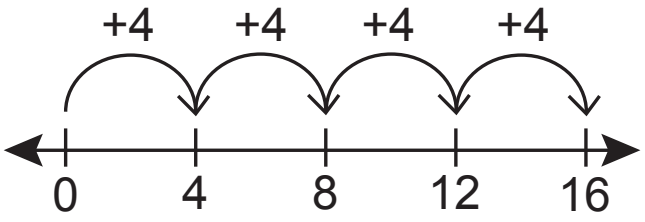
$$5 + 5 + 5 + 5 + 5 = 25$$

$$4 + 4 + 4 + 4 = 16$$

$$3 + 3 + 3 = 9$$

$$2 + 2 = 4$$

- ***"What do you notice about the equations (number sentences) you created?"***
- ***"How could you represent these equations on a number line?"***



Session 8: Fluently Adding Within 100

At A Glance

In this session, children consider place value when solving addition problems within 100. Children share their strategies with others and explain how they work. Children also represent their strategies on a number line. Strategies are recorded for future reference. The context is thin and only for the purpose of making the operations more meaningful, not more complex.

Materials

- Linking cubes
- Hundreds chart

Common Core State Standards focus

Operations and Algebraic Thinking

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

[3] Explanations may be supported by drawings or objects.

Measurement and Data

Relate addition and subtraction to length.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

MP 7 Look for and make use of structure.

Activate

Introduce the Activate section with the following:

- Say, ***"Joe is in a squadron of 25 bugs. What do you notice about the number 25?"*** (Possible responses: It is made up of 20 and 5; There are 2 tens and 5 ones.)
- ***"How about the number 32? How is it like 25? How is it different?"*** (Possible responses: Both are 2 digit numbers; 32 is larger; 32 has 3 tens and 25 has 2 tens; 32 has 2 ones and 25 has 5 ones.)
- ***"Let's imagine two squadrons of bugs joined for a parade. There were 25 bugs in Joe's squadron and 32 bugs in Nina's squadron. How many bugs were in the parade?"***
- ***"What equation can we write to represent this situation?"***

Write or show the equation *25 + 32 = □.*

- ***"Use what you know about these numbers to solve the problem."***

Have a few children share their strategies. They can also show their thinking using linking cubes or a hundreds chart. As they do, record their strategies.

Ideally, you want children to express strategies that preserve place value and come from their own thinking, not directed by you. However, you may want to highlight certain ideas if they do not come out naturally by asking questions such as the following:

- How about if we added the tens first [20 + 30]? How does that help us solve the problem? (Add tens, add ones, then combine)
- How about if we started with 25 and added on the tens [25 + 30]? How does that help us solve the problem? (Add on tens, then add ones)

Add Tens, Add Ones, Then Combine	Add On Tens, Then Add Ones
25 + 32 20 + 30 = 50 5 + 2 = 7 50 + 7 = 57 57	25 + 32 25 + 30 = 55 55 + 2 = 57 57

Engage

Introduce the Engage section with the following:

- ***"Let's imagine two different squadrons of bugs joined for a parade. There were 58 bugs in Joe's squadron and 26 bugs in Nina's squadron. How many bugs were in the parade?"***
- ***"What equation can we write to represent this situation?"***

Write or show the equation *58 + 26 = □*.

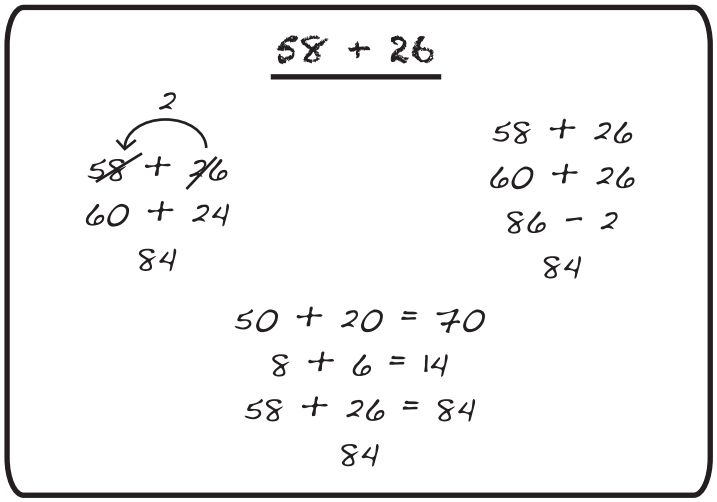
- ***"Use what you know about these numbers to solve the problem."***

Have a few children share their strategies. They can also show their thinking using linking cubes or a hundreds chart. As they do, record their strategies.

Ideally, you want children to express strategies that preserve place value and come from their own thinking, not directed by you. However, you may want to highlight certain ideas if they do not come out naturally by asking questions such as the following:

- How can we make one of these numbers easier to work with? (Move some to make tens)

You may want to make one poster for the expression that was solved containing several strategies.



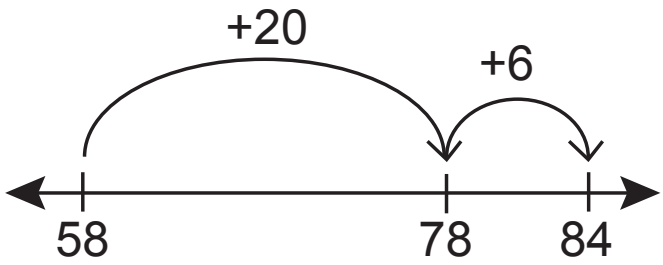
Develop

Introduce the Develop section with the following:

- ***"Take a look at some of the strategies we've talked about today. What would these strategies look like represented on a number line?"***

Have children consider one of the strategies and discuss how to represent that strategy on the number line.

You may want to include the number line representation with strategies you have posted.



Session 9: Fluently Subtracting Within 100 (Think Addition)

At A Glance

In this session, children consider place value and the relationship between addition and subtraction when solving subtraction problems within 100. The focus is on using adding-up strategies to solve. Children share their strategies with others and explain how they work. Children also represent their strategies on a number line. Strategies are recorded for future reference. The context is thin and only for the purpose of making the operations more meaningful, not more complex.

Materials

- Linking cubes
- Hundreds chart

Common Core State Standards focus

Operations and Algebraic Thinking

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

[3] Explanations may be supported by drawings or objects.

Measurement and Data

Relate addition and subtraction to length.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

MP 7 Look for and make use of structure.

Activate

Introduce the Activate section with the following:

- Say, ***"After the parade, the bugs have a celebration. Joe brings 37 leaves for the other bugs to munch on. Carlos was the only other bug to bring leaves. There were 72 leaves in all at the celebration. How many leaves did Carlos bring?"***
- ***"What equation can we write to represent this situation? Is there another one we could write?"***

Write or show the equations *72 – 37 = □* and *37 + □ = 72.*

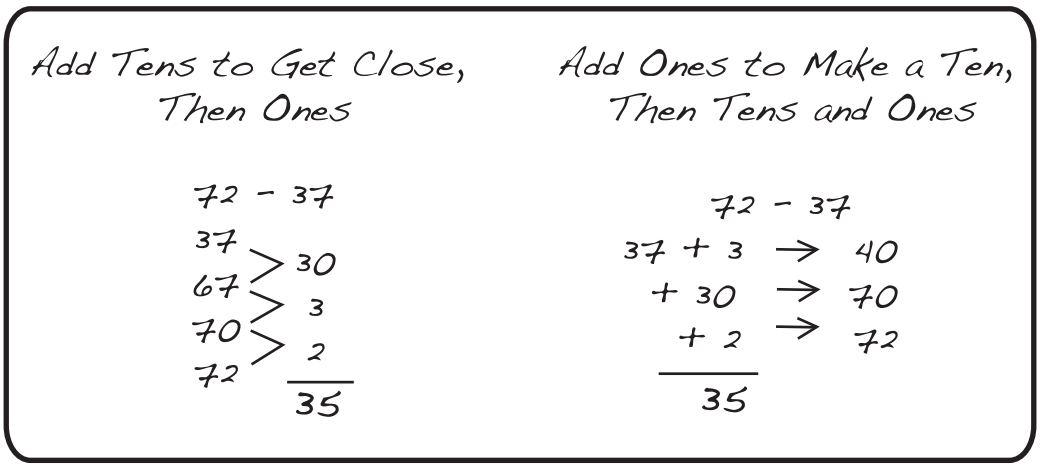
- ***"How can we use addition to help us think about this problem? Today, I'd like for you to think of strategies for adding up from 37 to 72."***

Keep the focus on children using what they know about addition in this subtraction situation. The next session addresses take-away strategies.

Have a few children share their strategies. They can also show their thinking using linking cubes or a hundreds chart. As they do, record their strategies.

Ideally, you want children to express strategies that preserve place value and come from their own thinking, not directed by you. However, you may want to highlight certain ideas if they do not come out naturally by asking questions such as the following:

- What if we started with 37 and added tens to get close to 72? How does that help us solve the problem? (Add tens to get close, then ones)
- What if we started by adding ones to make a 10?" (Add ones to make a 10, then tens and ones)



Engage

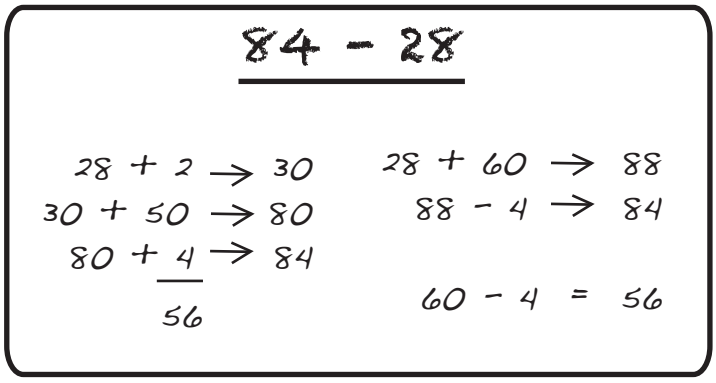
Introduce the Engage section with the following:

- ***"Before the celebration began, Joe wanted to fill all the bowls with pollen. There are 84 bowls in all. 28 have already been filled with pollen. How many more bowls does Joe need to fill?"***
- ***"What equation can we write to represent this situation? Is there another one we could write?"***

Write or show the equations *84 – 28 = □* and *28 + □ = 84.*

Have a few children share their strategies. They can also show their thinking using linking cubes or a hundreds chart. As children do, record their strategies.

You may want to make one poster for the equation that was solved containing several strategies.



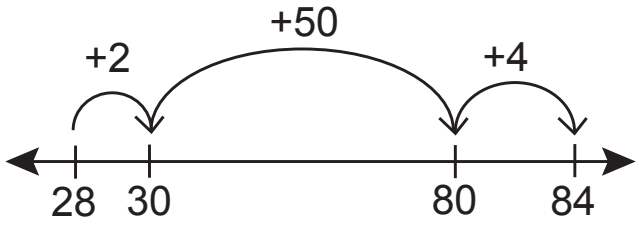
Develop

Introduce the Develop section with the following:

- ***"Take a look at some of the strategies we've talked about today. What would these strategies look like represented on a number line?"***

Have children consider one of the strategies and discuss how to represent that strategy on the number line.

You may want to include the number line representation with strategies you have posted.



Session 10: Fluently Subtracting Within 100 (Take Away)

At A Glance

In this session, children consider place value and the relationship between addition and subtraction when solving subtraction problems within 100. The focus is on using take-away strategies to solve. Children share their strategies with others and explain how they work. Children also represent their strategies on a number line. Strategies are recorded for future reference. The context is thin and only for the purpose of making the operations more meaningful, not more complex.

Materials

- Linking cubes
- Hundreds chart

Common Core State Standards focus

Operations and Algebraic Thinking

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

[3] Explanations may be supported by drawings or objects.

Measurement and Data

Relate addition and subtraction to length.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

MP 7 Look for and make use of structure.

Activate

Introduce the Activate section with the following:

- Say, ***"There were 93 bugs at the celebration. 26 bugs left before the dance. How many bugs were at the celebration when the dance started?"***
- ***"What equation can we write to represent this situation?"***

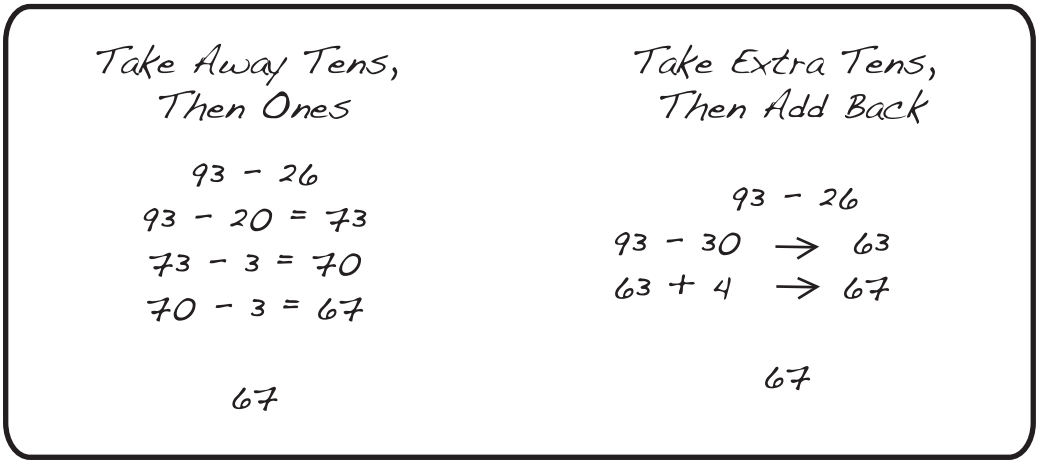
Write or show the equation *93 – 26 = □.*

- ***"In the last session, we thought of subtraction 'adding up.' Today I'd like you to think of ways you can 'take away' to find the solution."***

Have a few children share their strategies. They can also show their thinking using linking cubes or a hundreds chart. As children do, record their strategies.

Ideally, you want children to express strategies that preserve place value and come from their own thinking, not directed by you. However, you may want to highlight certain ideas if they do not come out naturally by asking questions such as the following:

- What if we start with 93 and take away the tens first [93 – 20]? How does that help us solve the problem? (Take away tens, then ones)
- What if we take extra tens away from 93 [93 – 30]? How does that help us solve the problem? (Take extra tens, then add back)



Engage

Introduce the Engage section with the following:

- ***"There were 67 bugs at the celebration when the dance started. By the end of the dance, there were 48 bugs at the celebration. How many bugs left the celebration during the dance?"***
- ***"What equation can we write to represent this situation?"***

Write or show the equation *67 – □ = 48*.

- ***"Solve this problem using a 'think addition' strategy and then solve it using a 'take-away' strategy."***

Have a few children share their strategies. They can also show their thinking using linking cubes or a hundreds chart. As children do, record their strategies.

You may want to add some take-away strategies to the poster with the expression *84 – 28*.

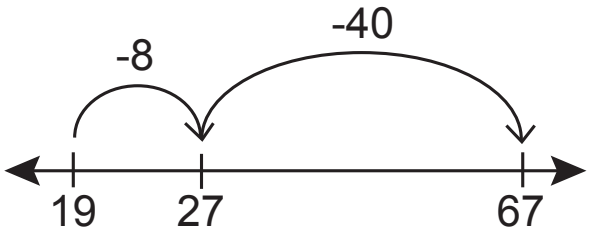
Develop

Introduce the Develop section with the following:

- ***"Take a look at some of the strategies we've talked about today. What would these strategies look like represented on a number line?"***

Have children consider one of the strategies and discuss how to represent that strategy on the number line.

You may want to include the number line representation with strategies you have posted.



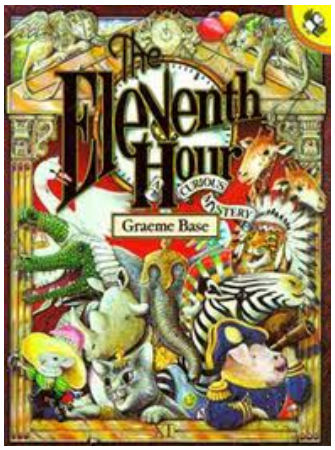
Sessions 11 and 12: Solving Word Problems (Part I)

At A Glance

In these sessions, children solve one- and two-step word problems that include two problem types (add to/take from and put together/take apart). All quantities are within 100. Most two-step problems can be represented with a single equation, but some may be better represented with a combination of a drawing or diagram and an equation. It is important to allow time for children to test and share different ways of representing the situations presented. On Day 1 (Session 11), children work as pairs. On Day 2 (Session 12), they work independently. On both days, some children may require support with two-step problems.

Materials

- One copy of Problem Set A for each pair of children
- One copy of Problem Set B for each child
- *The Eleventh Hour* by Graeme Base



Common Core State Standards focus

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Glossary, Table 1.

MP 1 Make sense of problems and persevere in solving them.

Activate

Day 1

Show children the cover of *The Eleventh Hour.*

- Ask, ***"What do you think this story is going to be about?"***
- Probe for comments about the picture on the cover and the title of the book.
- Read *The Eleventh Hour.* Horace, the elephant, invites his friends to his eleventh birthday party, for which Horace has made a feast and planned many games. After the games, they unfortunately discover that the feast is gone, and Horace makes them all sandwiches instead. They all have a nice picnic and leave satisfied.
- Say, ***"All of the animals had so much fun at Horace's eleventh birthday party that they decided that they would get together and throw a big birthday picnic for Horace on his twelfth birthday. As Horace's twelfth birthday approaches, the animals begin to prepare for the big party.***

  The Pig made 9 tuna sandwiches, and the Zebra made 7 tomato sandwiches for the picnic. When it was time to place the sandwiches in the picnic basket, 8 sandwiches had already been eaten. How many sandwiches were left when it was time to place them in the picnic basket?"

Show this problem in written form as well.

- ***"What's going on in this problem? What do you see in your head (visualize)?"***
- ***"How can we represent or show what's going on in the problem?"***
- ***"Now how would we solve it?"***

Have children share strategies and record them. Be sure that there are examples of using a drawing, an equation, a line diagram, or a combination.

- ***"Does our answer make sense? Let's look back to the problem to be sure."***

Day 2

Use No. 3 from Problem Set A.

There were 7 gingerbread cookies and 4 sugar cookies in the picnic basket. The Cat put in 9 more cookies. How many cookies are in the picnic basket altogether?

- ***"What strategy did you use to solve this problem?"***
- ***"Why did you decide to do that instead of something else?"***
- ***"What other strategies could be used?"***

Have children share strategies and record them. Be sure that there are examples of using a drawing, an equation, a line diagram, or a combination.

- ***"Does our answer make sense? Let's look back to the problem to be sure."***

Engage

Day 1

Say, ***"Here are some word problems about the party preparations. Make sure you show the strategies you used to solve."***

Hand out Problem Set A to pairs of children. Have them solve the three problems and show their thinking/strategies.

Day 2

Remind children that the animals are preparing a picnic for Horace's twelfth birthday.

Hand out Problem Set B to each child. Have them solve the three problems and show their thinking/strategies.

Develop

Day 1

Focus the conversation on Nos. 1 and 2 from Problem Set A. No. 3 is used in the next session.

Ask questions such as the following:

- What did you do first?
- What strategy did you use to solve this problem?
- Why did you decide to do that instead of something else?
- What information helped you decide what to do?
- How did you decide that your answer made sense?
- What other strategies could be used?

Day 2

Focus the conversation on Nos. 1 and 2 from Problem Set B. Ask the questions above to draw out student thinking. No. 3 is used in the next session.

Problem Set A

1. There were 19 celery sticks in a bag for the picnic. The Rhino ate 6 of those celery sticks. Then the Swan put 8 celery sticks in the bag. How many celery sticks are now in the bag?

2. The Mouse is making the invitations for the picnic. There are 75 invitations that need to be written. The Mouse has already written 38 invitations. How many more invitations does the Mouse need to write?

3. There were 7 gingerbread cookies and 4 sugar cookies in the picnic basket. The Cat put in 9 more cookies. How many cookies are in the picnic basket altogether?

Problem Set B

1. The Rhino made 12 sandwiches, and the Crocodile made 6 sandwiches. The Bengal Tiger put 14 of the sandwiches in the picnic basket. How many sandwiches are not in the picnic basket?

2. The Mouse is bringing paper cups to the picnic. He keeps his paper cups on two shelves. The Mouse has 36 paper cups on the bottom shelf and 48 paper cups on the top shelf. How many paper cups does he have altogether?

3. The Swan buys 17 lemons for lemonade. She squeezes 8 lemons in the morning and 6 lemons in the afternoon. How many lemons have not been squeezed?

Sessions 13 and 14: Solving Word Problems (Part II)

At A Glance

In these sessions, children solve two-step word problems that include all three problem types (add to/take from, put together/ take apart, and compare). All quantities are within 100. Most two-step problems can be represented with a single equation, but some may be better represented with a combination of a drawing or diagram and an equation. It is important to allow time for children to test and share different ways of representing the situations presented. On Day 1 (Session 13), children work as pairs. On Day 2 (Session 14), they work independently. On both days, some children may require support with two-step problems.

Materials

- One copy of Problem Set C for each pair of children
- One copy of Problem Set D for each child
- *The Eleventh Hour* by Graeme Base

Common Core State Standards focus

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Glossary, Table 1.

MP 1 Make sense of problems and persevere in solving them.

Activate

Day 1

Show No. 3 from Problem Set B in Session 12.

> The Swan buys 17 lemons for lemonade. She squeezes 8 lemons in the morning and 6 lemons in the afternoon. How many lemons have not been squeezed?

- Ask, ***"What strategy did you use to solve this problem?"***
- ***"Why did you decide to do that instead of something else?"***
- ***"What other strategies could be used?"***

Day 2

Use No. 3 from Problem Set C.

> The only bears at the picnic were 9 black bears and some brown bears. In all, 18 bears were at the picnic. Then some more brown bears came to the picnic after the volleyball game. Now there are 16 brown bears at the picnic. How many brown bears came to the picnic after the volleyball game?

- ***"What strategy did you use to solve this problem?"***
- ***"Why did you decide to do that instead of something else?"***
- ***"What other strategies could be used?"***

Have children share strategies and record them. Be sure that there are examples of using a drawing, an equation, a line diagram, or a combination.

- ***"Does our answer make sense? Let's look back to the problem to be sure."***

Engage

Day 1

Introduce the Engage section with the following:

- ***"Here are some word problems about what happened at the picnic. Make sure you show the strategies you used to solve."***

Hand out Problem Set C to pairs of children. Have them solve the three problems and show their thinking/strategies.

Day 2

Remind children that the animals are having a birthday picnic for Horace.

Hand out Problem Set D to each child. Have them solve the three problems and show their thinking/strategies.

Develop

Day 1

Focus the conversation on Nos. 1 and 2 from Problem Set C. No. 3 is used in the next session.

Ask questions such as the following:

- What did you do first?
- What strategy did you use to solve this problem?
- Why did you decide to do that instead of something else?
- What information helped you decide what to do?
- How did you decide that your answer made sense?
- What other strategies could be used?

Day 2

Focus the conversation on Nos. 1 and 2 from Problem Set D. Ask the questions above to draw out student thinking. No. 3 is used in the next session.

Problem Set C

1. There were 4 sugar cookies and some gingerbread cookies on the blanket. There were 12 cookies on the blanket. Then the Bengal Tiger placed 6 more gingerbread cookies on the blanket. How many gingerbread cookies are now on the blanket?

2. The Zebra took 43 pictures at the picnic. The Mouse took 7 fewer pictures than the Zebra. How many pictures did they take in all?

3. The only bears at the picnic were 9 black bears and some brown bears. In all, 18 bears were at the picnic. Then some more brown bears came to the picnic after the volleyball game. Now there are 16 brown bears at the picnic. How many brown bears came to the picnic after the volleyball game?

Problem Set D

1. The Pig poured 64 cups of lemonade. The Cat poured 37 more cups of lemonade than the Pig. How many cups of lemonade did they pour in all?

2. There were 6 salty crackers and some buttery crackers on the table. There were 23 crackers on the table. The Zebra handed out 8 buttery crackers. How many buttery crackers are now on the table?

3. The only frogs at the picnic were 8 bullfrogs and some tree frogs. In all, there were 22 frogs at the picnic. Then some of the tree frogs left. Now there are 9 tree frogs. How many tree frogs left the picnic?

Sessions 15 and 16: Solving Word Problems (Length)

At A Glance

In these sessions, children solve one- and two-step word problems that include all three problem types (add to/take from, put together/take apart, and compare) and involve length. All quantities are within 100, and all lengths are within the same units. Most two-step problems can be represented with a single equation, but some may be better represented with a combination of a drawing or diagram and an equation. It is important to allow time for children to test and share different ways of representing the situations presented. On Day 1 (Session 15), children work as pairs. On Day 2 (Session 16), they work independently. On both days, some children may require support with two-step problems.

Materials

- One copy of Problem Set E for each pair of children
- One copy of Problem Set F for each child
- *The Eleventh Hour* by Graeme Base

Common Core State Standards focus

Measurement and Data

Relate addition and subtraction to length.

2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

MP 1 Make sense of problems and persevere in solving them.

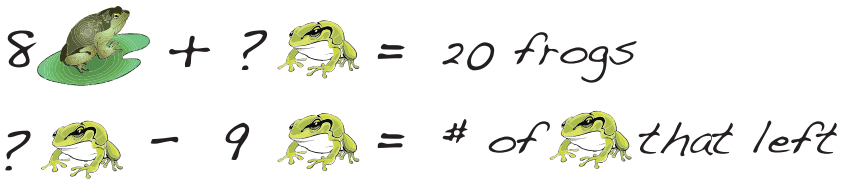
Activate

Day 1

Show No. 3 from Problem Set D in Session 14.

> The only frogs at the picnic were 8 bullfrogs and some tree frogs. In all, there were 22 frogs at the picnic. Then some of the tree frogs left. Now there are 9 tree frogs. How many tree frogs left the picnic?

- Ask, ***"What strategy did you use to solve this problem?"***
- ***"Why did you decide to do that instead of something else?"***
- ***"What other strategies could be used?"***



Day 2

Use No. 3 from Problem Set E.

> The Swan flew straight toward the Fly who was sitting on a rock 15 meters away. The Fly, not paying attention, began to fly straight toward the Swan. When the Swan had flown 10 meters, the Fly was 4 meters away from the rock. How close are the Swan and the Fly?

- ***"What strategy did you use to solve this problem?"***
- ***"Why did you decide to do that instead of something else?"***
- ***"What other strategies could be used?"***

Have children share strategies and record them. Be sure that there are examples of using a drawing, an equation, a line diagram, or a combination.

- ***"Does our answer make sense? Let's look back to the problem to be sure."***

Engage

Day 1

Introduce the Engage section with the following:

- ***"While all the animals are relaxing at the picnic, the pesky Fly shows up. The Fly sees the yummy food and moves toward it. Here are some word problems about what happened after the Fly came to the picnic uninvited. Make sure you show the strategies you used to solve."***

Hand out Problem Set E to pairs of children. Have them solve the three problems and show their thinking/strategies.

Day 2

Remind children that the animals are having a picnic when the Fly shows up uninvited.

Hand out Problem Set F to each child. Have them solve the three problems and show their thinking/strategies.

Develop

Day 1

Focus the conversation on Nos. 1 and 2 from Problem Set E. No. 3 is used in the next session.

Ask questions such as the following:

- What did you do first?
- What strategy did you use to solve this problem?
- Why did you decide to do that instead of something else?
- What information helped you decide what to do?
- How did you decide that your answer made sense?
- What other strategies could be used?

Day 2

Focus the conversation on the problems from Problem Set F. Ask the questions above to draw out student thinking.

Problem Set E

1. The Rhino ran 56 meters chasing the Fly. The Crocodile was chasing the Fly too. Altogether, the Rhino and the Crocodile ran 92 meters chasing the Fly. How many more meters did the Rhino run than the Crocodile?

2. The Fly on the Pig's plate crawled 5 inches toward the pie and stopped. Then he crawled 7 inches toward the pie and stopped. The pie is still 8 inches away from the Fly. How far is it from the Pig's plate to the pie?

3. The Swan flew straight toward the Fly who was sitting on a rock 15 meters away. The Fly, not paying attention, began to fly straight toward the Swan. When the Swan had flown 10 meters, the Fly was 4 meters away from the rock. How close are the Swan and the Fly?

Problem Set F

1. The cake was 92 centimeters away from the Fly. The Fly crawled 68 centimeters toward the cake. How much farther does the Fly need to crawl to get to the cake?

2. The Bengal Tiger chased the Fly 25 feet away from the picnic basket. Then the Fly flew 19 feet straight back toward the picnic basket. How far is the Fly from the picnic basket now?

3. The Fly decided to take a ride on the Frisbee. The Crocodile tossed the Frisbee 23 feet with the Fly on it. The Pig caught the Frisbee and tossed it to the Zebra. The Zebra then tossed the Frisbee 29 feet to the Cat. When the Cat caught the Frisbee, the Fly had ridden the Frisbee for 86 feet. How far did the Fly ride on the Frisbee from the Pig to the Zebra?

Session 17: Understanding Place Value (Represent)

At A Glance

In this session, children use linking cubes to represent the value of each digit in the hundreds, tens, and ones places. They then combine the sets to represent the number written. Children describe why the number of cubes they selected represents the written number.

Materials

- Linking cubes
- Decahedron die (with digits 0–9) per three children (Spinners with the numbers 0–9 may be used instead.)

Common Core State Standards focus

Number and Operations in Base Ten

Understand place value.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens—called a "hundred."

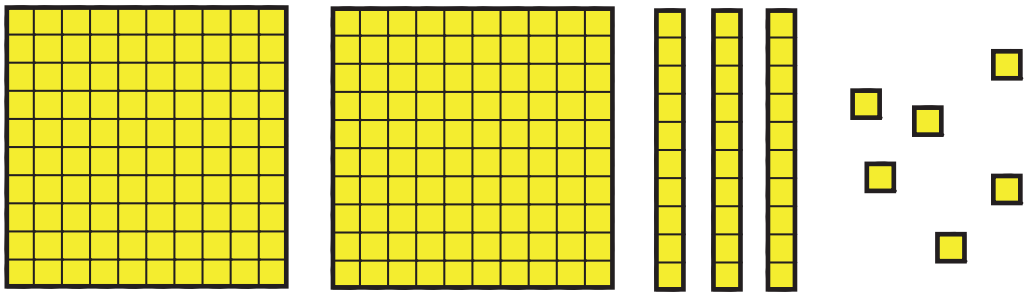
b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

MP 6 Attend to precision.

Activate

Show the number 236.

- Ask, ***"What do we know about the number 236?"***
- ***"What is the value of the digit 6 in the number 236?"*** (6 or 6 ones)
- ***"How can we show that value using the linking cubes?"***
- ***"What is the value of the digit 3 in the number 236?"*** (30 or 3 tens)
- ***"How can we show that value using the linking cubes?"***
- ***"What is the value of the digit 2 in the number 236?"*** (200 or 2 hundreds)
- ***"How can we show that value using the linking cubes?"***
- ***"So, how can we show the value of the number 236 with linking cubes?"***



Engage

Organize children into groups of three. One child rolls the die and places that digit in the hundreds place. The next child rolls the die and places that digit in the tens place. The last child rolls the die and places that digit in the ones place.

- Each child gathers linking cubes for the value of the digit they placed. Then they combine the sets to form the number rolled.
- All three children check the total number to be sure it matches the written number. All three say out loud together the number they created.
- Children rotate roles so that each child has at least one turn at the hundreds, tens, and ones places.

Develop

Meet as a whole group.

- Ask one group, ***"What is a number your group created?"***

Record the number for all children to see.

- ***"How did you decide how many linking cubes you would need?"***
- ***"How did you represent the number with linking cubes?"***
- ***"Describe how the cubes you selected represent this number."***

Have other groups share while asking them questions similar to those above.

Session 18: Understanding Place Value (Read and Write)

At A Glance

In this session, children consider various representations of a number. They connect the number written with numerals to the number written with words. Children also connect their previous work with linking cubes to represent the number in expanded form. They use the representations to accurately describe the number.

Materials

- Linking cubes
- Decahedron die (with digits 0–9) per three children (Spinners with the numbers 0–9 may be used instead.)

Common Core State Standards focus

Number and Operations in Base Ten

Understand place value.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens—called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

MP 6 Attend to precision.

Activate

Show the number 382.

- Ask, ***"How would we say this number?"***
- ***"How would we write this number in words?"***
- ***"How could we describe the value of this number?"***
- ***"How could we represent this number using the linking cubes?"***
- ***"How could we represent this number using expanded form?"***

Repeat with other three-digit numbers as needed.

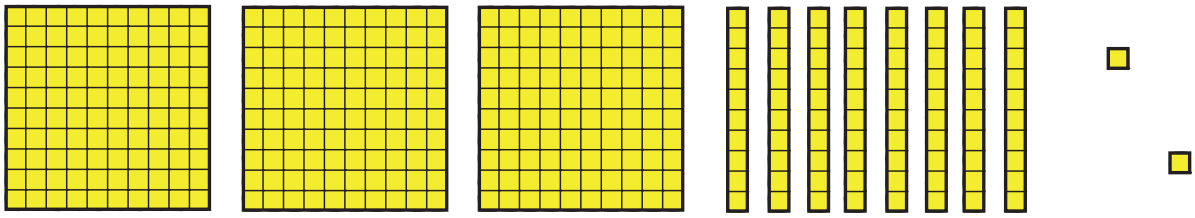
Three hundred eighty-two

300 + 80 + 2

Engage

Organize children into groups of three. One child rolls the die and places that digit in the hundreds place. The next child rolls the die and places that digit in the tens place. The last child rolls the die and places that digit in the ones place.

- All three children write the number in words and compare with each other. All three write the number in expanded form and compare with each other.
- Children use the linking cubes to represent the number they created.
- Children then compare the linking cubes representation or drawing of the number to the expanded form and the written number to ensure they all accurately represent the same number.
- Children rotate roles so that each child has at least one turn at the hundreds, tens, and ones places.



Develop

Meet as a whole group.

- Ask one group, ***"What is a number your group created?"***

Record the number for all children to see.

- ***"How did you represent this number in words?"***
- ***"How did you represent this number in expanded form?"***
- ***"How did you represent the number with linking cubes?"***

Have other groups share while asking them questions similar to those above.

Session 19: Understanding Place Value (Compare)

At A Glance

In this session, children describe numbers and think about their similarities and differences. Then they discuss the value of number to compare. When comparing, children use the symbols <, >, and = in an appropriate way. Finally, children prove that the relationship they have expressed is true using linking cubes and/or drawings.

Materials

- Linking cubes
- Decahedron die (with digits 0–9) per three children (Spinners with the numbers 0–9 may be used instead.)

Common Core State Standards focus

Number and Operations in Base Ten

Understand place value.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

MP 6 Attend to precision.

Activate

Show the number 429.

- Ask, ***"How would you describe this number and its value?"***

Show the number 434.

- ***"How would you describe this number and its value?"***
- ***"What similarities do you notice between these two numbers? What differences do you notice?"***
- ***"How can you tell which number is greater since both have the digit 4 in the hundreds place?"***
- ***"Why is 429 less than 434 even though 439 has a 9, the largest digit?"***
- ***"What symbol could we use to show the relationship between these two numbers?"***

$429 < 434$

Engage

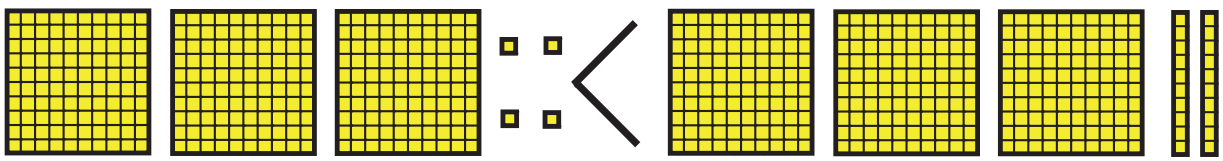
Organize children into groups of three. Hand out the blackline, "Understanding Place Value (Compare)."

- Children take turns rolling the die and placing the digit in the blanks provided.
- Children use the appropriate symbol to compare the two numbers.

Develop

Meet as a whole group.

- Ask groups of children to share the numbers they used and the symbol they chose to place between the numbers.
- Have children prove the relationship using linking cubes and/or drawings.
- ***"Are there any that you could compare even before you roll the die? How do you know?""***



Understanding Place Value (Compare)

Place the symbol **<**, **>**, or **=** between the numbers to make a comparison.

1. 58 74

2. 95 92

3. 136 88

4. 269 296

5. 304 320

Roll the die for each blank and record. Place the symbol **<**, **>**, or **=** between the numbers to make a comparison.

6. 4 ___ 2 4 ___ 2

7. ___ 71 ___ 71

8. 82 ___ 81 ___

9. ___ ___ ___ ___ ___ ___

10. 6 ___ ___ 6 ___ 9

Session 20: Understanding Place Value (Mentally Compute)

At A Glance

In this session, children use what they know about the numbers and their values to mentally add and subtract both 100 and 10. By analyzing the patterns that develop, they discover the similarities and differences between the use of addition and subtraction and the use of 10 rather than 100. Children determine how this operation changes the value and representation of the number to develop mental strategies.

Common Core State Standards focus

Number and Operations in Base Ten

Understand place value.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Use place value understanding and properties of operations to add and subtract.

2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

MP 8 Look for and express regularity in repeated reasoning.

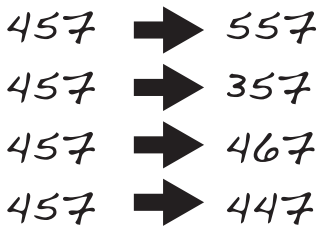
Materials

- Adding machine tape
- Decahedron die (with digits 0–9) per three children (Spinners with the numbers 0–9 may be used instead.)
- Linking cubes

Activate

Show the number 457.

- Ask, ***"What do we know about this number?"***
- ***"What would this number look like if we added 100? How do you know?"***
- ***"Why does only the digit in the hundreds place change?"***
- ***"Can you prove it with linking cubes or a drawing?***
- ***"Let's go back to 457. What would this number look like if we subtracted 100? How do you know?"***
- ***"Can you prove it with linking cubes or a drawing?***
- ***"Let's go back to 457. What would this number look like if we added 10? How do you know?"***
- ***"Why does only the digit in the tens place change?"***
- ***"Can you prove it with linking cubes or a drawing?***
- ***"Let's go back to 457. What would this number look like if we subtracted 10? How do you know?"***
- ***"Can you prove it with linking cubes or a drawing?***

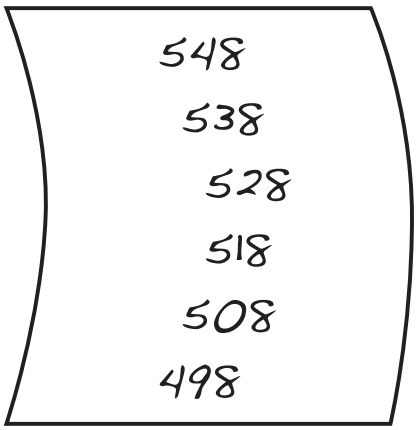


Engage

Organize children into groups of three. Hand each group a roll of adding machine tape.

- Children use the dice to roll a three-digit number and place it at the bottom of the strip.
- Children mentally add 100 to the number and record it above the number they wrote on the strip. They should line up the numbers by place value so that the patterns are more obvious.
- Children continue adding 100 to the last number and recording until the next number is more than 1,000.
- Children then tear off the strip and repeat with a new number as time allows.
- Next, have children create a new three-digit number and place it a foot away from the end.

- Children mentally subtract 100 from the number and record it below the number they wrote on the strip.
- Children continue subtracting 100 from the last number and recording until the next number is below 0.
- Use longer 2- to 3-foot strips of adding machine tape to repeat the same activity with adding and subtracting tens.
- Children continue adding 10 to the last number and recording until the next number is over 1,000 or they run out of room on the strip.
- Children continue subtracting 10 from the last number and recording until the next number is below 0 or they run out of room on their strip.



Develop

Meet as a whole group.

- Ask one group, ***"What did you discover making your lists of numbers?"***
- ***"How is adding hundreds different than subtracting hundreds? How is it similar?"***
- ***"Is that true of tens as well? How so?"***
- ***"What are some strategies we can remember to help us add or subtract 10 or 100 from a number mentally?"***

Have other groups share while asking them questions similar to those above.