

Performance Assessment Task
Graphs (2007) Grade 9
This task challenges a student to use knowledge of linear and quadratic functions and predict shapes of graphs from the equation. A student must be able to understand intersections of graphs to solve problems. A student must be able to solve two equations graphically and algebraically. A student must be able to draw a linear graph from the equation. A student must use knowledge of quadratic equations to choose a solution strategy to solve the equation.
Common Core State Standards Math - Content Standards
<p><u>High School – Functions – Interpreting Functions</u></p> <p>Interpret functions that arise in applications in terms of the context.</p> <p>F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal descriptions of the relationship. <i>Key features include: intercepts, intervals where the function is increasing, decreasing, positive or negative, relative maximums and minimums, symmetries and behavior; and periodicity.</i></p> <p>F-IF.5 Relate the domain of a function to its graph and where, applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i></p> <p>Analyze functions using different representations.</p> <p>F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic functions and show intercepts, maxim, and minima</p> <p>F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.5 Use appropriate tools strategically.</p> <p>Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.</p> <p>MP.7 Look for and make use of structure.</p> <p>Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about</p>

the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

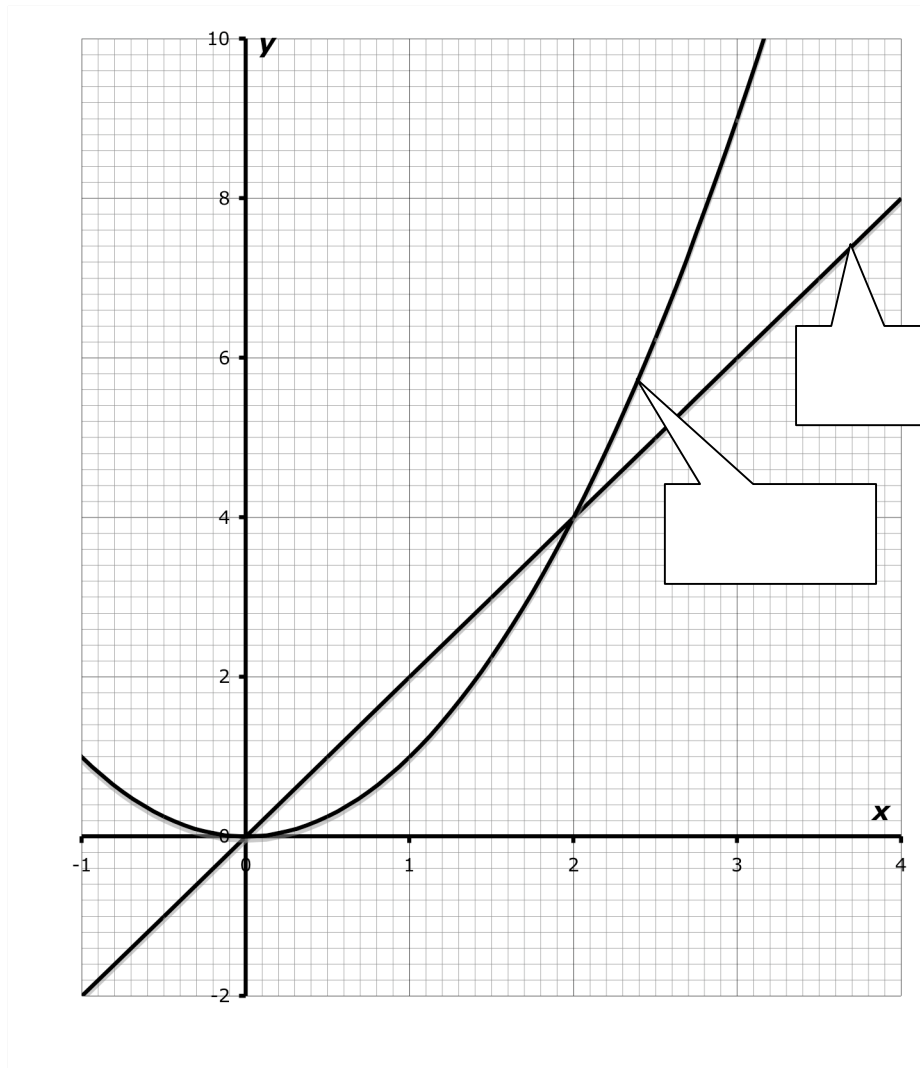
Grade Level	Year	Total Points	Core Points	% At Standard
9	2007	9	4	45%

Graphs

This problem gives you the chance to:

- work with linear and quadratic functions their graphs and equations

This diagram shows the graphs of $y = x^2$ and $y = 2x$.



1. Fill in the labels to show which graph is which. Explain how you decided.

2. Use the diagram to help you complete this statement:

$2x$ is greater than x^2 when x is between _____ and _____

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

a. Write down the coordinates of these two points. _____

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

4.

a. On the diagram, draw the graph of $y = 3x$.

b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? _____

d. Use algebra to prove your answer.

Algebra 1 – Task 1 – Graphs

Work the task. Look at the rubric. What are the key mathematical ideas the task is trying to assess? What other insights might the task provide in student understandings and misconceptions about graphing and intersections?

Look at student work on labeling the graph. How many of your students:

Labeled graph with good explanation	Reversed the labels	Tried to identify point where the arrow was	Had never seen a squared line or quadratic	Other

Some students used graphing techniques, like making a table of values or using $y=mx + b$ to distinguish between the two graphs. This was not considered by the rubric. Do you think this is an adequate justification? Why or why not?

Look at student work on part 2. How many of the students put

0 and 2	1 and 2	0 and 4	(0,0) and (2,4)	0 and 1	No response	Other

What misconceptions might of led to these types of errors? What are students not understanding?

Many students were able to get the coordinates for part 3a. Look at the types of strategies that they used to find the coordinates. How many of your students:

- Could set the two equations equal to each other and then correctly solve the quadratic equation?
- Could set the two equations equal but couldn't or didn't solve the equation?
 - Tried to divide both sides by x ?
 - Tried to take a square root of $2x$
- Used substitution or guess and check?
- Made a table of values for each equation to find out which values would match for both equations?
- Said to look at the intersection points on the graph?
- Tried to use slope: $y=mx + b$ or $\text{slope} = (y_2 - y_1)/(x_2 - x_1)$
- Made no attempt to explain how they got their coordinates?

Why do you think so many students had difficulty applying algebra to this situation?
Why do you think there was such a disconnect for students?

Look at student graphs for $y = 3x$. How many of your students:

- Graphed the equation correctly?
- Did not attempt the graph?
- Made a parabola?
- Made a graph that did not go through the origin?
- Made a graph that was too low?
- Made a graph that was too high?

While the task didn't require students to show how they made the graph, how many of your students showed:

- some evidence of making a t-table for x and y ?
- some evidence of using $y = mx + b$?

In earlier grades, teachers talk about using calculators to help students in problem-solving situations, but not when learning or practicing a procedure. How are graphing calculators used in your classroom? Do you think students have ample opportunity to understand the big ideas of graphing before relying on graphing calculators as tools?

What are the implications for instruction to help your students develop a better understanding of graphing and connecting it to other algebraic representations?

In 4d some students were able to find the coordinates by looking at patterns from the earlier numerical examples. Did you see any evidence of students looking for patterns? In 4c and d, many students only gave a response with the origin and explanations about all lines go through 0. How many of your students have this misconception? What kinds of experiences do these students need? What other misconceptions did you see in student work?

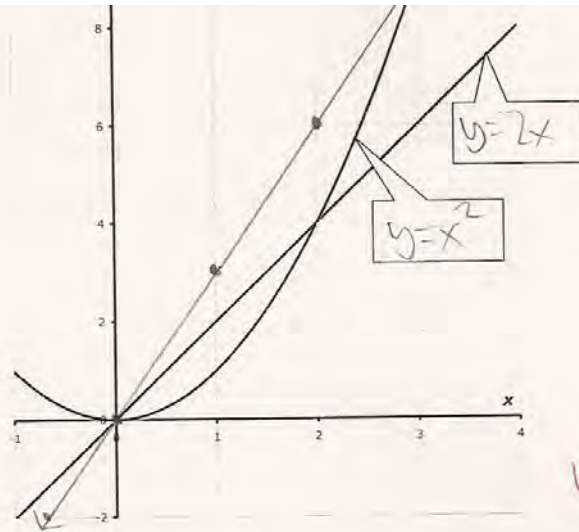
Looking at Student Work on Graphs

What does it mean to understand Algebra and be able to apply it to a problem-solving situation? A good assessment task should be rich enough to allow students to apply or choose between many of the tools they have learned. In looking at the task Graphs, many adult thinkers will choose to distinguish between the two graphs using a definitional approach, quadratics or equations with exponents form a parabola and functions without exponents are linear. But, are there other tools available for distinguishing between the two graphs? What other methods or ways of thinking about the situation could help to distinguish between the two graphs? Is there a hierarchy or value system attached to ranking strategies or are all strategies equally valid? These are tough questions for us to grapple with as a mathematical community.

Now think about using algebra to find the coordinates for the intersections of two equations. Many of us who learned algebra in a traditional approach focused on factoring think of those intersections as the solution to a quadratic equation. First set the two equations equal to each other. Then, put the equation in the form $ax^2 + bx + c = 0$. Next factor the left side and set one term equal to 0 or use the quadratic equation. But are there other approaches or algebraic tools available for making sense of this situation? Are some approaches “more algebraic” than others? What does it mean to understand and apply algebra?

Student A uses slope to make sense of finding the graph for $y=2x$. In part 3, Student A understands that to find the intersections the two equations should be set equal to each other. In earlier grades teachers complain then students take a problem like $100 - 98$ and have to set it up with regrouping to find the solution. Teachers ask, “Don’t we want students to just know that the answer is 2?” So here the student doesn’t appear to use any formal procedures to solve the task, but just uses number sense to find values for x and y and then uses substitution to check the results. Does this student apply algebra or not? In part 4 the student looks at the pattern of problems given. If all three problems have $y = x^2$ and then something in the form $y=nx$, then the coordinates for the intersection should also follow a pattern. However in this case the student has made an assumption that isn’t true. The student needs to think about justifying the generalization, $n=x$. “Does n always equal x ? When does $n=x$?”

Student A



1. Fill in the labels to show which graph is which. Explain how you decided.

The one I chose as $y=2x$ was straight, as it went up 2 for every 1 it went sideways. The one I chose as $y=x^2$ isn't straight, and gets steeper the further it gets, making it a parabola.

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

a. Write down the coordinates of these two points.

$(0,0)$ $(2,4)$ ✓ 1

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

$x^2 = 2x$ $x=0$ and $y=0$ $y=0$
 $y=2^2$ $y=4$ $y=2(0)$ $y=0$ $y=2(2)$
 $y=4$ $(0,0)$ and $(2,4)$ ✓ 1

4.

a. On the diagram, draw the graph of $y = 3x$.

b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

$(0,0)$ $(3,9)$ ✓ 1

c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet?

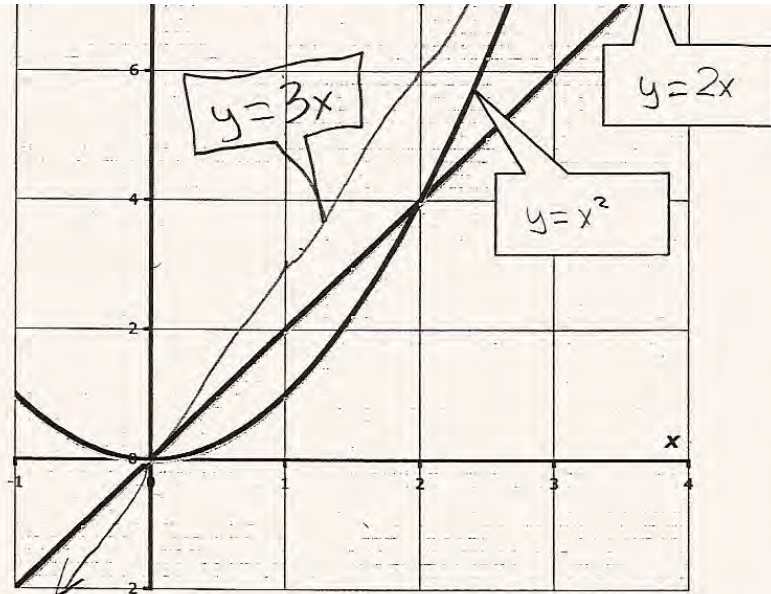
$(0,0)$ (n, n^2) ✗ 0

d. Use algebra to prove your answer.

$y=x^2$ $y=3x$ $x=3$ $y=9$ ($a=3$)
 $y=x^2$ $y=2x$ $x=2$ $y=4$ ($a=2$) ✓ 1
 $n=x$ $y=n^2$ and x^2

Student B has used a definitional approach to distinguishing the two graphs. In part 3 the student has made a table of values to show when the values of x will give the same value for y in both equations. Does this imply an equality? Does this idea link the representations between graphs and equations? What does this student understand about algebra? In part 4, the student uses patterning to find the solution for the coordinates. Is looking for patterns part of algebraic thinking? Is it a valid way to think about finding the solution?

Student B



1. Fill in the labels to show which graph is which. Explain how you decided.

A quadratic function, when graphed, is always a parabola. Naturally I chose the quadratic equation as the parabola. Linear functions, when graphed, are always a straight line. Consequently I chose the straight line as my linear equation.

Student B, part 2

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

✓ a. Write down the coordinates of these two points. $(0, 0)$ $(2, 4)$

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

✗ You can create a t-chart, and find coordinates of the equations.
When each equation has the same coordinate as the other, they intersect at that point.

✓ intersect each other →

x	$y = x^2$	y	→		$y = 2x$	
0	0^2	0		0	$2(0)$	0
1	1^2	1		1	$2(1)$	2
2	2^2	4		2	2^2	4

intersect each other →

✓ 4. a. On the diagram, draw the graph of $y = 3x$.

✓ b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

$(3, 9)$ $(0, 0)$

✓ c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? (n, n^2) $(0, 0)$

d. Use algebra to prove your answer.

✗ $y = x^2, y = 2x$ $(2, 4)$
 $2^2 = 4$

$y = x^2, y = 3x$ $(3, 9)$
 $3^2 = 9$

$y = x^2, y = nx$ (n, n^2)
 $n^2 = n^2$

(7)

9

Student C uses a graphing approach to help distinguish between the two graphs. If I know the x coordinate, I can substitute to find the y coordinate. Then if the line does through that point, it must fit that equation. In part 2 the student confuses the values of x that make the statement true with the coordinates on the graph where the statement is true. In part 3 the student uses a number theory or case law approach. If x is any number not equal to 0, both sides of the equation can be divided by x. If $x = 0$, then only substitution is necessary to find y. *This does not use the traditional factoring or quadratic equation method for finding the solution. Do you think this student understands and applies algebra? Why or why not?* The student uses the same system for finding the solution in part 4, but doesn't complete the substitution correctly to find the value of y.

Student C

1. Fill in the labels to show which graph is which. Explain how you decided.

I know that x^2 = a parabola. (Any number squared = parabola). Then I decided that $2x$ was the straight line because if x is 1 then $y = 2$ making the coordinates (1,2) and the line going straight went through (1,2) so I know that the linear line is $y = 2x$.

2. Use the diagram to help you complete this statement:

$2x$ is greater than x^2 when x is between (0,0) and (2,4) X

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

- a. Write down the coordinates of these two points. (0,0) (2,4) ✓
b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

For the point (2,4);

$$x^2 = 2x \xrightarrow{\text{Divide by } x} x = 2 \Rightarrow y = 2x \Rightarrow y = 2(2) \Rightarrow y = 4 \Rightarrow (2,4) \checkmark$$

For the point (0,0);

$$x^2 = 2x \xrightarrow{\text{Substitute } x \text{ with } 0} 0^2 = 2(0) \Rightarrow x = 0 \Rightarrow y = x^2 \Rightarrow y = 0 \Rightarrow (0,0) \checkmark$$

- 4.

- a. On the diagram, draw the graph of $y = 3x$. okay.
b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

(0,0) (3,9) ✓

- c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? (0,0) (n,nx) X

- d. Use algebra to prove your answer.

$$x^2 = nx \xrightarrow{\text{Divide by } x} x = n \Rightarrow y = nx$$

$$x^2 = nx \xrightarrow{\text{Substitute } x \text{ with } 0} 0^2 = n(0) \Rightarrow x = 0 \Rightarrow y = x^2 \Rightarrow y = 0 \checkmark$$

Student D uses a point on one of the graphs as a counter example to eliminate one of the equations. In part 3 and 4, Student D is using mathematical reasoning to attempt a generalization. In the beginning of 4d the student gives an argument about why 0 will work. In an equation in the form $y=mx + b$, if there is no "b" then the line will go through the origin. *Is this showing an understanding of algebra? Do you think the student has presented a clear argument?* The student has also made a generalization about the situation, similar to the pattern thinking of previous students.

Student D

1. Fill in the labels to show which graph is which. Explain how you decided.

I decided on $y=x^2$ because I know by the equation that it is a parabola. The coordinates (3,6) is a solution to $y=2x$ not $y=x^2$. ✓

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

- a. Write down the coordinates of these two points. (0,0) (2,4) 1
b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

You know that zero works if you plug it in. 0
You know the second point because the x-coordinate is the linear equation's coefficient and the y-coordinate is the same number squared. ✓

4. a. On the diagram, draw the graph of $y = 3x$.

- b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet? (0,0) (3,9) 1

- c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? (0,0) (n,n^2) 1

- d. Use algebra to prove your answer.

You know that zero works because every equation like this, $y=nx$, has a solution of zero. The second point is found by using the linear equation's numerical coefficient, so since the x-coordinate is the coefficient & the y-coordinate is the coefficient squared, the next point is (n,n^2) . ✓

Student E is able to find the coordinates for the intersections, probably by reading the graphs. The student has a very accurate graph for $y=3x$. The student uses substitution to show that the coordinates fit both equations. However, the student can't apply these strategies to a more general form of the equation because the thinking comes from visual representation.

Student E

2. Use the diagram to help you complete this statement:

$2x$ is greater than x^2 when x is between 0,0 and 2,4 + 0

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points. ✓

a. Write down the coordinates of these two points. 0,0 2,4 + 1

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

$$0 = 0^2$$

$$2 \cdot 2 = 4$$

$$0 = 2 \cdot 0$$

$$2^2 = 4$$

+ 0

+ 0

- 4.

a. On the diagram, draw the graph of $y = 3x$. + 1

b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

$$0,0$$

$$3,9$$

+ 1

c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet?

$$0,0$$

$$x$$

+ 0

d. Use algebra to prove your answer.

$$0 = 0^2$$

$$0 = n \cdot 0$$

+ 0

Student F gives a generalization for how to solve for the intersections, including the idea that the two equations should be made equal to each other. (The paper is mis-scored on this mark). *Do you think the student makes the generalization? Is making generalizations a part of algebraic thinking? What do you think the student understands about algebra? How would you score this student?*

Student F

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

a. Write down the coordinates of these two points. $(0,0)$ and $(2,4)$ ✓

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

I can use algebra to find the point of intersection by using a method called substitution. ^ 0

You put each equation in y form and then remove the y and set the two leftover pieces to each other. 0

Next you solve for x. That answer is your x coordinate.

Then you replace the x in one of the equations with the answer you got, and solve for y. Your two answers together make up the coordinates of the intersection point.

4. a. On the diagram, draw the graph of $y = 3x$.

b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet? ✓

$$\begin{aligned} x^2 &= nx \\ x &= n \end{aligned}$$

$$y = nx^2$$

$$(0,0)$$

$$(3,9)$$
 ✓

c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? $(0,0)$ (n,n^2) ✓

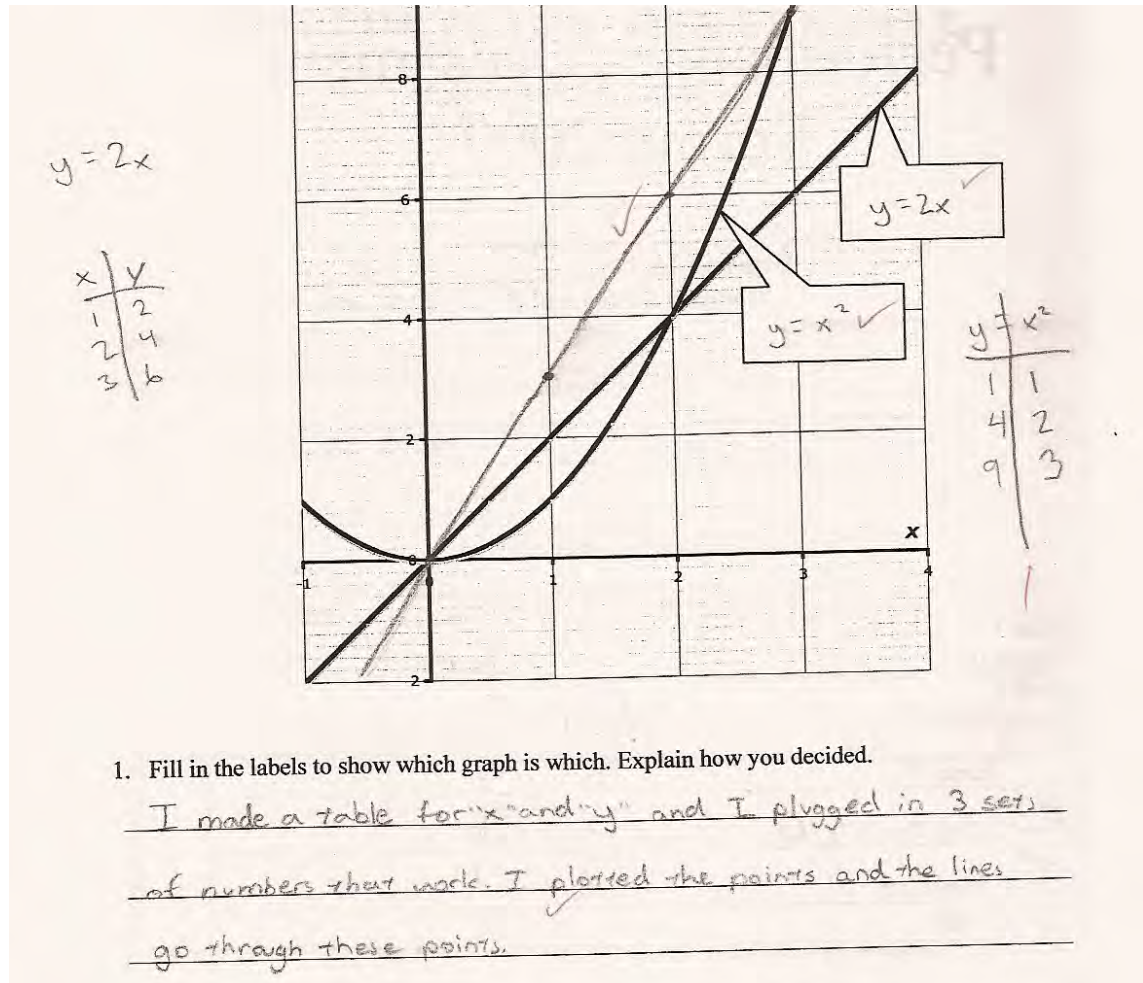
d. Use algebra to prove your answer.

Algebra says that if you use substitution (explained above), you can find intersection points. $(0,0)$ is one ✓ because anything multiplied by zero is zero. Also $(0,0)$ is a point on the $y=x^2$ graph. (n,n^2) is also one because it fits on the $y=x^2$ graph and because in $y=nx$, n and its multiples (such as n^2) would always work, so $(0,0)$ and (n,n^2) are the points of intersection.

9

Student G uses a table of value to verify which graphs match the equations. In part 2 the student misses the lower end of the values but tries to show that the numbers need to be lower than 2. In part 3 the student gives more a description of the graph than directions to find the intersections. In part 4, the student might be on the way to understanding that the intersections are when the two equations are equal. What does the student understand? What is the student struggling with?

Student G



Student G, part 2

2. Use the diagram to help you complete this statement:

$2x$ is greater than x^2 when x is between 1^x and 1.9 ✓

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

- a. Write down the coordinates of these two points. $(0,0)$ ✓ $(2,4)$ ✓ 1

- b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

$y = 2x$ means the equation will continually go up and right. $y = x^2$ 0
means there will be squares meaning 4, 9, 16 which become
a parabola. 0

4. a. On the diagram, draw the graph of $y = 3x$.

x	y
1	3
2	6
3	9
-1	-3

- b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

$(0,0)$ ✓ $(3,9)$ ✓ 1

- c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? $(0,0)$ ✓ all squares 0

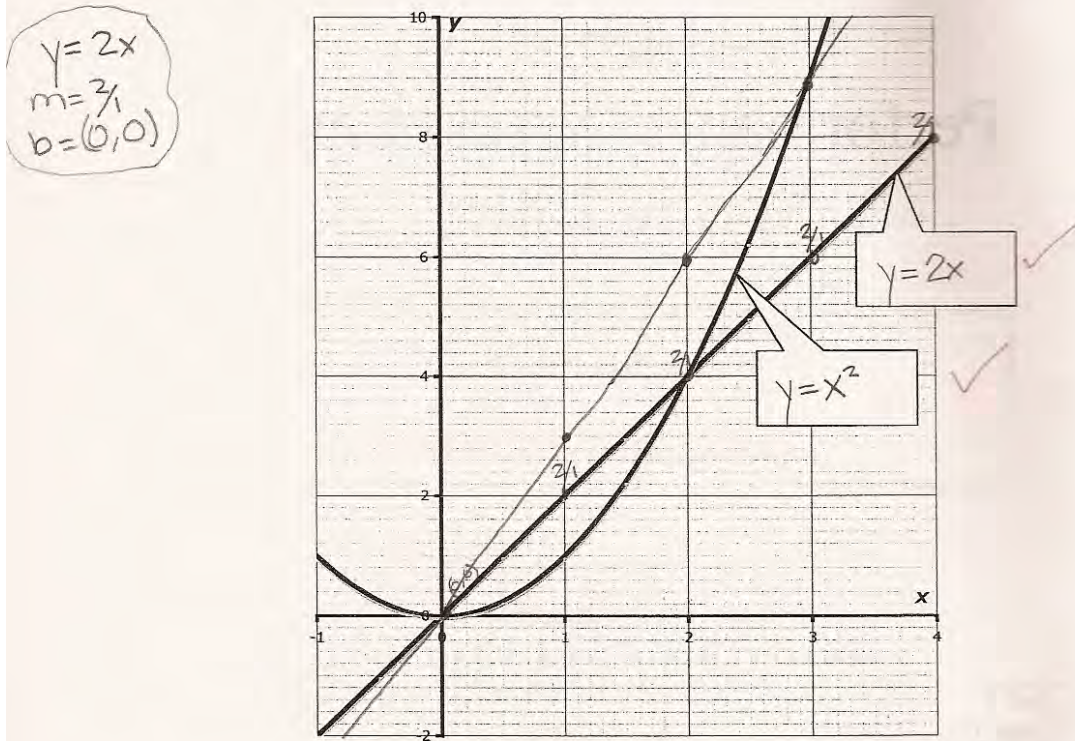
- d. Use algebra to prove your answer.

$y = x^2$ means a number multiplied by itself. $y = nx$ means
 n and x could have equal value such as this example.
 $(4 = 2^2) = (4 = 2(2))$ if n and x have the same 0
value it is exactly the same as squaring.

Student H appears to have a clear understanding of the equation $y=mx+b$ and uses this information to identify the graphs in part 1 and to draw the graph in 4a. However the student seems to be confused about the slope in 3b and then can't think about the generalization in 4c. What does this student need to help clarify ideas about slope? What do you think the student understands or doesn't understand about the intersections of two equations? What further instruction or experience does this student need?

Student H

This diagram shows the graphs of $y = x^2$ and $y = 2x$.



1. Fill in the labels to show which graph is which. Explain how you decided.

On the equation $y=2x$, slope = $2/1$ and y-intercept = $(0,0)$
 That leaves the other equation $y=x^2$ to be the
 curved line. ✓

Student H, part 2

2. Use the diagram to help you complete this statement:

$2x$ is greater than x^2 when x is between (0,0) and (2,4) X

0

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

a. Write down the coordinates of these two points.

(0,0) (2,4) ✓

1

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

You can ~~add~~ combine the slope (2,4) and combine X
the y-intercept (0,0).

0

X

0

4.

a. On the diagram, draw the graph of $y = 3x$.

$m = 3, b = (0,0)$ ✓

1

b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

(0,0) (3,9) ✓

1

c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet?

(0,0) (2,4) X

0

d. Use algebra to prove your answer.

I added 1 onto the x-point and 5
onto the y point. X

0

Student I tries to combine the two equations and use factoring, but uses a positive instead of a negative in both 3b and 4d. However the student ignores that work and is able to find the coordinates using substitution and number sense or guess and check. *What does the student know about algebra? What is the student confused about?*

Student I

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

a. Write down the coordinates of these two points. $(2,4)$ $(0,0)$ ✓

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

1. solve $y = 2x$ $x^2 + 2x = 0$ $\frac{0}{2} = 0$ $(x+2)(x+0)$ 0
 $y = x^2$ x

2. plug in. $y = 2 \cdot 2$ $y = 4$ $y = 2 \cdot 0$ $y = 0$ 3. put x and y together 0
 $y = 2^2$ $y = 4$ $y = 0^2$ $y = 0$ $= (2,4)(0,0)$ ✓

4.

a. On the diagram, draw the graph of $y = 3x$.

b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet? ✓

$(0,0)$ $(3,9)$ ✓

c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? $(0,0)$ (n,n^2) ✓

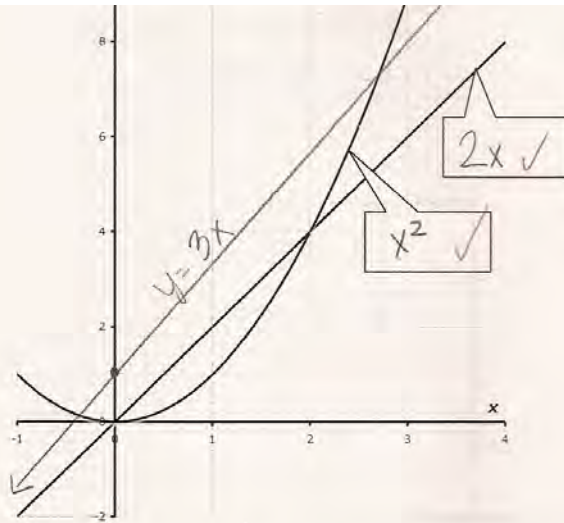
d. Use algebra to prove your answer.

$y = x^2$ $x^2 + nx = 0$ $x = n, x = 0$ 10
 $y = nx$ ✓

$y = n \cdot 0$ $y = 0$ $y = n \cdot n$ $y = n^2$
 $y = 0^2$ $y = 0$ $y = n^2$ $y = n^2 = (0,0)(n,n^2)$

Student J does not understand that the graph for $y=3x$ should go through the origin. While it is unclear how the student made the graph, it does appear to go through the point (1,3). The student is able to get the coordinates for 3a, probably by looking at the graph. The student is able to make the 2 graphs equal, but doesn't know how to use algebraic skills to solve the equation. It is interesting that the student doesn't use the graphs to find the coordinates in 4b. *What types of experiences does this student need? Where does his understanding of graphing break down? Where does his understanding of solving equations break down?*

Student J



1. Fill in the labels to show which graph is which. Explain how you decided.

I decided the line that's going like this \checkmark is a parabola the x^2 because it's on a negative and positive line and it curves like a parabola. The 1 that's just going straight up is a line because in every graph function if there is a number and an x we know it will be a line.

Copyright © 2007 by Mathematics Assessment Resource Service. All rights reserved.

Page 2

Graphs Test 5

2. Use the diagram to help you complete this statement:

$2x$ is greater than x^2 when x is between 1 and 2 ✓ 0

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

a. Write down the coordinates of these two points. 0, 0 2, 4 ✓ 1

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

$$\begin{array}{r} x^2 = 2x + 0 \checkmark \\ -2x \quad -2x \end{array}$$

x 0

- 4.

a. On the diagram, draw the graph of $y = 3x$. x 0

b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

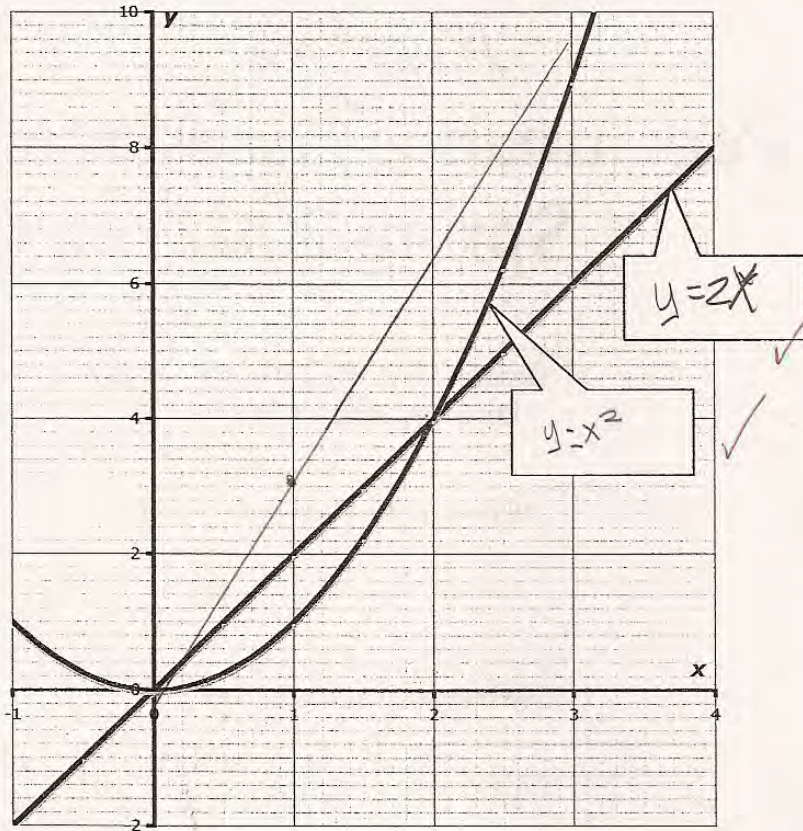
-1, 1 2.25, 3 x 0

c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet? 0, 0 3.7, 14 x 0

d. Use algebra to prove your answer.

Student K has only 2 of the 9 marks on this task, but there is evidence of understanding. In part 1 the student makes a t-table to help identify the graphs. However the student doesn't give enough information about how it was used. In drawing the graph for $y=3x$, there appears to be evidence of thinking about $y=mx+b$. (The graph is off slightly, but clearly attempts to go through $(0,0)$ and $(1,3)$. However by the end of the line it appears the line might go through $(3,10)$ instead of $(3,9)$. The student doesn't pick up on this discrepancy in answering 4b. The student seems to think that looking at the graph should give you the solution, so why bother with algebra. But, in 4d can think about the equations in 4d going through the origin, but can't or isn't willing to attempt a second solution. *How can you help build on these understandings?*

Student K



1. Fill in the labels to show which graph is which. Explain how you decided.

~~Handwritten scribbles~~ $y = x^2$ $y = 2x$

1	1
2	4
3	a

Student K, part 2

2. Use the diagram to help you complete this statement:

$2x$ is greater than x^2 when x is between 2 and x^2

x^2 0

3. The graphs of $y = x^2$ and $y = 2x$ cross each other at two points.

a. Write down the coordinates of these two points.

2,4

0,0

✓ 1

b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

Will look where the intersect and write it down

Where they cross

x^2

x^2

4.

a. On the diagram, draw the graph of $y = 3x$.

$y = 3x$

x^2 0

1

b. What are the coordinates of the points where $y = x^2$ and $y = 3x$ meet?

10

3 x^2 0

c. Where do you think that the graphs of $y = x^2$ and $y = nx$ meet?

0,0

0,0 x^2 0

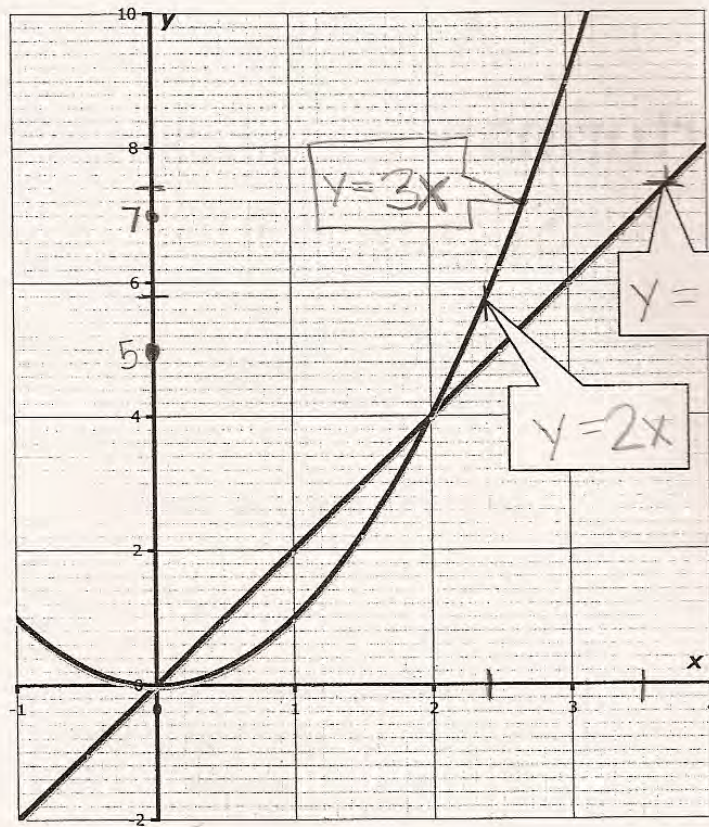
d. Use algebra to prove your answer.

$y = 0$ $x = 0$ so they would meet at x^2

0,0

Students L and M both struggle at just the entry level question on graphing. Student L knows the definition of linear and quadratic equations and their lines, but can't link that information with the equations. The student tries to make $y = 3x$ part of parabola. Student M does not see the link between the equations and the graphs, but instead tries to pinpoint the value of x or y at the exact place on the graph that the arrow is pointing. Neither of these students is in high school (thereby repeating algebra for the second time) and both are from districts that have participated in MAC since the beginning of the project.

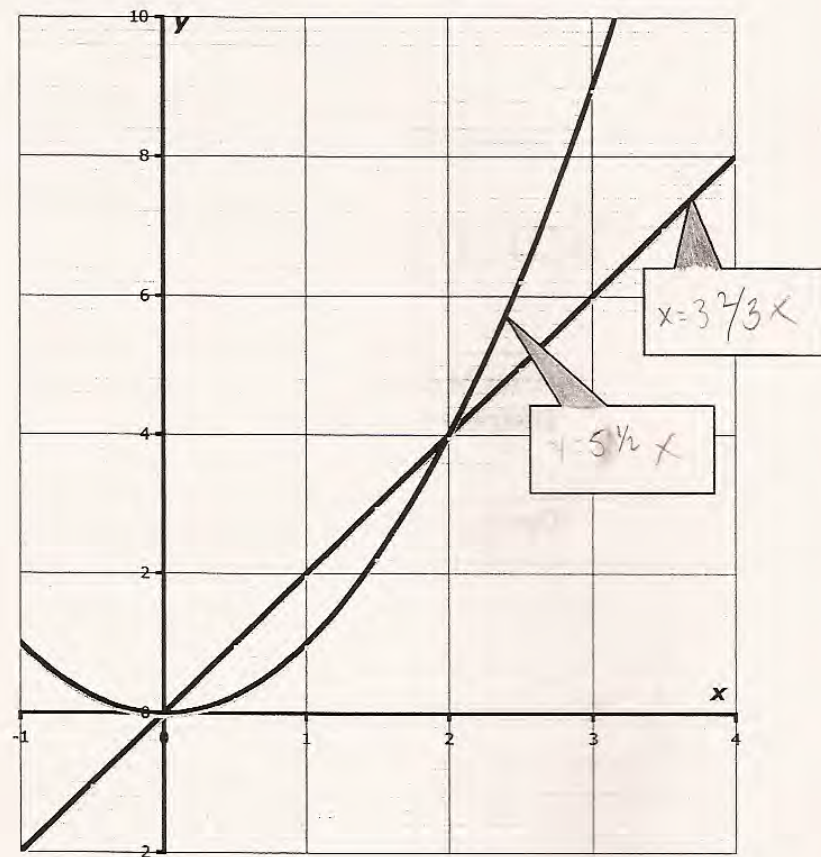
Student L



1. Fill in the labels to show which graph is which. Explain how you decided.

I decided because one line is linear (the straight one) ✓
 and the quadratic (the curved one). I think that
 it makes more sense because the number can be
 made to even on the linear line. (0)

Student M



1. Fill in the labels to show which graph is which. Explain how you decided.

Because they are lined up on that number

Algebra

Algebra

Task 1

Graphs

Student Task	Work with linear and quadratic functions, their graphs, and equations
Core Idea 1 Functions and Relations	Understand patterns, relations, and functions. <ul style="list-style-type: none">Analyze functions of one variable by investigating local and global behavior including slopes as rates of change, intercepts, and zeros.
Core Idea 3 Algebraic Properties and Representations	Represent and analyze mathematical situations and structures using algebraic symbols. <ul style="list-style-type: none">Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations.Write equivalent forms of equations, inequalities and systems of equations and solve them.Use symbolic algebra to represent and explain mathematical relationships.

Mathematics in this task:

- Distinguish between linear and quadratic equations and their graphical representations
- Ability to graph a linear equation
- Ability to locate points on a graph and interpret their meaning
- Use algebra to find the intersections of two equations

Based on teacher observation, this is what algebra students knew and were able to do:

- Find the coordinates where the graphs intersect
- Give a reason for connecting equations with their graphs
- Draw a graph of $y=3x$

Areas of difficulties for algebra students:

- Finding values for x , where one graph or equation is less than another
- Using algebra to find the points of intersection for two equations
- Knowing that the equations should equal each other at the points of intersection
- Using factoring as a tool to solve a quadratic equation
- Understanding that you can't divide by 0

Strategies used by successful students:

- Making a table of values to help them graph
- Understanding $y=mx+b$ and using it to help them graph
- Substitution

MARS Test Task 1 Frequency Distribution and Bar Graph, Course 1

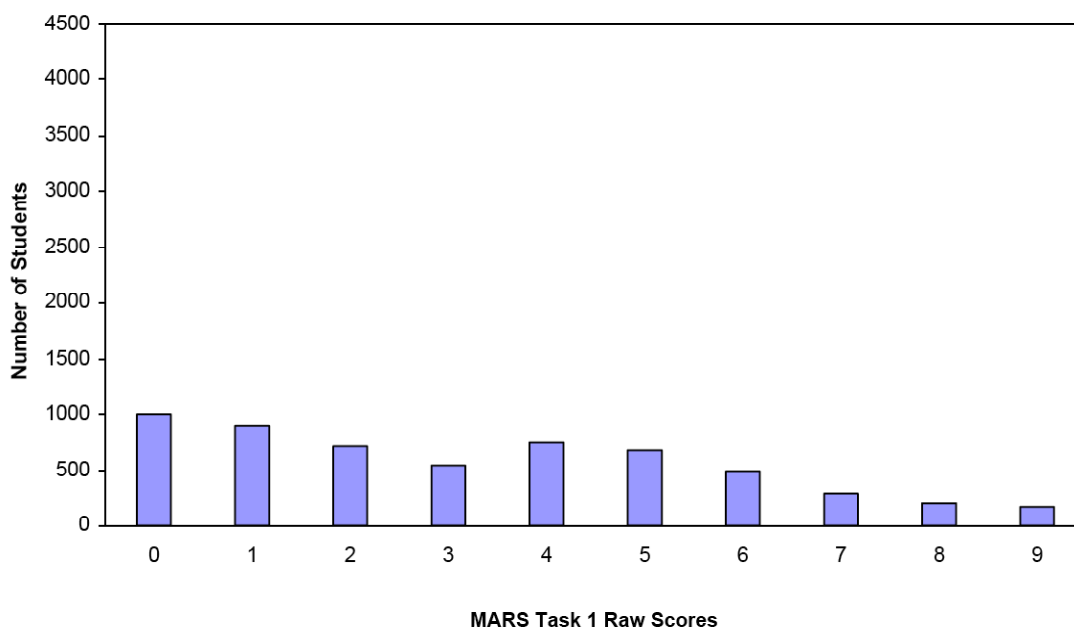
Task 1 - Graphs

Mean: 3.23 StdDev: 2.54

Table 45: Frequency Distribution of MARS Test Task 1, Course 1

Task 1 Scores	Student Count	% at or below	% at or above
0	1003	17.5%	100.0%
1	892	33.0%	82.5%
2	714	45.4%	67.0%
3	543	54.9%	54.6%
4	746	67.9%	45.1%
5	676	79.6%	32.1%
6	500	88.3%	20.4%
7	292	93.4%	11.7%
8	202	96.9%	6.6%
9	176	100.0%	3.1%

Figure 54: Bar Graph of MARS Test Task 1 Raw Scores, Course 1



The maximum score available for this task is 9 points.

The minimum score for a level 3 response, meeting standards, is 4 points.

Most students, 83%, could identify the coordinates for the points of intersection of the two graphs. More than half the students, 67%, could identify the equations with their graphs and the points of intersection. Almost half the students, 45%, could also draw a graph of $y=3x$ and the coordinates where it intersects with $y=x^2$. About 9% of the students could meet most of the demands of the task, but struggled with locating the coordinates of intersection for $y=x^2$ and $y=3x$ giving values for x in part 2. Almost 18% of the students scored no points on this task. 95% of the students in the sample with this score attempted the task.

Graphs

Points	Understandings	Misunderstandings
0	95% of the students with this score attempted the task.	Students did not understand the blanks and put the coordinates on separate lines, not giving both intersections (6%) Some reversed x and y coordinates (3%)
1	Students could find the coordinates for the intersections of the two graphs	Students had trouble giving reasons for matching the equations and their graphs 11% reversed the graphs Some tried to pinpoint the location of the arrows.
2	Students could match equations with their graphs and find the points of intersection.	25% did not attempt to draw the graph for $y=3x$. 6% made graphs that were too low. 5% drew parabolas. 5% made graphs that did not pass through the origin.
4	Students could identify the graphs and intersections, draw a graph from an equation, and find the intersection of their graph with $y=x^2$.	Students had difficulty using symbolic algebra to find the points of intersection. 10% said just look at the graph. 12% made a table of value to find where the values were the same for each equation. 12% just substituted values from the graph into the given equations to show the values were equal. 12% tried to use slope to make a justification.
6	Students could meet all the demands of the task except use algebra to justify the points of intersection.	10% could set the two equations equal to each other but couldn't solve the equation. They may have divided by x (x could and does equal 0). They tried to take the square root of $2x$ or $3x$.
8	Students could match equations to graphs, make their own graph of an equation, find the intersection points for 2 equations including 2 generalized equations. Successful students looked at patterns to solve the problem, used number theory, and number sense and mathematical reasoning.	No student in the sample solved a quadratic equation.

Implications for Instruction

Students need more opportunities to work rich tasks that provide them with the opportunity to synthesize and to use all the knowledge that they are accumulating about a particular algebraic topic. The problems should allow them to use a variety of tools to make sense of the situation. Teachers need to think about which tools the students feel comfortable enough to use and to use well. Also, which expected tools are never used by students? What types of instruction do students need in order to feel comfortable applying these tools to new situations?

Action Research

Sit down with colleagues to think about what are the big ideas in Algebra. Many in the mathematics world are talking about redefining what it means to learn Algebra. In Europe for example and in many junior colleges, books are focusing on a functional approach to algebra, often eliminating all those sections on factoring and starting quadratics with the quadratic formula. In the task Graphs, many issues about what is valued in learning and understanding graphs are raised. How do we teach algebra in a way that allows students to develop practical skills as well as procedures? What are the underlying ideas that we want students to understand about graphs and their equations? about graphs and their intersections? What does the solution to a quadratic equation actually mean in practical terms?

Use the tool kit for graphs to probe the thinking of your group as you grapple with defining the issues and implications.

What are some of the other issues that this task raised within the group? What tasks can you give to students that might allow you think more deeply about these issues or allow you to gather more data?