Performance Assessment Task

Carol's Numbers
Grade 2

The task challenges a student to demonstrate understanding of concepts involved in place value. A student must understand the relative magnitude of whole numbers from the quantity of a digit in a particular place in the number and use this understanding to compare different numbers. The student must make sense of the concepts of sequences, quantity, and the relative position of numbers.

Common Core State Standards Math - Content Standards

Number and Operations in Base Ten
Understand place value.
2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
a. 100 can be thought of as a bundle of ten tens — called a "hundred."
b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones)

2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Common Core State Standards Math – Standards of Mathematical Practice

MP.5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
2	2006	8	4	78%

Carol's Numbers

Carol has three number cards.

4	7	2

1. What is the largest three-digit number Carol can make with her cards?

2. What is the smallest three-digit number Carol can make with her cards?

Explain to Carol how she can make the smallest possible number using her three cards.

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Carol's teacher drew a number line on the board.

3. About where would 85 be? Place 85 on the number line where it belongs.

4. About where would 21 be? Place 21 on the number line where it belongs.

5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

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$\boxed{8}$

Carol's Numbers
Mathematics Assessment Collaborative
Performance Assessment Rubric Grade 2

	Carol's Numbers: Grade 2	Points	Section Points
	The core elements of the performance required by this task are: • Understand the relative magnitude of whole numbers and the concepts of sequences, quantity, and the relative position of numbers • Use strategies to estimate and judge the reasonableness of results • Communicate reasoning using words, numbers or pictures Based on these credit for specific aspects of performance should be assigned as follow:		
1	Gives correct answer of : **742**	**1**	**1**
2	Gives correct answer of: **247** Gives correct explanation such as: Put the smallest number on the left, then the next smallest number and the largest number last	**1** **1**	**2**
3	Places 85 approximately twice the length of 42	**1**	**1**
4	Places 21 approximately one half the length of 42 (use a range from approximately 15 -25)	**2**	**2**
5	Places 31 approximately on half the length between 21 and 42 Dependent upon the correct placement- Gives correct explanation such as: 31 is almost in the middle of 21 and 42. Or Because 31 is 10 more than 21 or Because 31 is 11 less than 42	**1** **1**	**2**
Total			**8**

2nd grade – Task 5: Carol’s Shapes

Work the task and examine the rubric.
What do you think are the key mathematics the task is trying to assess?

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Part 1 and Part 2 – Place Value

There are two elements to this part of the task. First, students must attend to the constraint of using the digits provided as 4, 7, and 2. Also, students have to demonstrate an understanding of place value, and how the place and digit combine to create a value that can be compared. In order to think about where your students are on this spectrum of understanding, sort students into the following matrix:

	Student uses the 4, 7, and 2	Student uses digits not on the provided list
Makes correct placements based on place value requirements.		
Makes one or more incorrect placements based on the place value requirements.		

Looking at where your students fall on the matrix, what activities or experiences will you provide in your classroom so students have the opportunity to improve their understanding of place value and/or their understanding of constraints in a problem-solving situation?

Reference: Problem of the Month, Noyce Foundation
http://noycefdn.org/math/members/POM/POM-GotYourNumber.pdf

Part 3 – 5 - Placing Numbers on the Number Line

Did your students:

- Place the numbers 85, 21, and 31 without regard to the 0 and 42 that were already placed?
- Place the 85 correctly?
- Place the 21 incorrectly, but then put the 31 about halfway between 21 and 42?
- What evidence is there that your students did or did not attend to scale or proportion or relative position when placing the three numbers?

Can you sort the errors in these parts as either errors in using the conventions of the number line, *or* errors in understanding the relative magnitude of numbers? In what ways did your students quantify their thinking?

The Language of Mathematical Explanation

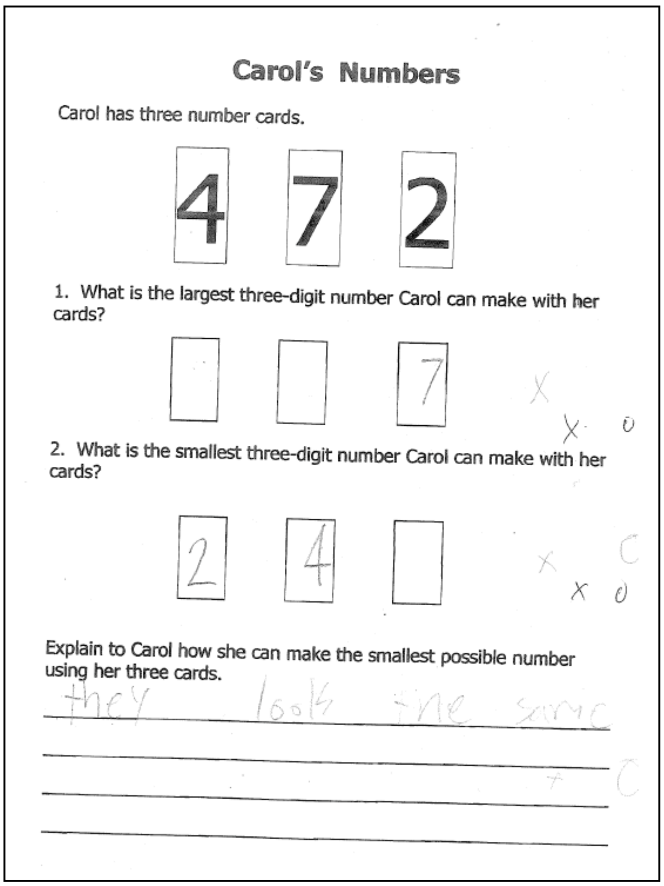
How specific were students being in their explanations for Part 2?

Specificity	Example
Very General or Vague	"Use the cards."
Directions for greatest or least only	"Put the 7."
Reasoning in every day language	"Start with the 2…next is the 4…and last the 7"
Reasoning with relative language	"Put the 2 first because it is the smallest, then 4 is next smallest and 7 is next smallest."
Reasoning with place value language	Put the 7 in the 100s place.

Are there student papers that represent each of these areas of specificity? Can this student work be used to introduce the students to a more precise language or explanation of their thinking?

The following student work reflects an example of the struggles and successes second graders experienced as they made sense of place value. Which of the **Students A – E** seem to understand that the value of the digit is based on its place in the number? Which students understand the constraints of the problem, and how to use the digit cards? What can you learn about how students are thinking about place value through their explanations of how to make the smallest number? Are there opportunities, where a student has enough understanding of the concepts, to introduce a more specific mathematical vocabulary that could add to the students ideas or clarify some beginning thoughts?

Student A



Carol's Numbers

Carol has three number cards.

4 7 2

1. What is the largest three-digit number Carol can make with her cards?

_ _ 7

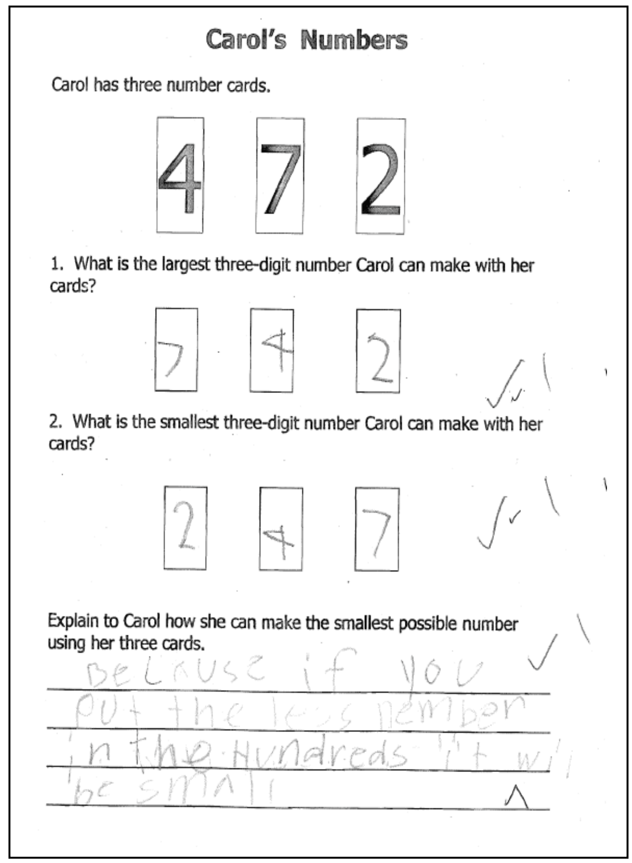
2. What is the smallest three-digit number Carol can make with her cards?

2 4 _

Explain to Carol how she can make the smallest possible number using her three cards.

they look the same

Student B



Carol's Numbers

Carol has three number cards.

4 7 2

1. What is the largest three-digit number Carol can make with her cards?

7 4 2

2. What is the smallest three-digit number Carol can make with her cards?

2 4 7

Explain to Carol how she can make the smallest possible number using her three cards.

BeCAUSe if YOU PUt the less nember in the Hundreds it will be small

Student C

Carol's Numbers

Carol has three number cards.

4	7	2

1. What is the largest three-digit number Carol can make with her cards?

7	4	2

2. What is the smallest three-digit number Carol can make with her cards?

2	4	7

Explain to Carol how she can make the smallest possible number using her three cards.

She can put 2 on the first card because it is the smallest card. Then she can put 4 because it is a little higher from the first card. Last she can put 7 because it is a little more higher then the other two cards.

Second Grade – 2008

Student D

Carol's Numbers

Carol has three number cards.

4	7	2

1. What is the largest three-digit number Carol can make with her cards?

7	4	2

2. What is the smallest three-digit number Carol can make with her cards?

2	4	7

Explain to Carol how she can make the smallest possible number using her three cards.

Put the 2 in the hundreds place and put the 4 in the tens place and put the 7 in the one's place.

Student E

Carol's Numbers

Carol has three number cards.

4	7	2

1. What is the largest three-digit number Carol can make with her cards?

4	0	0

X

2. What is the smallest three-digit number Carol can make with her cards?

	0	2

X

Explain to Carol how she can make the smallest possible number using her three cards.

use 2 hundreds, 4 tens, and 7 on

Student F

Carol's Numbers

Carol has three number cards.

4	7	2

1. What is the largest three-digit number Carol can make with her cards?

2	4	7

X

2. What is the smallest three-digit number Carol can make with her cards?

2	4	2

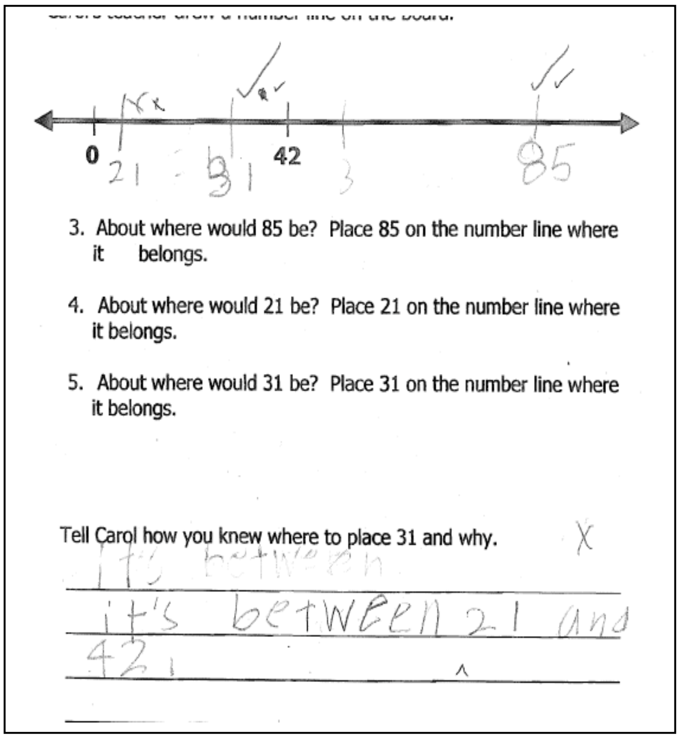
X

Explain to Carol how she can make the smallest possible number using her three cards.

I wlod put 9 fow becase it is the smallest numder.

Students G – L How does the language of these explanations, combined with the placement students made on their number lines, help clarify or confuse the notions of relative magnitude? What do words like “middle”, “between”, “after”, “before”, or “in order” mean in this context? What evidence in the student work is there that any of these words is describing a relationship, or measurement, between numbers? What questions might you ask a student to get clarification, or to surface confusions?

Student G



0 21 31 42 85

3. About where would 85 be? Place 85 on the number line where it belongs.

4. About where would 21 be? Place 21 on the number line where it belongs.

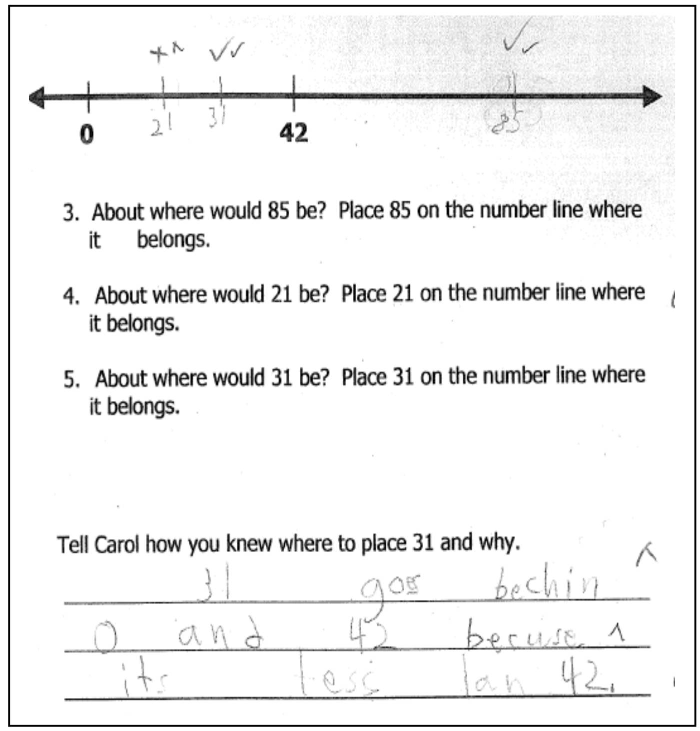
5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

~~it's between~~

it's between 21 and 42.

Student H



0 21 31 42 85

3. About where would 85 be? Place 85 on the number line where it belongs.

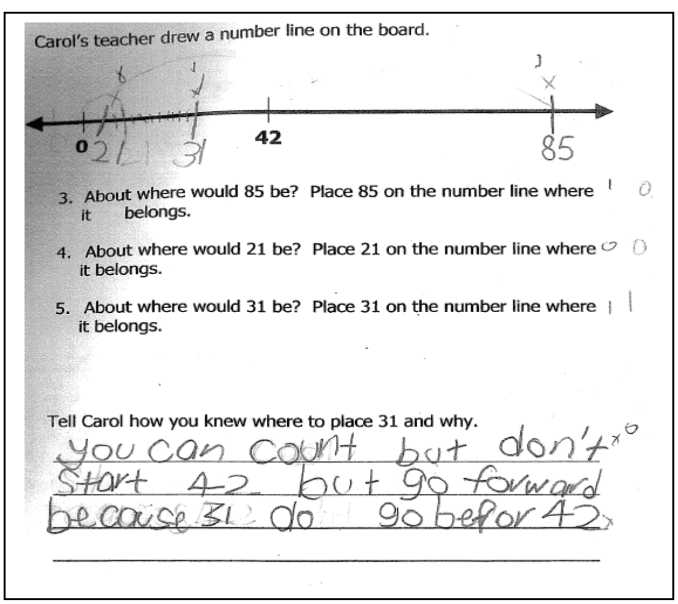
4. About where would 21 be? Place 21 on the number line where it belongs.

5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

31 gos bechin 0 and 42 becuse its less tan 42.

Student I



Carol's teacher drew a number line on the board.

0 21 31 42 85

3. About where would 85 be? Place 85 on the number line where it belongs.

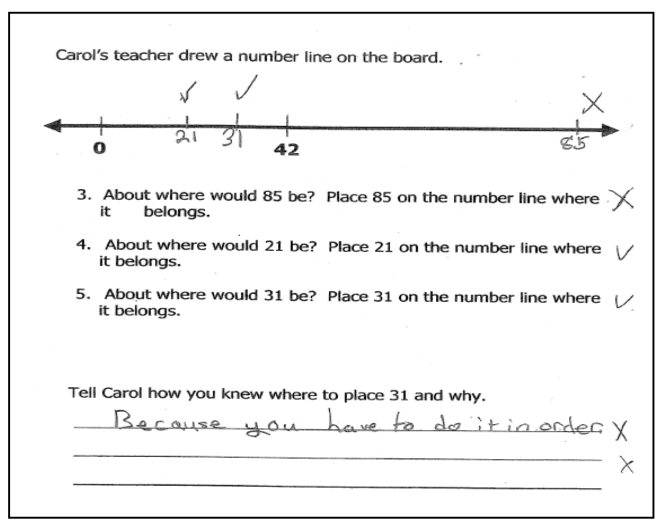
4. About where would 21 be? Place 21 on the number line where it belongs.

5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

you can count but don't start 42 but go forward because 31 do go befor 42

Student J



Carol's teacher drew a number line on the board.

0 21 31 42 85

3. About where would 85 be? Place 85 on the number line where it belongs.

4. About where would 21 be? Place 21 on the number line where it belongs.

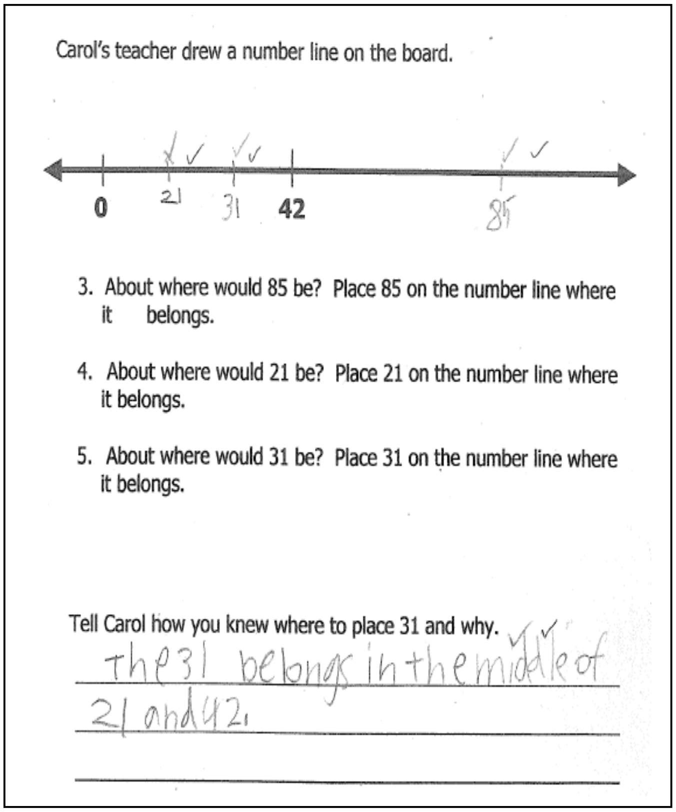
5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

Because you have to do it in order

Students K and L both use the word "middle" in their explanations. In these totally different contexts, what does this word mean? What evidence is there that students had a particular meaning in mind?

Student K


Carol's teacher drew a number line on the board.

0 21 31 42 85

3. About where would 85 be? Place 85 on the number line where it belongs.

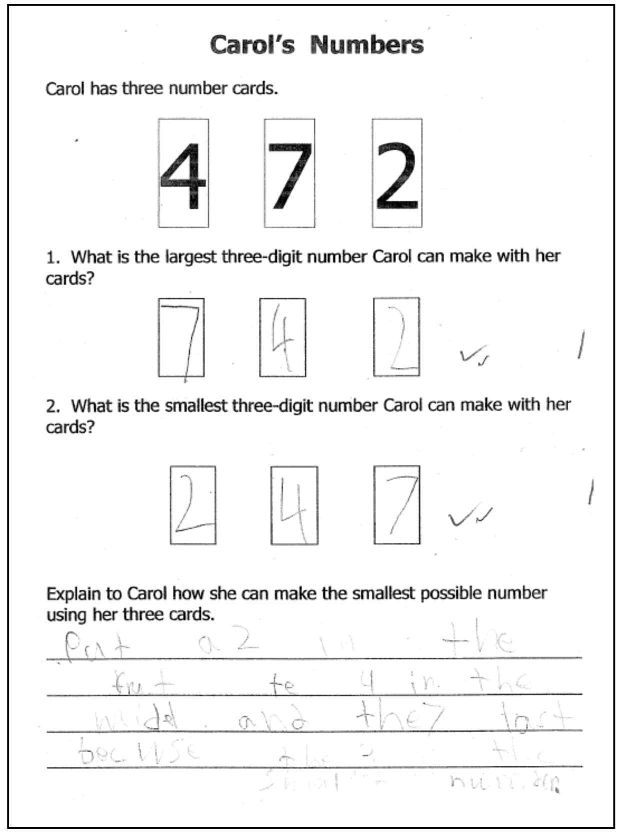
4. About where would 21 be? Place 21 on the number line where it belongs.

5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

the 31 belongs in the middle of 21 and 42.

Student L


Carol's Numbers

Carol has three number cards.

4 7 2

1. What is the largest three-digit number Carol can make with her cards?

7 4 2

2. What is the smallest three-digit number Carol can make with her cards?

2 4 7

Explain to Carol how she can make the smallest possible number using her three cards.

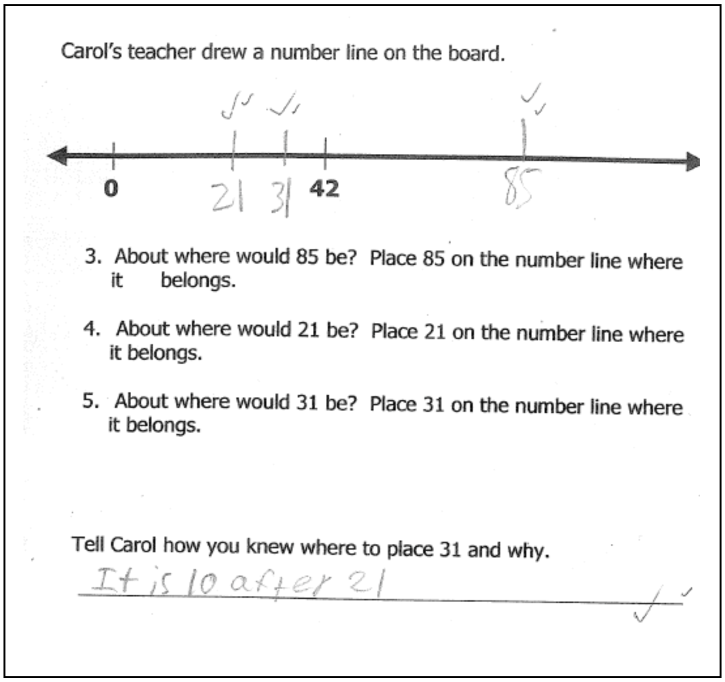
Put a 2 in the frut te 4 in the midd and the 7 last becuse the 2 the smal[illegible] numbr

"The 31 belongs in the middle of 21 and 42."

"Put a 2 in the first the 4 in the middle and the 7 last because that's the smallest number."

Students M – O use quantifying language to explain their placement of numbers on the number line. In what ways is this strategy helpful for clarifying their thinking? Does it provide more evidence that they understand the relative magnitude of these quantities?

Student M

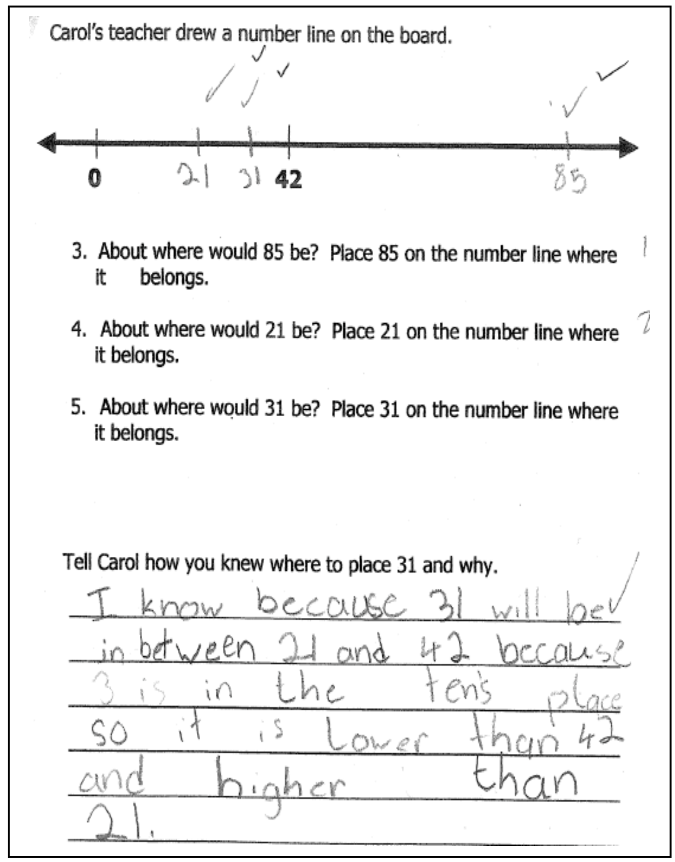

Carol's teacher drew a number line on the board.

3. About where would 85 be? Place 85 on the number line where it belongs.
4. About where would 21 be? Place 21 on the number line where it belongs.
5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

It is 10 after 21

Student N

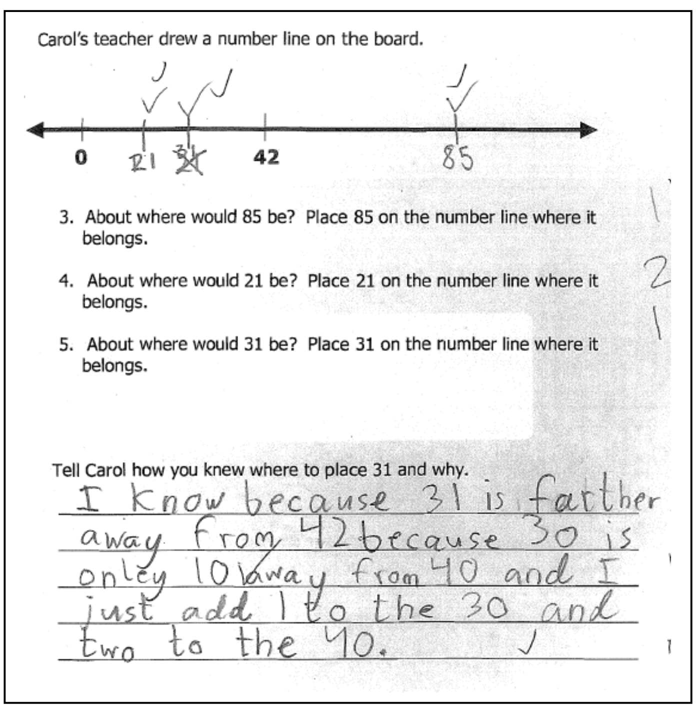

Carol's teacher drew a number line on the board.

3. About where would 85 be? Place 85 on the number line where it belongs. 1
4. About where would 21 be? Place 21 on the number line where it belongs. 2
5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

I know because 31 will be in between 21 and 42 because 3 is in the ten's place so it is Lower than 42 and higher than 21.

Student O


Carol's teacher drew a number line on the board.

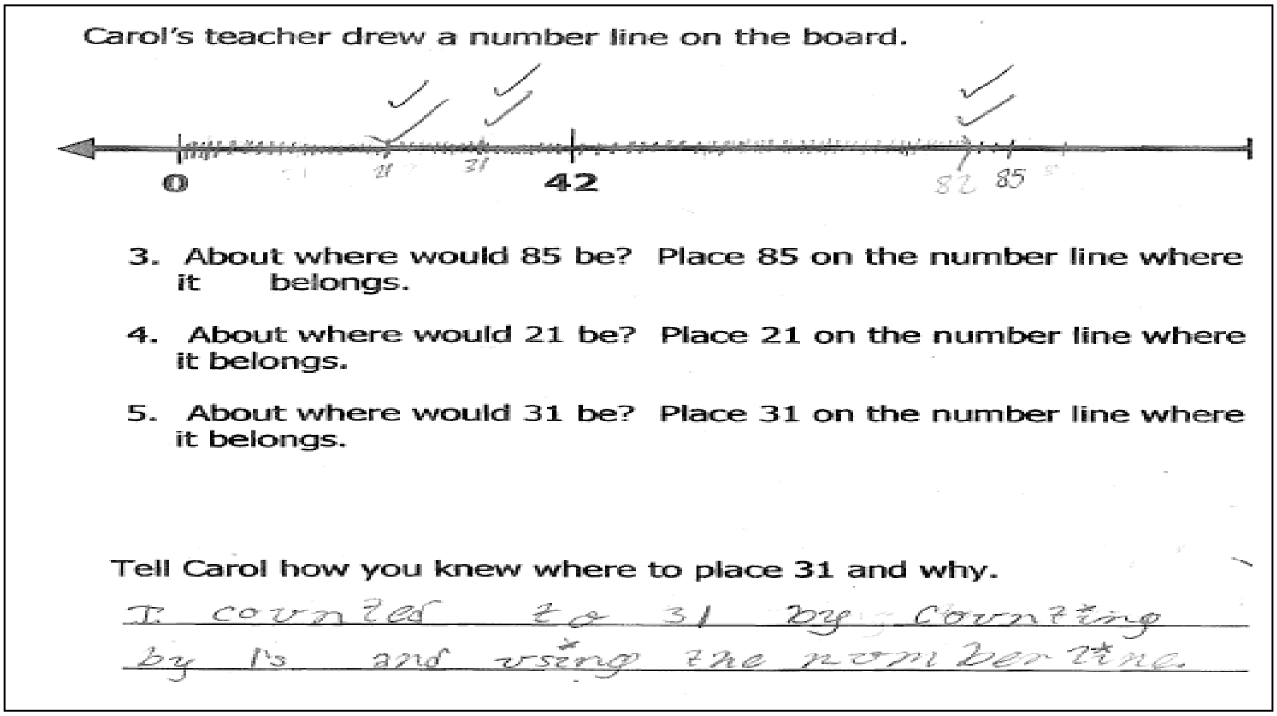
3. About where would 85 be? Place 85 on the number line where it belongs. 1
4. About where would 21 be? Place 21 on the number line where it belongs. 2
5. About where would 31 be? Place 31 on the number line where it belongs. 1

Tell Carol how you knew where to place 31 and why.

I know because 31 is farther away from 42 because 30 is onley 10 away from 40 and I just add 1 to the 30 and two to the 40.

Students P and Q used a counting strategy that involved drawing in a hashmark onto the number line for each number that would be between the placed numbers. How does the notion of scale and proportion relate to what they did?

Student P


Carol's teacher drew a number line on the board.

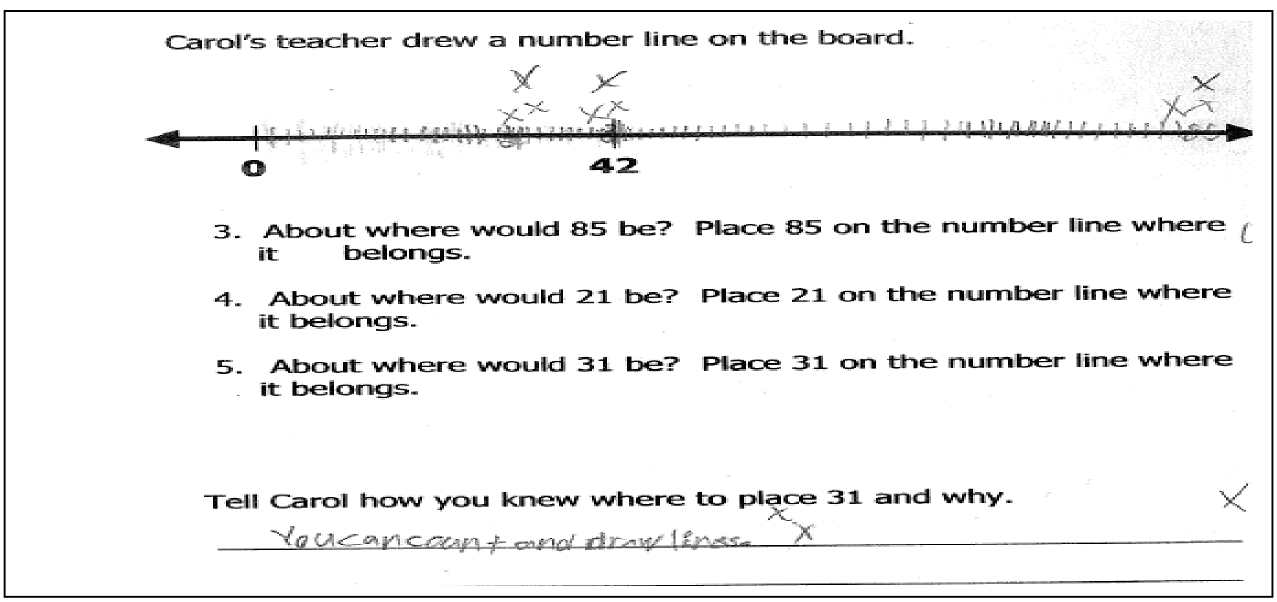
0 21 31 42 82 85

3. About where would 85 be? Place 85 on the number line where it belongs.
4. About where would 21 be? Place 21 on the number line where it belongs.
5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

I counted to 31 by counting by 1's and using the number line.

Student Q


Carol's teacher drew a number line on the board.

0 42

3. About where would 85 be? Place 85 on the number line where it belongs.
4. About where would 21 be? Place 21 on the number line where it belongs.
5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

You can count and draw lines.

(If it's hard to read, 21 is under the first cluster of x's and 31 is on top of 42)

2nd Grade Task 5 Carol's Numbers

Student Task	Arrange three digits to make the largest and smallest combination. Place numbers on an open number line, using scale. Give a mathematical explanation for their work.
Core Idea 1: Number Properties	**Understand numbers, ways of representing numbers, relationships among numbers, and number systems.** • Understand the relative magnitude of whole numbers and the concepts of sequences, quantity, and the relative position of numbers. • Demonstrate an understanding of the base-ten number system and place value concepts.

Mathematics in this task:

- Make the largest and smallest numbers possible using the digits 7, 4, and 2.
- Explain why 247 is the smallest number possible using the digits 7, 4, and 2.
- Place the numbers 85, 21, and 31 onto an open number line already positioned with 0 and 42. Numbers must be placed using a sense of scale and proportion.
- Explain why 31 would be placed just under the halfway mark between 21 and 42.

Based on teacher observations, this is what second graders know and are able to do:

- They can make the largest and smallest numbers with the digit cards.
- They understand the sequence of numbers in a range, knowing which numbers come before, after, or between others.

Areas of difficulty for second graders:

- Making accurate placements on the number line when the scale and proportion had to make sense for the numbers.
- Making written explanations of their mathematical reasoning around place value and relative magnitude of numbers.

Strategies used by successful students:

- Quantify distances between numbers (31 is 10 more than 21; is is 11 les than 42).
- Use scale markings when measuring between numbers on the number line.
- Think of the number line as a measurement.
- Recognize relative locations of numbers that are about *halfway* or in *the middle of* two numbers.

MARS Test Task 5 Frequency Distribution and Bar Graph, Grade 2

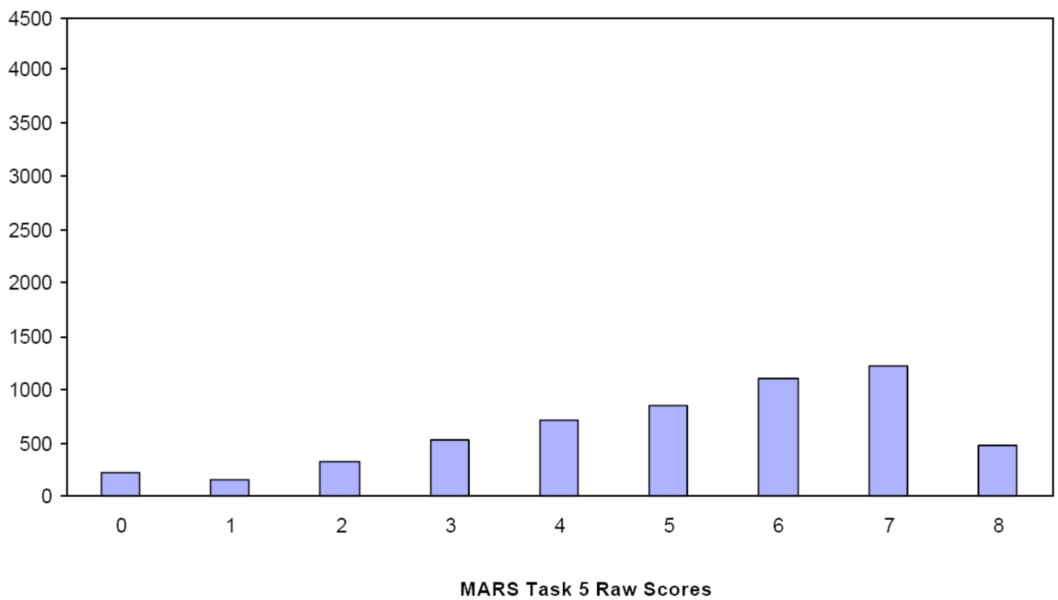
Task 5 - Carol's Numbers

Mean: 5.09 StdDev: 2.08

Table 14: Frequency Distribution of MARS Test Task 5, Grade 2

Task 5 Scores	Student Count	% at or below	% at or above
0	229	4.1%	100.0%
1	159	6.9%	95.9%
2	317	12.6%	93.1%
3	525	21.9%	87.4%
4	716	34.7%	78.1%
5	844	49.8%	65.3%
6	1106	69.5%	50.2%
7	1231	91.4%	30.5%
8	481	100.0%	8.6%

Figure 23: Bar Graph of MARS Test Task 5 Raw Scores, Grade 2



Carol's Numbers

Points	Understandings	Misunderstandings
0	None of the students in the sample with this score attempted the task.	
2	Students working at this level could correctly place one or more digits in the correct place to meet the place value requirements of the task. Some students could accurately place 85 on the number line. Some students could place the 31 between the 21 and the 42 on the number line.	Many students did not use the digits 2, 4, and 7 from the problem for Parts 1 and 2. Students scoring in this range did not attend to the relative magnitude of the numbers, or the placement of the **0** and the **42.**
3	58.3% could correctly make the largest and smallest 3-digit numbers from 2, 4, and 7. Around 40% could accurately place the 85 and the 31 on the number line. Students used sequencing words such as "comes before" or "comes after" to explain placements on the number line.	Only half the students who successfully completed building the numbers in Parts 1 and 2 could adequately explain their reasoning. 25% ignored the **0** and **42** that were already placed, resulting in placing 31 between 42 and 85.
4	84% made the greatest and least three-digit numbers correctly using the 2, 4, and 7. These students were able to make a statement that the 2 should come "first" for the smallest number, and/or the 7 should be "first" for the largest number. 75% accurately placed the 85 and the 31 on the number line.	Only 25% were able to place the 21 as halfway between **0** and **42**. Mathematical explanations were incomplete or vague. Students who explained the place value of the three-digit number stopped after explaining the 100s place. Students working at this level described 31 as being "between" 21 and 42, but generally made no reference to a *distance* from either, nor to a relative position of "about halfway".
6	Students could make the greatest and least three-digit numbers correctly using the 2, 4, and 7. Almost 50% could make a complete mathematical explanation of their strategy for doing so. Students could accurately place the 21, 31, and 85 onto the number line, including placing the 21 about halfway between **0** and **42** and placing the 31 about halfway between 31 and **42.**	Students continued to struggle with making an accurate and complete explanation, that demonstrates an understanding of relative magnitude, for placing the new numbers on the number line.
8	Students used a knowledge of 100s, 10s, and 1s to explain their greatest and least numbers. Students used language of measurement, either quantifying the distance between 21 and 31 (10) or using words like "middle" or "halfway between", when describing their work on the number line.	

Teaching Implications

Students who worked this task struggled with the vocabulary need to adequately and accurately explain their reasoning around place value and quantity.

"Hundreds place" was referenced as "first". "Digits" were called "numbers". 19.7% of the students worked this task at the level of 6 out of 8 points, and they described numbers as coming "before", "after", or "between" numbers, instead of quantifying the distance measured between the numbers as "10 more", "11 less", "about halfway", or "in the middle".

Panchyshyn and Monroe, in their article *Vocabulary Considerations for Teaching Mathematics* (Childhood Education, 12-22-1995), describe the complications of teaching vocabulary in the elementary math classroom.

They describe the mathematics vocabulary as *technical*, *subtechnical, general, or symbolic*.

- "Technical" vocabulary has only one meaning, and it's a specific mathematical meaning. *Examples:* quadratic; integer.
- "Subtechnical" vocabulary can have more than one meaning (even within math, such as degrees in temperature versus degrees in an angle measurement) but most importantly these are words that have a specific mathematical meaning in math class, but also has one or more meanings in general or other content-specific vocabulary. *Examples:* the volume of a cube; the volume on the television; the volume of trade on the stock market. The word "table" is another word that has a specific meaning in math, but an entirely unrelated meaning in the general usage vocabulary of most students.
- "General" vocabulary are words that are not technical or subtechnical but still make up the majority of language in elementary math textbooks. Even though it's considered "general" language, the authors did a 1992 study in which they discovered that more than ½ the words included in the math texts were *not* among those most frequently used in children's reading materials.
- "Symbolic" language includes numerals (symbols that represent quantities), exponents, and the symbols for inequality and operations, etc.

Students working this task struggled to find the specific math language to explain their thoughts. Where do words like between, before, after, middle, half, first, last, smallest, "next smallest", "tens", etc. fit into this continuum of vocabulary words. How might the multiple meanings of most of these words be contributing to the students' confusion?

Marilyn Burns did an interview with Instructor in April, 2006 ("An Expert's Guide to Teaching Math's Unique Vocabulary") in which she described the purpose of learning the language of mathematics. The purpose is "communicating about mathematical ideas and it's necessary to first acquire knowledge about the ideas that the mathematical language describes. Only when I understand mathematical ideas do I have a reason for learning the correct language of mathematics to communicate about those ideas."

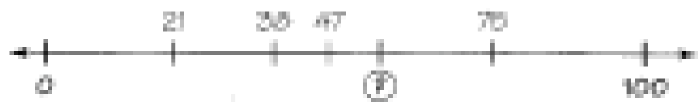
Looking back at the requirements of this task, what opportunities have the students had to develop their ideas of relative magnitude? Have they used or created open number lines? Are they expected to logically deduce where new numbers would go on an existing number line, with regard to scale and proportion? If not, are there activities in the Ideas for Action Research section that follows that can provide these activities, where the language can then begin to grow?

Ideas for Action Research

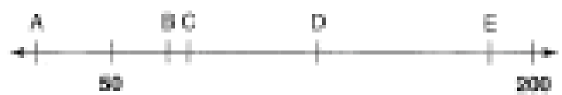
John van de Walle describes "relative magnitude" as a reference to the "size relationship one number has with another – is it much larger, much smaller, close, or about the same?" (*Teaching Student-Centered Mathematics K-3*, Pearson, 2006 p. 142)

How might the following open number line activities be worked into classroom routines to provide students with the opportunity to develop these big ideas:

- **Who Am I?** (p. 142) "Sketch a line labeled 0 and 100 at opposite ends. Mark a point with a ? that corresponds to your secret number. (Estimate the position the best you can.) Students try to guess your secret number. For each guess, place and label a mark on the line. Continue marking each guess until your secret number is discovered." Vary the endpoints. Can students give a reason for their guess?



- **Who Could They Be?** (p. 143) "Label two points on a number line (not necessarily the ends). Ask students what numbers they think different points labeled with letters might be and why they think that. In the example shown here, B and C are less than 100 but probably more than 60. E could be about 180. You can also ask where 75 might be or where 400 is. About how far apart are A and D? Why do you think D is more than 100?"



- **Close, Far, and in Between** (p. 143) "Put any three numbers on the board. For first and second grade, use two-digit numbers and modify the questions accordingly. With the three numbers up (example: 219, 457, 364), ask questions such as the following and encourage discussion of all responses:
  - Which two are closest? Why?
  - Which is closest to 300? To 250?
  - Name a number between 457 and 364.
  - Name a multiple of 25 between 219 and 364.
  - Name a number that is more than all of these.
  - About how far apart are 219 and 500? 219 and 5000?
  - If these are "big numbers", what are some small numbers? Numbers about the same? Numbers that make these seem small?