PAM BROUSSEAU: Melissa, what concept are you going to be teaching today?

MELISSA NIX: So, today we're going to be looking at, uh, area of polynomials. And students have looked a little bit about the exterior dimensions already of an area model, and they've been able to play a little bit with getting the actual dimensions of the area, but this is going to take it one step further in that we're going to be providing the area and have them then try to derive what the two dimensions would be. So, the exterior length and width for example. And there's an added twist to that too that today's lesson is going to be a bit of a challenge because it's going to ask them to do that with multiple variables, so x and y or y and z, and an integration of fraction and decimal coefficients. So, they've had some practice already doing some polynomial multiplication, length and width, finding the area, but this is going to be one layer, um, of a challenge more.

PAM BROUSSEAU: So, you're hoping that this is going to solidify the knowledge that they already have and then push them forward?

MELISSA NIX: Yeah. Ultimately, my -- my goal is to then kind of show them how they can use this model to apply to any multiplication of polynomials as well as then factoring, so that they can use their knowledge of this visual to work backward. You know, integrating their previous understanding of the distributive property, which they first learned in 6th grade, worked with again last year in 7th grade, and we've been kind of practicing the last few days. Just reminding them that this is something that they've been exposed to and now we're going to take it that one step further.

PAM BROUSSEAU: So, it sounds like there's some connectedness to it and some connectedness to their prior knowledge and where they're going to move to.

MELISSA NIX: Right.

PAM BROUSSEAU: And, um, can you talk about other connections that you would hope to see them make today?

MELISSA NIX: Well, one of the things I thought about is, um, when planning today's lesson, you know … So, I want to start with a number talk to get them kind of primed. It is an early morning class and they come in somewhat sleepy. So, to get them thinking about mathematics, I'm doing a double-digit multiplication, um, as an opener, but I'm hoping that somebody will show how to do that multiplication problem using the area model. So, breaking it down into partial products and multiplying. And then my second number talk is going to be integrating decimals and fractions just to make sure that they have that as a going into the task at hand …

PAM BROUSSEAU: So, you chose both of these number talks very, very purposefully.

MELISSA NIX: Correct.

PAM BROUSSEAU: And, you're hoping with the double--digit multiplication number talk that someone's going to come up with the area model that will segue nicely into the lesson?

MELISSA NIX: Yeah. One of the, um, possibly misconceptions that I think students might see is when they're trying to multiply the dimensions of my rectangle that they're going to erroneously take the two parts of the length, for example, and multiply them. So, instead of it being like 2*z* and 8, they're going to try to say that that's 16*z* rather than seeing it linearly that they're adding those two, but if we can tie it into that number talk and say, look, you know, when you did a number talk, how did we get, for example, 12 was 10 and 2, you know. What would be the connection there when they're doing the dimensions of the rectangle? So, I'm hoping that that'll tie into their prior knowledge and -- and kind of help support them, or we can reference back to that to -- if somebody gets stuck as a more concrete model for then the algebraic abstract …

And then the second number talk ties in the multiplication of decimals and fractions just to resurrect that skill that can get dusty with middle schoolers. [Laughs.] So.

PAM BROUSSEAU: [Laughs.] Absolutely.

MELISSA NIX: And high schoolers, and adults.

PAM BROUSSEAU: [Laughs.] Um, thinking of the true dimension, what, which of these dimensions might your lesson hit the most?

MELISSA NIX: Yeah, um, I'm glad you said the most, because I can see elements of all of these dimensions in the lesson. You know, from access with those number talks. I am going to do an exit ticket so I can, will be able to have some formative assessment by the end of the lesson. Um, but I think because of where we're at right now with the trajectory of how I've been teaching it through the last week and a half, I think today's lesson's going to be a good example of cognitive demand. It's sort of, uh, at that sweet spot where it's going to be challenging for some, and it's going to be, um, more easily accessed for others. And I think they're not going to be totally lost at sea, but they're going to have enough of a push that's -- it's -- that's one leap forward considering what we've been doing. We've done some work already with the distributive property, using some variables and coefficients. But they haven't done a lot of multiplying polynomials in general. So, and certainly working backward to look at the product, and then try to find the factors, is going to be one shift more challenging. So I'm looking forward to seeing what they do with that, and then having students share out where they're at with their learning, so that others can learn from those students.

PAM BROUSSEAU: Okay, so let's talk about the mathematics and the cognitive demand. So talk a little bit about how you sequence the lesson. And then also, what's going to happen, what strategies are you going to use, or what strategies are you expecting them to use when they start struggling?

MELISSA NIX: Um, so the lessons was sequenced -- is sequenced such that, you know, I'll start with my number talk, but that's going to segue way into just letting them look at some, two rectangles, and identify what the dimensions are and how they're related and if they're related, and could they come up with the area of those? Um, and then I'll combine those two rectangles to kind of see, you know, what does area ... What does it do? How does it change? What's the area now of that? And let them try to come up with, um, what that answer would be. I keep, I continuously turn it back to the students to come up with what the work is without me trying to tell them. See what they come up with.

PAM BROUSSEAU: So they have to do the thinking?

MELISSA NIX: Yeah. Um, absolutely. And if they are stuck, then I always turn it back onto, you know, try to do the thinking first on your own, check in with your partner, and then if you need to square the pair or to go to a group of four, we'll do that as well.

PAM BROUSSEAU: And is that kind of built in, where they do the personal think time and then they share with a partner and then they square the pair?

MELISSA NIX: It's depending on the level of the task or how much it is ... In fact, in today's lesson, when I'm doing some of the, kind of the concept development, when I'm building the lesson a little bit, we'll be doing more pair work. So individual, then pair. But when they're in the bulk of the lesson and they're trying the challenge of finding all the different areas and dimensions of the, um, carnival. Then I will have them work in groups of four. And you asked what do they do if they get stuck?

PAM BROUSSEAU: Mm-hmm. [affirmative]

MELISSA NIX: So throughout that lesson, I'll be popping around and kind of checking. If somebody's already one step further, then I might interrupt the whole lesson and say, hey, you know, our student over here found a really good strategy that's moving him, moving this particular student forward. Let's just look at that and see if that gives you any ideas or suggestions or hints to move you forward. So I'll try to do that accordion of, like, okay, I'm going to let you out a little bit, now I'm going to pull up back in and give you some ideas, and then I'll let you out again. Um, so we'll see.

PAM BROUSSEAU: So facilitating it, and it's not coming from you, it's coming from them.

MELISSA NIX: Right. Um, it, they'll be able to try to work that out, I hope, in their pairs or in their fours. But some students might be on the other side of the room, and might not know that this student did something that moved them forward, and they might still be stuck. So if I see that, as the facilitator, I kind of want to maybe move them forward with that information.

PAM BROUSSEAU: Okay. And what opportunities will they have to engage deeply in the mathematics?

MELISSA NIX: Well, I think ... I mean, the level of the task itself is kind of deep.

PAM BROUSSEAU: Okay.

MELISSA NIX: Um, it's not a very structured, rote practice. It's more open-ended, where they're applying it to an actual problem. So I think seeing how, you know, polynomials are applying to real life is on a deeper level. And then trying to, like, solve this challenge, this puzzle, um, they'll be engaging with the mathematics. And really, trying to manipulate those numbers to see, how does the distributive property work? How does it work in both directions so that I can multiply into and create my product, and I can factor out of, and, and find those factors? So that, just how those are connected. I mean, it's really hitting on, you know, SMP seven and eight, where they're looking for structures and that repeated reasoning that, to kind of look for that, how they're all intertwined and connected. And I -- I'm hoping that the way it's sequenced will make those connections. Um, and we'll find out.

PAM BROUSSEAU: Well, I'm excited to see the lesson.

MELISSA NIX: And so I want you to, like all number talks, not use any pen or pencil. So, you can go to the whiteboard in style, but put your pen down please. Okay. And this is all mental. Think about how you're going to solve this. And be prepared, please, to share your strategies for your thinking.

When you have the answer, give me a quiet thumb at your chest. And then test yourself to see if your strategy worked, and see if you can get another strategy, and if you do get a second strategy, put out a second finger. Or a third, or a fourth.

What do you all think the answer is?

STUDENTS: 180.

MELISSA NIX: Do you need to look at it again? You can call it out to me, is what I'm saying.

STUDENTS: 180.

MELISSA NIX: 180?

STUDENTS: 180.

MELISSA NIX: 180? All right, 180. Does anyone think it's something other than 180? No? All right, so, how about a way to solve this? Ellie, what'd you come up with?

STUDENT: Um, I know that 12 times 12 is 144, so then, I … 3 plus 12 is 15, so I added 3 times 12, which is 36, to 144.

MELISSA NIX: So, you knew that 12 times 12 was 144.

STUDENT: Yeah.

MELISSA NIX: And you said that 12 plus 3 was your 15. Tell me why you knew you needed to do 15 of these.

STUDENT: Because the, it's 12 times 15, not 12 times 12.

MELISSA NIX: So, it's 15 groups of 12?

STUDENT: Yeah.

MELISSA NIX: So, here are your 15 groups of 12, and you added this up? And when you added this up, how did you figure out that that equaled 180?

STUDENT: Um, 4 plus 6 is 10, and 3 plus 4 is 7, so then you add the 1 to the 7 … [crosstalk]

MELISSA NIX: So, did you stack it like this in your head?

STUDENT: Yeah.

MELISSA NIX: Okay, so you were able to stack it, and come up with it that way? Thank you. Did anyone else solve it like Ellie did? How about someone who did it differently? Heaven, what'd you come up with?

STUDENT: Um, 12 times 10 is 120. And you have to have the extra 5 from the 15, so 12 times … 12 times 5 is 60, and I added those, the 120 and 60.

MELISSA NIX: So, you took this 15, and broke it down? Turn to your partner for a minute, I think we've done this, uh, a couple day ago. Is there a property in math that is allowing you to --

Turn to your partner and talk about it for a second.

STUDENTS: [crosstalk]

MELISSA NIX: So, the property that's allowing her to break this apart, and multiply this in one piece, and this in another, is, on the count of three, called the one, two three …

STUDENTS: Distributive property.

MELISSA NIX: Right on, that's, I thank you for knowing that. Heaven, thanks for your idea, that's awesome. Did anyone do it differently? Ethan, what'd you got for me?

STUDENT: Um, I did 15 time 3, which was 45.

MELISSA NIX: You did 15 times 3?

STUDENT: Yeah.

MELISSA NIX: Whoopsie, that color will not show up. 15 times 3, which is 45.

STUDENT: And then, I multiplied, like, the 45 by 4 [inaudible].

MELISSA NIX: And then you multiplied your 45 by 4.

STUDENT: Which was 180.

MELISSA NIX: Which was 180. And how did you know, two questions, how did you know, one, that 45 times 4 was 180?

STUDENT: Um, because 45 plu-, times 2, is 90.

MELISSA NIX: And 90 and 90 got you your 180. But why 4?

STUDENT: Because 3 times 4 is 12.

MELISSA NIX: Because 3 times 4 is 12.

STUDENT: Right.

MELISSA NIX: Is there anybody else who can explain what Ethan's thinking there? So he said he did 15 times 3, and he got 45, and then he took that product and took it times 4. And I asked him, "Why 4?" And he said, "Because 3 times 4 is 12." Does that make sense to anyone? Makenna, what are you thinking?

STUDENT: Um, so, 15 times 3 equals 45, meaning that 45 is the -- like, 15 times 3, 45, are the same thing. So, it would've been like doing 15 times 3 in parentheses, and then having the 4.

MELISSA NIX: Maybe like that?

STUDENT: Yeah. And then that would equal 180. And the 3 times the 4 equal the 12 in the original problem.

MELISSA NIX: Interesting, thank you. Does that capture your thinking, Ethan?

STUDENT: Yep.

MELISSA NIX: Okay.

MELISSA NIX: I'm gonna move on to another number problem, to see how you're doing with that. You shared all this good thinking. Here's another one for you. Again, chest -- finger on your chest when you come up with an answer. And then push yourself to see if you can come up with a solution another way.

There are many ways to solve a problem. And I value your creative thinking.

Honestly, do you find this one a little bit more challenging? Yeah. It's okay. Challenging's good. Right? It's making our -- it's making our brains grow. It's okay to push yourself a little bit. Now, let's move these around and see what we come up with!

Do you wanna suggest what answers you think it is? So, on the count of three -- one, two, three -- what do you think it might be? One, two, three?

STUDENTS: 6.

MELISSA NIX: Okay. So we think it's 6. Anybody thought maybe it was not 6? Okay. Does anyone wanna share? Anyone! Whole room's welcome. Anyone wanna share? Somebody came up with 6 somehow. Ellie, you wanna be first again?

STUDENT: Um. First I ... Times 0.5 is the same as divided by 2. So I did 16 divided by 2 is 8.

MELISSA NIX: So you said, 0.5 is the same thing as one half, which is the same thing as dividing by 2. So, knowing that information, you said, "I'm gonna do 16 divided by 2," and you got 8. And then you took 8, and then what next?

STUDENT: Um, to, to multiply by three-fourths, first I divided by 4, which is 2.

MELISSA NIX: Mm-hmm. [affirmative]

STUDENT: And then I multiplied by 3, which is 6.

MELISSA NIX: You could have written that out a little bit differently. If I had written it, such as this ... Is that the same thing? How do we know that's the same thing? Who can tell me why that's the same thing?

These are equivalent expressions? I got nods, but no one's willing to tell me why? Itzel, tell me why that's the same thing.

STUDENT: Well, because, 8 divided by 1 is 8, so it's the same as 8.

MELISSA NIX: Okay. So, it's 8 divided by 1 is the same thing as 8. So I am allowed to write it like that. And then am I allowed to write it where I multiply my numerator, and I multiply my denominator? Does that still feel like the same thing as this one above it?

And so then what Ellie's doing is moving these in the different directions, so she's then dividing by 4. Does that feel legal? Is there a property that allows us to move the numbers like that?

STUDENT: [inaudible]

MELISSA NIX: Say it to me again, Miranda?

STUDENT: Commutative?

MELISSA NIX: Yeah, you're right. Everyone, say that -- what's the property that allows us to move those numbers if it's multiplication or addition? It's the ...

STUDENTS: Commutative.

MELISSA NIX: Commutative property. So she commuted those numbers to make the math easier to envision.

Anyone else do it similar to the way Ellie did it? Awesome, that means somebody else did it differently.

STUDENT: [Laughs]

MELISSA NIX: Do tell! Now it's crickets ... No one? You know, I heard more than one person scream out 6.

Okay, let's try it together then. What happens if ... I didn't take one half of 16 first? Could I take three-fourths of 16?

STUDENT: [inaudible]

MELISSA NIX: So using what Ellie did here, is there something I could do to make the math easier, if I wanted to take three-fourths of 16? Or what do you think you would do if you were to take three-fourths of 16?

What are you thinking, Nicky?

STUDENT: Um, you can do 4 times 4 to get 16. And so ...

MELISSA NIX: So ... So 16's the same thing as 4 times 4?

STUDENT: Mm-hmm. [affirmative] Then, if you do, um, three-fourths of that, it would be 12.

MELISSA NIX: So three-fourths of 16 would be 12. And now I have, five-tenths times 12 or 50% of 12.

STUDENT: 6.

MELISSA NIX: You get 6!

MELISSA NIX: Okay, your brains feeling okay? A little bit stretched? They're gonna get a little bit more stretched. So stretch your arms, you're gonna need a little bit of exercise here, because we're gonna do some math-aerobics. Um. Not quite! You're gonna be totally skilled at doing what we're doing, and you're gonna push yourself to known how to do this.

I intentionally add in some fractions and decimals, because you're gonna see them again in a few minutes. And I just don't want you to feel blindsided by them. I want you to think about the knowledge you already have, about how to move these things around to apply to what we're doing today. That sound okay?

All right!

MELISSA NIX: I'm gonna show you two figures right now, and I want you to think about what mathematical questions can we ask about these figures. So, you're thinking by yourself for a second, and then I wanna have you turn to your neighbor and chat it through. So, let's take 10 seconds of think time.

All right, go ahead and turn to your other partner, and discuss what mathematical questions can we ask about these figures.

STUDENT: Or the, the perimeter.

STUDENT: Yeah.

STUDENT: The other perimeter.

STUDENT: What is the perimeter of the figure?

STUDENT: I would also ask why the sides are the 2*z* [inaudible]. They're just kind of different.

STUDENT: We just, we need to see, to figure out what one is, is one, one variable of 8. We just need the extras of 2*z* to find out that one. But, we have nothing for that one, so.

MELISSA NIX: What'd you and your partner talk about?

STUDENT: We talked about, um one mathematical question we could ask is what's the, what amount is *z* ...

MELISSA NIX: What's the--

STUDENT: On the first figure.

MELISSA NIX: What, it's, the value of *z*?

STUDENT: Yeah, that.

MELISSA NIX: Thank you. *Z*, the value of *z* might not always be the same value, it might actually vary. What do you call *z*?

STUDENTS: A variable.

MELISSA NIX: Go ahead, you can say it to me. What is it called?

STUDENTS: A variable.

MELISSA NIX: All right, so what's the value of *z*, on the first figure. And then, Miranda, did you come up with, did you or your partner come up with another mathematical question?

STUDENT: What's the area of the second figure?

MELISSA NIX: What's the area of the second figure. Thank you. Ah! Qwentin, did you come up with something?

STUDENT: Um, no, we had the same idea of what is the value of z.

MELISSA NIX: That's the same some--, that's the same something, the same idea's a good idea. Thank you. Ellie, did you come up with something?

STUDENT: What are the perimeters of the figures?

MELISSA NIX: What are the perimeters of the figure. I appreciate that, when you look at a problem, you start thinking about, all right, what might the potential questions be of this particular problem? Whenever you're tackling into a problem, that's a good thing to ask yourself. Before you even read what are they asking me, start asking yourself, "What might I be asked?"

I'm gonna ask you now, if I showed you these answers, what do you think the question was? Sorry for those of you in the back who can't see. It says, "Blue: $2z(x)=2x$ squared," and it says, "Orange: $2z(8)=16z$."

Go ahead and chat with your partner, and then be prepared to convince me that you think that's really what I asked.

STUDENT: Like, what is the area of the -- of the rectangles?

STUDENT: Yeah, because they multiplied the two variables that they had, which is why, I would say, like, area.

STUDENT: Yeah.

STUDENT: By each other, which ...

STUDENT: $16z$.

STUDENT: Yeah, exactly.

STUDENT: [inaudible] be area.

STUDENT: It's z times z, so you have $2z$ squared.

STUDENT 8: Mm-hmm. [affirmative]

MELISSA NIX: Rolando, what do you think I asked when I show these answers, when I get these answers -- what do you think the question was?

STUDENT: What is the area of the rectangles?

MELISSA NIX: What is the, what is the area of the rectangles?

STUDENT: Mm-hmm. [affirmative]

MELISSA NIX: And can you convince me that the area of the blue rectangle really is 2*z* squared? Or what would you tell me if you were to convince me?

STUDENT: Well, *z* times *z* is *z* squared.

MELISSA NIX: Mm-hmm.

STUDENT: And then there's a 2, so ...

MELISSA NIX: And then there's a 2?

STUDENT: Yeah, so, like, two sets of them.

MELISSA NIX: Oh, two sets of them? Is that something you could draw to help me see that? Does anyone think they can draw on my little diagram up here to help see the two sets of *z*'s?

How many *z* squares would I end up with over here? Two. What shape would my *z* squares be?

STUDENTS: Squares.

MELISSA NIX: Why?

STUDENT: Cause it's ...

MELISSA NIX: Cause it's why?

STUDENT: Well ...

MELISSA NIX: Who thinks -- what do you think?

STUDENT: Because, um, you're multiplying *z* by *z*, so it's the same length.

MELISSA NIX: Okay, we'll find the same length, by the same width, so it's the same value, same value, so you're gonna get a what shape?

STUDENTS: Square.

MELISSA NIX: A square. So, what if I do that? Now what do you see in that picture?

STUDENTS: Two squares.

MELISSA NIX: I have two squares, and what are the names of those two squares?

STUDENTS: *Z* square.

MELISSA NIX: So, Rolando, what would you label this dimension?

STUDENT: *Z*.

MELISSA NIX: Why?

STUDENT: Because *z* times, *z* times *z* is *z* square.

MELISSA NIX: So, *z* here, and *z* here, and *z* times *z* is *z* -- is *z* squared. And *z* times *z* is *z* squared. Turn to your partner and decide, do you agree that this *z* and this *z* are the same thing as two *z*'s?

STUDENTS: Yes.

STUDENT: Two *z*'s.

STUDENT: That's all we're missing.

STUDENT: Multiplying 2 by *z*, so there's *z* at the end, there's *z* down there, and we know that, because it's *z* squared. And the square has four equal sides. So they all have to be *z*.

MELISSA NIX: How did you know that *z* and *z* got me 2*z*? Does that look like anything we've already done before?

I heard some people talking about it. Like, I heard this group talking about it. So I'm gonna call on Mason from this group and say, "Mason, does that look like anything we've done before?"

STUDENT: Yeah, like the, um, 15 times 4 problem?

MELISSA NIX: Oh, so when we did the area model of multiplication, and we did 15 times 12.

STUDENT: Or whatever two-digit ...

MELISSA NIX: Or whatever two-digit ... Like, it -- we did 15 times 12 I think, um, if not today then a couple days ago. But when we did 15 times 12, what did we break that down into?

STUDENT: Well, some people ... It was different from how we broke it up. Everybody did different, but, you can do 12 times 10 and 12 times 5 ...

MELISSA NIX: 12 times 10, and 12 times 5 ...

STUDENT: Yeah.

MELISSA NIX: Or if we did it the box method, it would be 10 and 5 and, 10 and ...

STUDENT: 2.

MELISSA NIX: 2. So, this quantity of my length is the 12, and what math am I doing to these two values here?

STUDENT: Adding them.

MELISSA NIX: So, who can write -- say that for me in a full sentence as to how I knew that that was ... 2*z* was *z* and *z*?

MELISSA NIX: [inaudible] ... picking up what I'm putting down?

STUDENT: Mm-hmm. [affirmative]

MELISSA NIX: Brave soul. Who thinks they can explain to me why that's 2*z* there? Maybe using this as an example?

Talk to your partners. See if you can come up with some bravery.

STUDENT: It's two *z*'s.

STUDENT: Two *z*'s.

STUDENT: It's not *z* times *z*, because that's *z* squared, since you multiply those.

STUDENT: Because 10 plus 2 is the 12 parts, and then if it was z plus z it would be $2z$.

MELISSA NIX: Did you and your partner come up with anyway that you could explain--

STUDENT: If you add the sidelines of the square it come up to $2z$.

MELISSA NIX: So that added the sidelines of my square or the dimensions of this figure, it would come up to $2z$. Great thinking! Check this one out ... On your whiteboard, can you write an expression for the area of this figure? Once you write an expression for the area of the figure, see if you can prove that it is correct by maybe drawing the picture like we just did on the slide before.

For the area of this figure ... is my expression as simplified as it can be?

There's a couple ideas that I want to show you just to kind of review a little bit while we're here. So on to what we just looked at, this person took this and split it into how many pieces?

STUDENT: Two.

MELISSA NIX: Two pieces. And why two pieces?

STUDENT: Because there's two ...

MELISSA NIX: Because there's two *z*'s. Now I'm going to ask real quick, though: Now that we've combined these two rectangles, are the two *z*'s only for this two -- this first shape?

STUDENT: No.

MELISSA NIX: Then how can I see that in your picture? Go ahead and keep trying.

Before we go to break, I'll show you one more. Let's all look up here for a second, at somebody else's thinking. See if we can understand what they're thinking. I stole this paper from Leila. Would be willing to help tell me sort of what you're thinking?

STUDENT: So the first, the smaller rectangle works better. It's two so you'd have to select it earlier, which is z squared and ... z squared. And the other rectangle would be, uh, just 8 more.

MELISSA NIX: Be 8 more. Now, up here, I see the length should be how much?

STUDENT: 8.

MELISSA NIX: 8. Is that where we're at here? What is the length up here?

STUDENT: 4.

MELISSA NIX: 4. So I'm going to excuse you to break, and I want you thinking, "If this is a gift, how can we modify it to make it express what we want it to when we come back in?" I'll see you in a few minutes.

MELISSA NIX: Leila, you came to me at break and said you wanted to do something. What would you like to do?

STUDENT: Uh, I'd like to fix the picture.

MELISSA NIX: Oh, okay. So, you revised your thinking. Go on up.

STUDENT: Yes. I did.

MELISSA NIX: Want to go on over to the whiteboard? Or to the back of the camera. Would you be so kind as you revise your thinking, to tell me what you're doing and why you're doing it.

STUDENT: Well, my picture shows actually only 8 here. When it should be 8 times 2, which is 16. So, I need to add about four more lines, I think. To fix it, so it shows 16. So, now it's actually turned to the equation.

MELISSA NIX: So, now the top -- this unknown dimension's value here is how much, everybody?

STUDENTS: *Z*.

MELISSA NIX: *Z*. And, this is equal to ... count with me.

STUDENTS: 1, 2, 3, 4, 5, 6, 7, 8.

MELISSA NIX: 8.

STUDENT: Yes.

MELISSA NIX: And, this one is equal to ... how much here?

STUDENTS: *Z*.

MELISSA NIX: And, how much here?

STUDENTS: *Z*.

MELISSA NIX: So, all together?

STUDENTS: *Z* squared.

MELISSA NIX: Oh.

STUDENTS: $2z$.

MELISSA NIX: Interesting. I heard different answers there.

STUDENT: $2z$.

MELISSA NIX: So, tell me what you think it is and why you think it's that?

STUDENT: I think it's 2*z* because if you -- wait -- if you divided that by half, it will be *z* and *z*. And, that will give you two squares. So, both the squares is *z* squared because *z* times *z* is *z* squared.

MELISSA NIX: So, *z* times *z* gets me *z* squared. And, so this length is just *z* and *z*, which is what again?

STUDENT: 2*z*.

MELISSA NIX: So, *z* and a *z* is how much everybody?

STUDENTS: 2*z*.

MELISSA NIX: So, 2*z*, *z*, and eight. And, all together you have down here -- Leila, tell me what was your equation simplified?

STUDENT: Um, 2*z* squared plus 16*z*.

MELISSA NIX: Give me a thumbs up if you agree with what she came up with. Thank you very much. I appreciate the gift and the modification, the edit of that. You may take your whiteboard back, yeah.

All right, you may clear off your whiteboards, please. I have another problem for you.

STUDENT: We just did one.

MELISSA NIX: You are going to draw a figure with the dimensions that I show you. And I'm going to start by asking, what do you know about this figure? What do you know about this figure? Y'all know something about this figure just by looking at it. So, I feel pretty comfortable pulling a stick and saying what is something you know about this, Scarlett. What is something you know about that figure?

STUDENT: Um, it's *a* squared *a*, so there's probably two 3*a*'s.

MELISSA NIX: Ah. Okay. Probably two 3*a*. It's 3*a* squared. Something else you know about this figure, Adonay.

STUDENT: It's got variables.

MELISSA NIX: It does have variables. And, the last one. Ella, what do you think? Oh, Ella is not here anymore, sorry. I'm going to change the letter at the end and call you, Ellie.

STUDENT: [laughs]

STUDENT: Um, I think that three *a* squared is the same as 3*a* squared.

MELISSA NIX: You think that three *a* squared units is the same as 3*a* squared. What do you think I'm going to ask you about this figure, which by the way is a rectangle? I was going to wait for that one, but no one said that one. Amanda was like, “Obvi.”

STUDENT: [laughs]

MELISSA NIX: All right. What do you think I'm going to ask you about this? Turn to your partner. What do you think I'm going to ask?

STUDENT: It's bigger.

STUDENT: It's bigger.

STUDENT: Yeah, what's the figure.

STUDENT: The figure.

MELISSA NIX: Think I'm going to ask what the area is?

STUDENTS: [crosstalk]

MELISSA NIX: Can you draw this figure instead?

STUDENT: Yeah.

STUDENT: If you -- like had a …

MELISSA NIX: If you haven't done so already, go ahead and draw this figure.

STUDENT: Make a square.

STUDENTS: [crosstalk]

MELISSA NIX: And, what do you think I'm going to ask you ... Come back to me in three ... two ... one.

STUDENT: And, then you would split it up into three parts.

MELISSA NIX: Makenna, what do you think I'm going to ask you about this question?

STUDENT: Um, maybe what the area is.

MELISSA NIX: Maybe what the area is. Well, let's think for a sec. Is area going to be ... What is area of a figure?

I asked you to solve for the ...

STUDENTS: Area.

MELISSA NIX: Area. And, when you solve for the area, as Leila showed us. She said it was 2*z* squared and 16*z*. Where in the figure does that area get recorded? Where in the figure does that area get recorded? Where do I write the area? What part of the figure did I write it in? Or on? Or into? Qwentin, where did I put that area? Where did I record it?

STUDENT: Uh, on the inside.

MELISSA NIX: On the inside. So, Makenna, do you think I'm going to ask you what the area is of this figure?

STUDENT: Probably not.

MELISSA NIX: Why not?

STUDENT: Because, um, it's like, lopsided figure. It's, um, lopsided.

MELISSA NIX: Yeah. Heaven, what are you thinking?

STUDENT: You already gave us the area.

MELISSA NIX: Right. I'm giving you the area right now. But, writing it on the inside of the figure, I'm trying to communicate a convention that says we tell you the area by telling you on the inside of the figure. Area is on the inside and what's on the outside?

STUDENTS: Perimeter.

MELISSA NIX: The perimeter or the dimensions of the figure. So, Heaven, if I'm telling you the area, what do you think I'm going to ask you about on this problem?

STUDENT: What are the dimensions of this figure?

MELISSA NIX: All right. Did you hear the question?

STUDENTS: Yes.

MELISSA NIX: What are the dimensions of the figure? That's your challenge.

MELISSA NIX: What are the dimensions of this figure, if you know the area is 3*a* squared plus 3*a*? And, by the way, there are many answers to this. And so if you can come up with one, see if you can come up with two. And then we'll share some about in a second. Do you have a clarifying question? Yeah?

STUDENT: Does it have to be a rectangle?

MELISSA NIX: Um, it doesn't have to be a rectangle. It doesn't have to be a rectangle. Uh, for sake of my examples in here today, because our next lesson is going to go into rectangles, I'm gonna have the first few that I share up be rectangles.

Go ahead and do a little private think time on your own, and see if you come up with, and I will let you know in about a minute when you can turn to your partner and check.

STUDENT: [inaudible] was squared but [inaudible]

MELISSA NIX: Is it? What makes ... How do you know that 3*a* times 3*a* equals 3*a* squared?

STUDENT: Um, because 3 times 3 is 3 squared, right? Or *a* times *a* could be a squared?

MELISSA NIX: It seems like you're thinking it through a little bit more. What would 3 times 3 give you?

STUDENT: 9.

MELISSA NIX: So is …

STUDENT: Mm-hmm. [affirmative]

MELISSA NIX: ... 9 in your answer at all?

STUDENT: No.

MELISSA NIX: Would it have to be if ...

STUDENT: Yes.

MELISSA NIX: Hmm. So what modifications can you make if you want your area to be 3*a* squared …

STUDENT: Hmm?

MELISSA NIX: ... but yet it's not working to have two 3*a*'s. I'm gonna come back and see what you go with that.

STUDENT: I think she said [inaudible].

STUDENT: I don't know.

MELISSA NIX: Go ahead and turn to your partner and think about, talk about what you were thinking.

STUDENTS: [crosstalk]

MELISSA NIX: Share your thinking with one another and see if together you can move your thinking forward.

STUDENT: ... *a* squared and *a* on the top, and then I did 3. So then it goes 3 times *a* squared to be 3*a* squared along, and then 3 times *a* is 3.

STUDENTS: [crosstalk]

STUDENT: Having *a* squares but this time it's going to be 3, because it's 3*a*. So you, you just have to make it so there's 3*a* squares.

STUDENT: If you make this sideline radical 3 and this sideline radical 3, then this is 3, multiple 3*a*, and then it'll go 3*a*. It says 3*a* squared. And if you made this length radical 3*a* and this length radical 3*a*, then this was also 3*a* squared.

STUDENT: Ah, [inaudible] that last one.

STUDENT: But what is square units?

STUDENT: Just ... square, I assume.

STUDENT: Square?

STUDENT: Well it's just the units, isn't it?

STUDENT: I don't know that [inaudible] square--

MELISSA NIX: Knowing that the 3*a* might have come from *a* and *a* and *a*, this person said *a* and *a* and *a*, by *a* [laugh], must be three *a* squares. What do you think of that? Does that feel right to you? *A*, *a*, *a*. Do you agree that that's 3*a*?

STUDENT: Mm-hmm. [affirmative]

MELISSA NIX: By *a*? So I got *a* squared, *a* squared, and *a* squared. Love the fact that this person put down what they're thinking, and then said, "I want to know this next piece." What does the question say up here? Will you all read it with me? Where does 3*a* come in?

STUDENTS: Where does 3*a* come in?

MELISSA NIX: Where does 3*a* come in? How is this similar to the one we just saw? How is this similar to the one we just saw? What's the same? And, so, what's the same as this one and the one that we just saw.

STUDENT: Well that, um, there are three *a* squares.

MELISSA NIX: All right. So again, is the left side *a* and *a* and *a*?

STUDENT: Yes.

MELISSA NIX: Is that the same?

STUDENT: Yes.

MELISSA NIX: Does this one answer that question? Where's the 3*a* coming from in this model? Where is the 3*a* coming from in this model? Do you all see where the 3*a* is coming from in this model?

STUDENT: Mm-hmm. [affirmative]

MELISSA NIX: Brisa, do you see where the 3*a* is coming from in this model?

STUDENT: Yeah.

MELISSA NIX: Can you explain to me where the 3*a* is coming from in this model?

STUDENT: [inaudible]

MELISSA NIX: How did they get the 3*a*?

STUDENT: Because 1 times 3*a* is 3*a*.

MELISSA NIX: So there's a little bit of a glare, but the top half of this rectangle looks like it's *a* and 1. And this left side of the rectangle is 3a. So 3*a* by *a* would get me, I -- 3 *a* squareds, and 1 would get me 1. What math would I want to articulate that this is a 1? Think back to what Mason was telling us earlier about that connection with that multiplication problem. What math is letting me see that this is *a*, and what is this, multiplication up here?

STUDENTS: [crosstalk]

MELISSA NIX: What math would that be?

STUDENTS: Addition.

MELISSA NIX: Addition, okay. This one is slightly different. Did we answer your question, where the 3*a* came from?

STUDENT: Yeah. [inaudible]

STUDENT: Uh-huh. [affirmative]

MELISSA NIX: Check out this one. Is this the same answer or is this a different approach? This one also simplifies to 3*a* squared plus 3*a*, but this person approached it differently. Do we agree with this math? What happens when 3 and a squared come together and multiply?

STUDENTS: 3a squared.

MELISSA NIX: What happens when 3 and *a* come together and multiply?

STUDENTS: 3*a*.

MELISSA NIX: Joseph, do you agree? Is this the same one?

STUDENT: Yeah.

MELISSA NIX: So if you were to write an expression for this one, what expression would you write? What are they multiplying?

STUDENT: They're multiplying 3 times *a* squared and then 3 times *a*.

MELISSA NIX: They're multiplying 3 times *a* squared and then 3 times *a*? Is there only one way to write that? Or can I write that 3 times *a* squared and 3 times *a*, and you said, "and." Do I need to, mathematically, show what “and” means?

STUDENT: [inaudible]

MELISSA NIX: What would I need to put in here?

STUDENT: Addition.

MELISSA NIX: Addition?

STUDENT: Yeah, addition.

MELISSA NIX: Is there another way I could write that same expression? Let's think back to what Mason said about that whole math problem, that 10 and 2 and 10 and 5, right? What do you think, Heaven?

STUDENT: 3 times *a* squared plus *a*.

MELISSA NIX: 3 times the quantity of *a* squared plus *a*? Are those equivalent expressions?

STUDENT: Mm-hmm. [affirmative]

MELISSA NIX: Okay. One more model to show. So this one might be articulating again, where that 3*a* is coming from. But I see that this has been compartmentalized here, or sectioned off. How many units do we now have?

STUDENTS: Three.

MELISSA NIX: Three. So this was *a* by a, which got me …?

STUDENT: *A* squared.

MELISSA NIX: *A* and *a* is?

STUDENT: *A* squared.

MELISSA NIX: *A* and *A* is?

STUDENTS: *A* squared.

MELISSA NIX: *A* and 1 is ... 1*a*, right? *A* times 1 is?

STUDENT: 1*a*.

MELISSA NIX: 1*a*. And *a* times 1 is …

STUDENTS: [inaudible]

MELISSA NIX: 1*a*. So adding those up, you got three *a*'s and 3*a* squared. All right.

MELISSA NIX: I'm going to pose a challenge problem to you that you're going to work on first by yourself and then with a partner. So I want you, right now, as you're cleaning off your whiteboards, just to talk. How do you feel about this? Multiplying, un-multiplying, area, and dimensions thing. Where are you sitting right now with all this information? Just chat for a second.

STUDENT: Do you understand this?

STUDENT: Yeah. Sort of.

STUDENT: I don't.

STUDENT: Oh, you don't understand it?

STUDENT: No.

MELISSA NIX: All right, so. What pieces do you feel comfortable with? When it comes to looking at rectangular figures.

STUDENTS: Oh, I need that.

MELISSA NIX: And looking at like areas or dimensions? Is this still kind of “ish”?

Okay. The reason I'm asking is because I'm gonna have you look at a problem right now and it's going to ask you to look at four dimensions and look for areas. And I'm going to give you some pieces but not all of them. So you need to make sure that you have your puzzle-solving skills out and ready to attack this problem as best you can. And like I've done this whole time, as people come up with different ideas, I'll come share them with you to move your thinking forward. So you're not going to stress, you're going to launch into this and do whatever you think you can to get as best as you can. That sound okay?

Let's figure out what the story’s about though because it's a word problem and whenever I tackle a word problem, I just want to know what is the word problem about? So, we're gonna take a volunteer, and Jack, you're right in front. Will you read the first sentence to me?

STUDENT: The carnival is given a rectangular area to set up at the local rodeo.

MELISSA NIX: All right, so we're at a local rodeo. And who is given a rectangular area?

STUDENTS: The carnival.

MELISSA NIX: The carnival. What do you mean by rectangular area? What do I mean by rectangular area? What am I thinking? Miranda, when I say rectangular area, what do I mean? They have a ...

STUDENT: They have uh... it's like an amount of space that's in a rectangle shape [inaudible].

MELISSA NIX: Exactly! The amount of space that's in a rectangular shape, right? Sound all right? Now, Adonay. There will be sections for ... Can you finish the sentence for me? Can you read from there?

STUDENT: Yeah. There will be a section for kids called Kids Corner, traditional fair rides called Country Fair, high-rise called High Rollers, cars and truck rides called Four-Wheeler Fun, and a food court called Carni-Eats.

MELISSA NIX: Yeah, the Carni-Eats. It's the carnival food court. So I want you in your head just to think about it, all right? It's a rectangular area that the carnival is going to set up in. So if you were to close your eyes, think about what you see in your eyes right now, in your mind. And there's a section for each of these different things. How many different sections will we have?

STUDENTS: Five.

MELISSA NIX: We'll have five, right? Let's label them all. We got one for the Kids Corner …

STUDENTS: Kids Corner.

MELISSA NIX: One for the County Fair.

STUDENTS: Fair rides.

MELISSA NIX: Or country fair, county fair. One for the …

STUDENTS: High Rollers.

MELISSA NIX: High Rollers. One for the car and truck rides. And one for the food court. And all of this squeezes into what shape?

STUDENTS: Rectangle.

MELISSA NIX: Okay. Are we good with the context of the story? That's a lot of words and that's just telling you what's happening. That's not even telling you any math yet. The plan for the layout is shown in the figure hidden under this box. Use the figure and your knowledge of polynomials, which is what you've been doing, those are the variables and the constants, knowledge of the distributive property and of area to write expressions that are representing each of the missing areas or lengths below. So, Ethan. Is this a rectangular area?

STUDENT: Yes. Maybe. Yes.

MELISSA NIX: Yes, okay.

Leila, do you see Kids Corner somewhere there?

STUDENT: Uh, yeah.

MELISSA NIX: Check. County Fair. Check? Brisa, do you see High Rollers? Yeah. Itzel, do you see Four-Wheel Fun? Neri, are you with me on Carni-Eats? See them all? Do we have all five?

STUDENT: Yes.

MELISSA NIX: All right. We have to figure out the missing dimensions, and you're not going to have to do any of this on your whiteboard. I have this for you as a worksheet. Okay. So I'm actually going to hand this out because I think as I'm doing this I want you to pay attention to where these missing dimensions are. As I hand this out, will you look up here and see, does every rectangle have all of the dimensions?

STUDENTS: No.

MELISSA NIX: Does every rectangle have all the area? Have the area label.

STUDENTS: No.

MELISSA NIX: Now remember, Qwentin, you told me the area is going to be on what part of the rectangle?

STUDENT: The inside.

MELISSA NIX: The inside. So I need to know the dimensions, which are where on the rectangle?

STUDENT: The outside.

MELISSA NIX: The outside. And I need to know the area, which is on the inside, right? So, I need a friend to come up and help me see this better and highlight for me, what are the dimensions that I'm missing.

I'll get you started and then I'm going to pull a fancy stick. Ready? For example, do I have this dimension?

STUDENTS: Yeah.

MELISSA NIX: Yes. Do I have this dimension?

STUDENTS: Yes.

MELISSA NIX: Yes. Do I have this dimension?

STUDENTS: No.

MELISSA NIX: No. So that would be one that I need to figure out. Okay?

MELISSA NIX: So, we're gonna play pass the stick real quick. Someone's gonna come up and help highlight all of the exterior dimensions that I need in order to do this. So Rolando, you are my, you're my *artiste* today. Let's help him out though. He's going to draw a yellow highlight on the outside dimensions of what we need to solve. So, help him out. Where do you think he needs to go?

STUDENTS: Right there.

MELISSA NIX: You can go ahead and share with him, yeah.

STUDENTS: On the top of the High Rollers.

MELISSA NIX: On the top of the High Rollers. Where else?

STUDENTS: Bottom of High Rollers?

MELISSA NIX: The bottom of the High Rollers. Where else?

STUDENTS: Bottom of Kids Corner.

STUDENTS: On the bottom of the Kids Corner.

MELISSA NIX: Bottom of Kids Corner. That's it? Holler.

STUDENTS: In between the Kids Corner and County Fair?

MELISSA NIX: In between the Kids Corner and County Fair. Thanks, [inaudible]. Help him out.

STUDENTS: In between Carni-Eats, Four-Wheel Fun.

MELISSA NIX: Carni-Eats, Four-Wheel Fun.

STUDENTS: And on both sides of the High Rollers.

MELISSA NIX: And on both sides of the High Rollers. All right, pass the pen to somebody else.

Go Jack, go! You are now going to change the color, Jack. And you are going to put a line inside the rectangle if we need to find the interior area. So which ones do we not know the area for? Help him out.

STUDENTS: Carni-Eats.

MELISSA NIX: Carni-Eats. So just draw a single line in there, or circle Carni-Eats if you want. Circle Carni-Eats. We need to find the area of Carni-Eats. What else?

STUDENTS: Four-Wheel Fun.

MELISSA NIX: Four-Wheel Fun.

STUDENTS: County Fair.

MELISSA NIX: County Fair.

STUDENTS: And Kids Corner.

MELISSA NIX: And Kids Corner. All right. That's Cameron and Amelia's corner. My littles.

Does everyone understand what I am asking you to do? Okay.

You are going to try to find the dimensions that are missing. Some information has already been given for you, given to you. And some of it you are going to have to figure out on your own. And that's what I'm setting you off to the challenge to do. So you are going to work a minute on your -- a couple minutes on your own, just launch into it and see what you can do. And then you will share with your neighbor and then turn around and share in your group. So you are going to work on this as a whole multiple minds put together. No one of us is as smart of all of us together. Okay?

Let's just get you to launch into it for a couple minutes on your own, and see what you come up with. Go for it.

STUDENT: Yeah, yeah, because we're trying to find High Rollers. Um ...

STUDENT: Because we have 2*x*.

STUDENT: Horizontal, nope diagonal.

STUDENT: Why?

STUDENT: Because you see? One of the sides we have to find is this one.

STUDENT: Yeah.

STUDENT: And that one. For this one, I got 6.5 because 5 -- I added the two 1's I get there, which is 5*x* and 1.5*x*, and that gave me 6.5*x*. And, but this one is 3*y*, and one-half of *y*. So, if you were to use the map they give you, you'd add them and get 3.5*y*, 3 and a half *y*. Even though it's different from this side.

STUDENT: Yeah.

STUDENT: Yeah, so ...

STUDENT: 6*z* in here

STUDENT: Because that's what they give us. So, it's not …

STUDENT: I'm not sure about this one.

STUDENT: Well, so ... If you see this piece right here, if you add these dimensions right here, 6*z* plus 6*z* and then minus 1, minus 1. Which would equal to -- you'll be minusing 2 from 12.

STUDENT: Yeah.

STUDENT: That would be 10. Plus a 3*y*, which would equal to 13*x*, but we don't know where that *x* comes … in place. It must be on this side though.

STUDENT: Yeah.

STUDENT: And, we'll figure this out later, so -- since we know all the dimensions of this square.

MELISSA NIX: Yeah.

STUDENT: So, 6*z* minus 1 times 3*y*.

STUDENT: That's what I got.

MELISSA NIX: Why you got that?

STUDENT: Oh, you got that.

STUDENT: I already did it.

STUDENT: Then you can do the same thing for Kids Corner.

STUDENT: I got 6*z* minus 1 times half *y*.

STUDENT: And then did you Four-Wheel?

MELISSA NIX: And, I really like what you've done here. If you multiply that out--

STUDENT: Uh, yeah. I got 2*y* plus 3 times 5*x*, but for 2*y* plus 3, I put it in parentheses.

STUDENT: So, what are we going to do?

STUDENT: Well, is it ... I don't know how we're going to do this one if we don't have this side. Like, if this is the same.

STUDENT: That's true.

STUDENT: Yeah, so if the value is a 2, that will be 12--

STUDENT: Do we have the ...

STUDENT: So, you can't subtract--

STUDENT: It would be better if they gave us like the actual full area.

STUDENT: … one, because you don't have this entire thing over here. We only have these two values.

STUDENT: But, those are two ...

STUDENT: We don't even have these.

MELISSA NIX: Uh-huh.

STUDENT: Yeah.

STUDENT: Or this one, or this one, or this one, or this one.

STUDENT: Yeah.

STUDENT: Or this one.

STUDENT: Unless these two are the same.

STUDENT: Maybe.

STUDENT: They might be.

MELISSA NIX: So, show me what you multiplied to get that.

STUDENT: Oh, yeah.

STUDENT: Um, 1.5 multiplied by 3.

MELISSA NIX: And is that the entire term? It's 1.5 …

STUDENT: This one, right--

STUDENT: I guess.

MELISSA NIX: So, does that show up in your answer?

STUDENT: No.

MELISSA NIX: So, what did you do differently?

STUDENT: I -- I kept the x because it's 3 times that, and we don't know what x is. [inaudible]

MELISSA NIX: Okay. So, using that same pattern to try the other ones, could you figure out what the other ones would be. And, [inaudible], I agree that your rectangular dimensions are going to have the same dimensions because it's a rectangle.

STUDENT: Yes.

MELISSA NIX: So, you have two sets of parallel lines, so the distance of those parallel lines will be the same value.

STUDENT: Okay. Thank you.

STUDENT: So, I, uh, these are the side lengths for the ones that we're going to try and split it up, but we're not really sure how to find it.

MELISSA NIX: Oh, okay. Um, what information do you know about.

STUDENT: The $26xy$ plus $13x$.

MELISSA NIX: Okay.

STUDENT: That's to get the area.

MELISSA NIX: Okay, that is the area. Okay, I see that because that's on the inside, right? We talked inside. So, what was your thinking about splitting it up?

STUDENT: Um, we just thought cause like, they already split it up for us pretty much. So, if we just found half of it. Like, if we just split up the area.

MELISSA NIX: Do you know if where you drew your split line is exactly half of this area though?

STUDENT: No.

MELISSA NIX: No. So, I'll be a little bit anxious about splitting it because I don't know for sure if these are split in half. How could you use the dimensions that you figured out, though, to know if you didn't split it in half?

STUDENT: Could we add them and then ...

MELISSA NIX: Could you add them? What would 5*x* and 1.5*x* be?

STUDENT: 5*x*.

STUDENT: 6.5*x*.

MELISSA NIX: Huh. So, this length here is 6.5*x*. Could you use that to figure out what you would have to multiply it by in order to get this area?

STUDENT: You could divide it by 2.

MELISSA NIX: So, maybe that's where you can work. So, I -- I think that thinking is very interesting. If this is 6.5*x*, as Miranda said. What would this dimension be to get you this area?

STUDENT: Um, okay.

STUDENT: 6.5*x*.

STUDENT: No, 3.25.

STUDENT: No, it's really 3.5.

STUDENT: Oh, yeah.

STUDENT: 3 and a half is equal to 6 and a half *x* [crosstalk]. So, that might be important to find the area.

MELISSA NIX: How did you figure that out?

STUDENT: Because this--

MELISSA NIX: What two pieces of information did you use to figure that out?

STUDENT: So, if you add these two side lengths, it would equal 3 and a half *y*, and then if you add these two side lengths it would equal 6 and a half *x*.

MELISSA NIX: Oh, so you then did what with that, 3 and a half *y* and 6 and a half *x*?

STUDENT: I'm trying to find, like, this--

MELISSA NIX: You set them equal to each other, and now you're simplifying to figure what the value of each of them will be. So, this is the ratio of how they are related to each other.

STUDENT: So, one -- one and three quarters *y* is equal to three and one quarter *x*.

MELISSA NIX: That's awesome.

STUDENT: And--

MELISSA NIX: Now, is that going to help you find your figure of your area? It might, right?

STUDENT: It might, yeah.

MELISSA NIX: Okay. How's it going? If we made this with algebra tiles, what would we pile on here?

STUDENT: .5 goes like this.

STUDENT: Um, of the long piece ...

MELISSA NIX: Five of the blue *x*'s, right?

STUDENT: Yeah.

MELISSA NIX: And if I could, what would I -- I would line up five of the blue algebra tiles here, and what would I line up here?

STUDENT: One and a half.

MELISSA NIX: One and a half of the algebra tiles? So, how many of those algebra tiles would I have --

STUDENT: Six and a half, 6.5.

MELISSA NIX: And, what are they called? The name of those algebra tile--

STUDENT: *X*'s.

MELISSA NIX: They're *x*'s. So, you're actually going to end up having six and a half--

STUDENT: Oh. You just have the *x*, like [inaudible].

MELISSA NIX: So, is it 6 and a half plus 2*x*?

STUDENT: Wouldn't it just be --

STUDENT: 2 and a half *x*.

STUDENT: 6 and a half *x*.

MELISSA NIX: I'm thinking it's just going to be 6 and a half *x*.

STUDENT: Oh. That makes sense.

MELISSA NIX: That's a great question. So, let's visualize it. Do you remember on Monday when we played with the algebra tiles?

STUDENT: Mm-hmm. [affirmative]

MELISSA NIX: And what do the *y*'s look like?

STUDENT: Like, small little rectangles.

MELISSA NIX: Yeah, the blue one, the skinny ones, right? And what about the -- what would 3 be made up from?

STUDENT: The individual cubes.

MELISSA NIX: Individual cubes. So, if you are going to combine those terms. Can you combine a blue long and skinny with a small yellow?

STUDENT: No.

MELISSA NIX: No, so likewise what would be equaling five *y*'s? Because Y is its own entity.

STUDENT: Mm-hmm. Beause we're trying to find the area--

MELISSA NIX: Oh, but I see what you're doing here. So, let's simulate your multiplying this dimension one and a half.

STUDENT: That's this one.

MELISSA NIX: Times 2*y* and what would --

STUDENT: But when you mix the variables there, what do you do?

MELISSA NIX: Well, what happened when we did the algebra tiles and we had that long and skinny *x* and the long and skinny *y*? Where we looked at Rectiles.

STUDENT: Would it be *xy*?

MELISSA NIX: Would it be *xy*? What do you think?

STUDENT: Yeah, I was thinking that too.

MELISSA NIX: Why would it be *xy*?

STUDENT: Because it -- because it ends up having one side *x* and one *y*?

MELISSA NIX: Oh, so what would that look like then if you wrote that as an expression?

STUDENT: So, 3*xy* plus 4.5x, because 4.5 goes right here.

MELISSA NIX: There you go.

STUDENT: Oh, okay, so that will be the expression for what the area is.

MELISSA NIX: By Jove, I think you got it.

MELISSA NIX: So, I see lots of hands going. But I think at this point, can you turn around and chat as a group of four? And then I'm gonna start pulling a couple examples to move the thinking forward. But let's see if now, collectively, your four-pods can come up with even more movement.

STUDENT: All the dimensions for these. And then what we did was we decided to, um, add these together and then get, we got 6.5 and then we divided them into each of the [crosstalk] dimensions.

STUDENT: We divided 6.5 by 26 and 6.5 by 13.

STUDENT: And we got 2*x* plus 4*xy*.

STUDENT: Um.

STUDENT: We just got, like, all the, like, dimensions of, like, all the sides.

STUDENT: We also were trying to figure out how, like, *x* equals, uh, *y* equals *x*. So we found out that 1.75*y* equals 3.25*x*.

MELISSA NIX: Ooh. Fascinating. Fascinating.

STUDENT: So that might help us find *z*.

STUDENT: The areas of the other ones.

STUDENT: And ultimately find the actual number.

STUDENT: *X* like over here. Then it equals the 15.

STUDENT: Ohhh yeah. Okay.

STUDENT: If you were gonna convert it to the area.

STUDENT: So what [crosstalk] if the 3.5 helped with the 26.

STUDENT: I wonder if the 3.5 comes over the 26. Like if you do that times, like, whatever that length is.

STUDENT: It'd have to be an *x* though. Because you get *xy* eventually, right?

STUDENT: Yeah.

STUDENT: Yeah.

STUDENT: Well, but you have *x* -- no, because that goes there.

STUDENT: Yeah. No, no, no. This one, it multiplies by 2 so you keep just the 1x. But this length would have to have a *y* in it. I mean. No, it'd have to have an *x* in it. Because, um, your answer's 26*xy*.

STUDENT: Yeah.

STUDENT: That means that you had an, um, this is your *y* you get. So this one is something *x*.

STUDENT: Yeah.

STUDENT: 5 and then we, um, divided it into the area. I think that's half of it. But I'm … I don't know.

STUDENT: Oooh.

STUDENT: Divide 2. And then add that 26*xy*.

STUDENT: Okay.

STUDENT: 6 and a half times 2. 13 and then *x*.

STUDENT: Can you find the conversion from 1*x*?

STUDENT: Yeah.

STUDENT: I just run, uh, *x* equals.

STUDENT: It's equal.

STUDENT: You just explained to me that this, that this part, this portion, cut in half is this portion up here because 4 times 6 and a half would equal, would … So 6 times 4 would equal, um, 24. And then half of, half of 4 is 2. So 24 plus 6 is 26*xy*. And then he explained to me that 13 times 3 point, that ...

STUDENT: He explained to me that -- what'd he say, Ethan?

STUDENT: [inaudible] 13 times 3 point something. 5?

STUDENT: Something. Right? It was like 4 times, like ... no, it wasn't 4 times 3 and a half though, because then that will be 14. How'd he find the other one?

MELISSA NIX: First one has to do with Carni-Eats and finding the area of Carni-Eats. Okay. So can we all hear Heaven as she explains how she found the area of Carni-Eats? All right, so let's make sure that we're giving her the floor.

STUDENT: Um. Since you can't add 2*y* plus 3 since they're different, um, things, you took 2*y* plus 3 in parentheses, multiply that by 1.5*x*, and, um, you can distribute 1.5*x* into it. So I did 1.5*x* times 3 and that gives you 4.5*x*.

MELISSA NIX: So the first thing you do is 1.5*x* times, either way.

STUDENT: 1.5x times 2, 2*y*, that gives you 3. And you just blend that two, um, variables *x* and *y*. So, because you end up getting one, one side like would be an *x* and one side like would be a *y*, that's why you do that. So you get 3*xy*. Oh, yeah, I missed that.

MELISSA NIX: Do you have a gift? You have a gift. What should this be down here?

STUDENT: Uh, 3*xy*.

MELISSA NIX: All right. So this is 3*xy* here. What she's saying in her head is she's saying that she multiplies this and this.

Now do we agree that 1 and a half times 2 is 3? Who wants to explain that one to me? Go for it, Ben.

STUDENT: … of 2. Which gives you 2 and then half of 2 is 1. And then you add them together and it gives you 3.

MELISSA NIX: Okay. So 1 times 2 is 2, and half of 2 is 1, and I add those to get 3. So Heaven said, "All right. This gets me 3." And *x* times *y*, does that seem reminiscent of what we did with those algebra tiles and when we looked at the activity of Rectiles that we had *x* and we had *y* and we had an *xy*? And what about 1 and a half times 3?

STUDENT: That gives you 4.5.

MELISSA NIX: 4.5.

STUDENT: And one of them doesn't have the, another extra variable, so it's just *x*.

MELISSA NIX: So because this one doesn't have an *x* you still need to recognize that *x*, and so you have an *x*.

Anyone else come up with that same area for that? If you didn't, see if you can, somebody else who got that too.

MELISSA NIX: So, let's see if you can apply that strategy to figuring out the area of Four-Wheeled Fun.

STUDENT: For what?

STUDENT: For the Four-Wheeled Fun.

STUDENT: Mm-hmm. [affirmative]

STUDENT: Because, does this say it's 5, and this is right up here? Like she did, it's …

STUDENT: So 5?

STUDENT: ... it's like 5 times 2*y*, which it, wouldn't that be 10*xy*?

STUDENT: 5*x*, times *y*, 2*y* plus 3? Like that?

STUDENT: Yeah. Yeah, then, then wouldn't that be, uh, where it's ... wouldn't that be since, like, it's 5 times 2 that's 10. And there's the *x* and *y*, wouldn't that be 10*xy*?

STUDENT: No, 5 times 2 is --

STUDENT: Plus 13*xy*.

STUDENT: Oh.

STUDENT: Because, here it is. So you do 6.5 times 2*y*, so then you get your ... That goes down here, because your 13*xy*. And then 6.5 times 3, so that gets you 19.5*x*.

STUDENT: Because we're writing it.

STUDENT: But we don't know what *y* is …

MELISSA NIX: Oh, kind of frustrating though, cause it's--

STUDENT: ... so, I don't know.

STUDENT: So weird?

STUDENT: So what about 6, 16?

MELISSA NIX: We can't right now.

STUDENT: [inaudible]

STUDENT: Or 15.

STUDENT: 15?

MELISSA NIX: We're close to being done with class my friend. That would be a better time.

STUDENT: Miss Nix?

MELISSA NIX: Yes.

STUDENT: Um …

STUDENT: Okay.

STUDENT: So how haven't we gotten her area? We don't really get how she got it, like …

MELISSA NIX: Okay.

STUDENT: What …

MELISSA NIX: So, what part? I see you've copied it down. What part do you, do you understand?

STUDENT: We understand the 1.5 times the 2 by plus 3 part, but--

MELISSA NIX: Okay. So can we write that out in a way that would be easier to look at? Okay. So you, what you're not sure about is ... which part are you not sure about?

STUDENT: Like the x and the y. I think that confuses us because--

STUDENT: Yeah. Since we don't know what x and y is.

STUDENT: No, like, we don't know how to place the--

STUDENT: Yeah, like replace them.

STUDENT: Oh.

STUDENT: Yeah.

MELISSA NIX: Gotcha. So, when we did the algebra tiles and we had the algebra tile that looked like an x …

STUDENT: Mm-hmm.

MELISSA NIX: ... and we multiplied it by a y, what shape did we end up with?

STUDENT: *Xy*?

MELISSA NIX: We ended up with that other shape that was an xy.

STUDENT: Mm-hmm.

MELISSA NIX: So you can multiply an x and a y together and you would end up with its own polynomial collection, called an xy.

STUDENT: Mm-hmm.

MELISSA NIX: This is its own term.

STUDENT: Okay.

MELISSA NIX: So if we thought of that, and said, "I'm going to multiply 1 and a half *x* times 2*y*," it would look something like ... There's 1*x* --

STUDENT: Mm-hmm.

STUDENT: Oh.

MELISSA NIX: ... and there's my half an *x*.

STUDENT: Mm-hmm.

MELISSA NIX: And there's a *y*, and there's a *y*, right? So this shape would look like what?

STUDENT: Another *x*?

MELISSA NIX: Go ahead and draw that for me. [crosstalk] And what would this look like?

STUDENT: That would look like, um, 2, one half?

MELISSA NIX: Half of one. So I normally would have drawn an *xy*, right?

STUDENT: Oh.

MELISSA NIX: But it's only half of one.

STUDENT: Mm-hmm.

MELISSA NIX: So if there was such thing as half an *xy* ... So what would you label those as?

STUDENT: That would be *xy*?

MELISSA NIX: Is it a full *xy*, Nicky, or has it …?

STUDENT: Uh, a half.

MELISSA NIX: It's only half of one, right? So it's half of this shape.

STUDENT: Mm-hmm.

MELISSA NIX: So I will label it as half of what?

STUDENT: *Xy*.

STUDENT: *Xy*.

MELISSA NIX: Half *xy*. And this one's what?

STUDENT: Half *xy* also.

STUDENT: It's half *xy*.

MELISSA NIX: So, if I were to smoosh them all together, what would I have?

STUDENT: 3?

MELISSA NIX: And what are those shapes called?

STUDENT: 3*xy*. *Y*?

MELISSA NIX: So let's go back to what she came up with.

STUDENT: 3*xy*'s plus 4.5*x*.

MELISSA NIX: So she came up with 3*xy*.

STUDENT: Mm-hmm.

MELISSA NIX: What did you come up with?

STUDENT: 3*xy*.

STUDENT: 3*xy*.

MELISSA NIX: Where are the 3?

STUDENT: Right there.

STUDENT: These. And then if you add these together--

STUDENT: They'll make a--

STUDENT: ... because they're half, they'd be gone?

MELISSA NIX: Do you see how it's okay to have an, a 1 and a half *x* …

STUDENT: Yeah.

MELISSA NIX: ... times 2*y*, and you would get how much?

STUDENT: 3*xy*.

STUDENT: 3*xy*.

MELISSA NIX: 3*xy*. 3*xy*. How did she get 1 and a half *x* times 3? Would you, could you draw a picture of that to see? One and a half *x*, so that's gonna look ... And now it's times three units. Three. So three little yellows.

STUDENT: Oh.

MELISSA NIX: Right?

STUDENT: 1 and half times 3?

MELISSA NIX: 1 and a half times 3.

STUDENT: So would I make, like really--

MELISSA NIX: So this is, like, one of those little yellow cubes--

STUDENT: Mm-hmm.

MELISSA NIX: Little yellow cubes, little yellow cubes. And so now if you're lining it up in this area here, what, what shape, when you multiply this shape and this shape, what shape do you get?

STUDENT: This and this shape together?

MELISSA NIX: Yeah, that's what ... This is one unit. So you would actually just get one of those units. We didn't play much with the one units. So one unit times this, is that same unit, and one more. And so, one and a half, one and a half. So one and a half, one and a half, and one and a half.

STUDENT: Oh! Okay.

STUDENT: Two is just like, each?

MELISSA NIX: You've got really far on your middle area, and then the last five minutes of class I want to show what you were thinking about this middle area, before we end for the day. Would you be willing to share a little bit what you're thinking about that middle area? Okay.

MELISSA NIX: Students, I know that we're going to be leaving soon, and I have a brief exit ticket for you. They're tackling this middle problem. Why is High Rollers different than the other rectangles? Why is it different, yeah?

STUDENT: Because we don't have the length or the width. We only have the area.

MELISSA NIX: You don't have the length or the width. You only have the area. And we're not going to finish this problem today, and that's totally okay. We're going to tackle more of it tomorrow. Um, we will have a bit of an exit ticket to kind of see where you're at right now, what thinking you, you are. But I want to, I want to visit this thinking. High Rollers, you have the area, but you don't have the length and the width, okay. Miranda and Nicky, you came up with something though. Can one of you articulate where you're thinking?

STUDENT: We were thinking that we wanted to do half of ... Like, we wanted to find the side lengths. And so, we added what we found together. So, we added 1.5 and 5*x*.

MELISSA NIX: Okay, so you got this side length. You said 1.5 and 5*x*, and you calculated that as ... What is your side length?

STUDENT: 6.5.

MELISSA NIX: 6.5.

STUDENT: And we knew that when you divided 6.5 by 26, and we knew that the [inaudible] goes into 26 four times.

MELISSA NIX: Okay, okay. So you just said something that's really big. And this is what I want people to hold onto for a second. You said you took 26*xy* and 13 and did what?

STUDENT: We divided it by 6.5.

MELISSA NIX: You divided it. Why are you dividing your 26*xy* and 13*x*? Why are you dividing it by 6*x*?

STUDENT: So we could find the dimensions.

MELISSA NIX: So you could find the dimensions. Why division?

STUDENT: Because if you divide the side lengths, then you, um, you can get the other side length of it. Like, half of it.

MELISSA NIX: Does that make sense to anyone else? Division's the inverse of what math? Multiplication. So if I multiply to get the area, she's basically --

STUDENT: Dividing to get the dimensions.

MELISSA NIX: Dividing to get the dimensions. She took it one step further and was dividing it. I want you to sleep on that tonight, and quickly do an exit ticket. So, quickly, like literally. Um, you may, yes, bring this paper home, except like, I know. Leave this paper here, and we'll revisit it tomorrow, sorry.

STUDENT: Should we put our name on it?

MELISSA NIX: Yes, please put your name on it, so I can give it back to you tomorrow. All right, Haley. Haley, don't stress. We'll do this as our opener tomorrow, okay?

PAM BROUSSEAU: That was so much fun. That was an exciting lesson.

MELISSA NIX: Why, yeah it was. Thank you. Thank you for being a part of it.

PAM BROUSSEAU: You're welcome.

MELISSA NIX: I'm certainly glad you were there.

PAM BROUSSEAU: Thank you for inviting me in. So, reflecting on the lesson ...

MELISSA NIX: Mm-hmm.

PAM BROUSSEAU: What things do you think went well?

MELISSA NIX: Reflecting on the lessons, things that went well were students definitely were following along, definitely kind of pushed them a bit so that they were engaged and it was not too easy for them at any given moment, I think in terms of cognitive demand, that was something that was there. And I'm glad I started with the number talk just to kind of get them primed and ready. And I'm really glad that some students were using the knowledge that they already had to kind of apply it to that last problem, the carnival problem. I didn't see any of the misconceptions that I had seen in the last couple of days about x and x being x squared, when it was the dimensions on the side of the rectangle.

PAM BROUSSEAU: Why do you think that was? Why do you think that you didn't see the misconceptions that you have seen recently?

MELISSA NIX: Well for starters, during the lesson I heard that that was gonna possibly come up and so I spent some extra time making those visual representations with the first blue and orange rectangles. When I made those blue and orange rectangles and I actually, you know, departmentalized it -- or compartmentalized it into that x and x to give me the x squared and the x squared, I think that helped them a little bit because there was still that underlying uncertainty about what happens when I'm combining terms. Am I actually squaring them or am I -- is it an x and an x to get me x squared, or an x and an x to get me $2x$? And their -- they just haven't had enough real experience with that, I mean they really haven't had much of any experience with that, just very cursory. So, I think it was nice to be able to visually show, you know, what is an x and an x gonna get you, and then tie it back into that multiplication array, sort of, bring back that same structure that they've seen. So, I kind of feel like I tried to see what might've been a mis-- a hiccup later on, and I wove that into the lesson. That wasn't intended, for me to show that -- it was the z. To show that $2z$ times z is actually $2z$ squared, and to show that $2z$ was z and z, wasn't a part of my initial lesson; that was something I did in the moment to try to avoid that hiccup coming out later on when they did the actual problem.

PAM BROUSSEAU: So, it's thinking about the misconceptions you've seen. Then in the moment, it was, "Okay, we're gonna focus on this, and try to illustrate that," so that helps deepen the understanding, the correct understanding, the conceptual understanding versus the misconception.

MELISSA NIX: Sure, we had done some work with algebra tiles, just on Monday, where they kind of got to see, you know, x and x, and x times x got me an x squared, and an x times y got me an xy, but it was literally just sort of a day of exploration of knowing what those names are before we started playing with them more on Tuesday and Wednesday. Today's Thursday, so there's still some of that beginning sense-making, and yeah, I thought the final problem was a good push to kind of apply some of that sense-making, "But what does happen when I multiple an x and a y? I see an x and a y over here, in the area of the middle region, but is that what happens when you multiple an x and a y?" I mean, I had a student ask me that and, "I don't know, what could we rely back on to do what we did earlier? What would happen when you multiply an x and a y?"

PAM BROUSSEAU: So, would it be fair to say that you mix up your lesson design? So, one day might be exploration, and this was a concept development lesson?

MELISSA NIX: Yes, so I would say that for sure. And before we even got into talking about what variables are and what happens when you multiply variables, that's when I pulled out some manipulatives, so that they could tangibly see a visual representation to anchor it in that concrete, because today gets to be a little bit more abstract in this application. I mean it has an application, but it's still pretty abstract.

PAM BROUSSEAU: So concrete to abstract?

MELISSA NIX: Concrete to abstract.

PAM BROUSSEAU: Tell me about -- you mention sense-making. So tell me about how their sense-making has evolved from kind of the exploration to where you saw it ... especially evolved today.

MELISSA NIX: Yeah, so I mean, we started kind of, I -- last week in kind of showing them about expressions in general, and that if I had like 3 times 5, plus 3, what visual representation could I make of that? So I showed them, you know, three groups of five and three just using units. And that kind of bridged us to using the algebra tiles. So now let's start talking about these concrete representations of, you know, abstract variables. So they kind of got to see what happens with an *x*, and an *x*, and a *y*, and a *y*.

Um, and we explored a little bit yesterday with a MARS [Mathematics Assessment Resource Service] task called Rectiles, and that explores that *xy* and the combination of those patterns. And then today, again trying to bridge this whole connection of ... you know, you start with these expressions, then we went to the algebra tiles, then we went to more of like a visual model, and now it's just sort of numbers. Still visual arrays, but they're getting that whole, like, "Oh, my dimensions multiply to get my area."

And that was a big conception that came out today, because it was a misconception that was highlighted when they thought with the pink rectangle that they were looking for the area. But they were actually asked to look for the dimensions ... Or I was going to be asking them to look for the dimensions. So to highlight, oh, the area as a convention, we label on the inside, and the dimensions on the outside. So now, what do you think I'm asking? And then at the very end, to have my last two students talk about well, wait a second, if this is my total area, then the way that I can un-area this is to …

PAM BROUSSEAU: [laughs]

MELISSA NIX: ... is to do my inverse operation. Like, if I multiply to get here ... I mean, they were working on that a lot. And to see that ... I mean they -- they actually got it back down to um, I think it's 4 and 2*y* or 4*y* and 2, what the dimensions were. They were able to take all of those components out and then multiple ... I said -- I was asking them if they can multiply it to check that I really got that area.

I'm excited, because I get to see them tomorrow and -- and kind of ... I planted that seed with the students to see what's going to come out more tomorrow when we visit it.

PAM BROUSSEAU: What teacher moves did you make to help them -- support them. What teacher moves did you make to support them in their sense-making?

MELISSA NIX: Um, well for starters, I think I -- I think it's really important to let every student individual think time. But there's also that relief when you're like, okay, I need to confer with you because I'm really lost, or I'm not lost. So I do really value that opportunity to kind of check in. Throughout the lesson I wanted to offer out examples of how other people were making sense of it to help any -- to nudge anyone along.

So, for example, when I came up with Heaven's example of the [inaudible], to say, "Oh, explain to me how you multiplied this to this to get this area. Now can you use that to get to the next area?" Use that same structure and, you know, application to find the area down here. Might that help push you forward?

PAM BROUSSEAU: So using their thinking.

MELISSA NIX: Mm-hmm. [affirmative]

PAM BROUSSEAU: So the agency ownership, or authority ownership identity piece, usually -- really using their thinking to move them along and help them make sense.

MELISSA NIX: Right, and kind of integrating that, you know, SMP of structure of like, you know, is this the same pattern? So if I know I can multiply this dimension to this dimension, to get this area, and I agree that this is $2y$ plus 3 because it's a rectangle, and these are parallel, and this is $2y$ plus 3, then can I use that $5x$ and $2y$ plus 3 to get this next area? So trying to connect all -- how it's all connected.

But yeah, not me just telling them, because I don't want to just tell them all the time. It's a lot more powerful to hear it from somebody else. And it's a lot more powerful for that student who showed that example to be able to explain it, as well.

PAM BROUSSEAU: Absolutely.

MELISSA NIX: And so the more that I can have them explain it, the more empowered they will be, the more ownership they will have of it, and um, if I can facilitate that happening as often as I can, I think it's a powerful math classroom.

Uh, I do wish it wasn't so much just them sitting. I try to do an accordion. I'll show you a little, and bring it back, and show you a little, and bring it back, and let you work together. Um, because it is kind of long. But they--

PAM BROUSSEAU: Mm-hmm. Well that had a nice flow.

MELISSA NIX: Oh, thank you.

PAM BROUSSEAU: You -- you did that. They worked some, you brought them back. You give them a little more, highlighted -- illustrated some of their thinking, planted another seed, worked some more. And so that was kind of a nice -- nice flow.

MELISSA NIX: You asked how did I help build some of the conceptual stuff with students. I went around and helped and a few students weren't sure, so on one student's page I like, turned it over and was kind of helping them draw. They were telling me what to draw, but I was trying to draw for them, what would the algebra tiles look like if you were to multiply this? But because we had such a cursory introduction and play time with that, I didn't quite have ownership of that visual.

PAM BROUSSEAU: Mm-hmm. [affirmative]

MELISSA NIX: But as I kind of illustrated it for them, they were able to evolve with that visual and kind of make sense of ... Because they asked me this, "I don't understand how Heaven got that first area. How did -- how did that one student get that area for [inaudible]?" And you know, 1 and a half *x* times 2*y*, how did that end up getting a 3*xy*? So by showing them with the concrete representation of the tiles, you're like, oh, I see, that's an *xy*, that's an *xy*, and those are both half *xy*'s. I can make that into a 3*xy*. So try to move their conceptual thinking forward using models.