Whitebeard's Treasure

Whitebeard, the notorious pirate of the West Bay, buried treasure on Tiki Island over 200 years ago. Archaeologists recently discovered a map showing the location of the treasure. The location has generated quite a bit of media attention, much to the dismay of the archaeologists. In order to allow both the media and archaeologists to work together, officials have decided to erect two fences around the location, allowing the media access to the site, while also allowing the archaeologists room to work. One fence encloses the actual area where the archaeologists will work. Another fence surrounds the enclosed dig area.

Descriptions of the fencing locations have been provided to the media so they may indicate accessible areas for their employees. Use the given information to draw and label a quadrilateral on graph paper indicating the location of the two fences.

1. Corners of the first fence are located at points A(11,3), B(3,-11), C(-13,-9) and D(-5,9). The media must stay within this fenced area. Connect the points in alphabetical order, and then join point D to point A.

2. Find and label the midpoints of each segment of quadrilateral ABCD, showing all work. Label the midpoints of the segments as follows:

   $\overline{AB}$ has midpoint Q,

   $\overline{BC}$ has midpoint R,

   $\overline{CD}$ has midpoint S,

   $\overline{DA}$ has midpoint T.

3. Connect the four midpoints in alphabetical order to create a new quadrilateral QRST. This quadrilateral represents the fence surrounding the archaeological dig site.

4. Quadrilateral ABCD was an ordinary quadrilateral, but QRST is a special one. Determine the special name for quadrilateral QRST, and justify your answer using coordinate geometry in two different ways.

"Whitebeard's Treasure" from Charles A. Dana Center at The University of Texas at Austin. (2013). Geometry Formative Assessments Through the Common Core. Austin, Texas. This material accompanies a video-recorded lesson on Inside Mathematics (www.insidemathematics.org): 8th Grade Coordinate Geometry: Public Lesson.

Teacher Notes

CCSSM in the Task

Content Standards

(HS.G-CO) **C. Prove geometric theorems.**

11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

(HS.G-GPE) **B. Use coordinates to prove simple geometric theorems algebraically.**

4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).

(HS.G-GPE) **B. Use coordinates to prove simple geometric theorems algebraically.**

6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Practice Standards

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

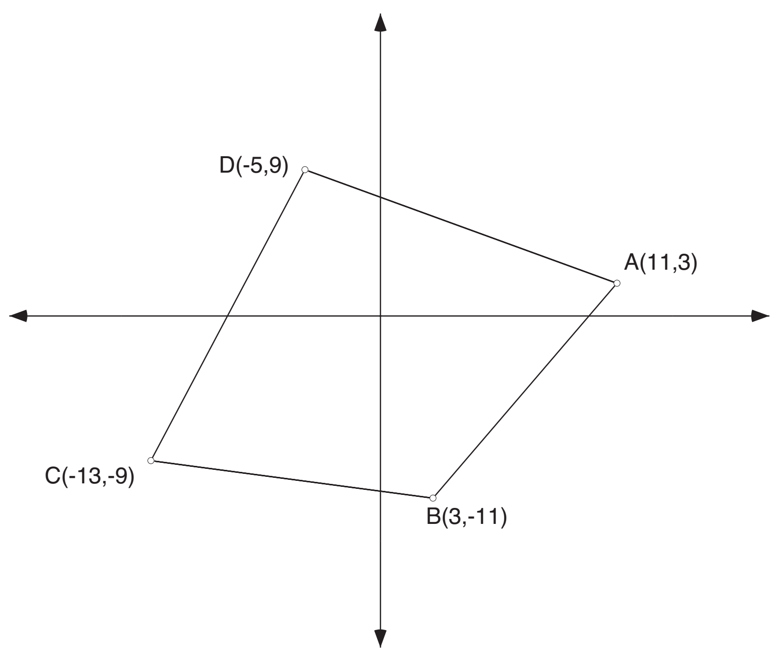
This performance task addresses the same mathematical concepts as the Wearable Art task later in this chapter. Whitebeard's Treasure gives the numerical coordinates. In Wearable Art the coordinates are given and the student must represent the situation using variable coordinates for the points. The teacher may choose to use one or both of these problems.

Scaffolding Questions:

- What is the formula for finding the midpoint of a line segment?
- Which of the quadrilaterals are special quadrilaterals?
- What are the characteristics of each special quadrilateral?
- What characteristics does quadrilateral QRST appear to possess that matches one of the special quadrilaterals?
- How can you prove these special characteristics?

Sample Solutions:

Quadrilateral ABCD is graphed as shown. This is the outer fence.



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To find the midpoint of each segment of quadrilateral ABCD, use the midpoint formula.

The midpoint of the segment with endpoints (x_1,y_1) and (x_2,y_2) is $\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)$.

To find the midpoint of each segment, substitute the *x* and *y* values from the endpoints of the segment into the formula as follows:

Midpoint of $\overline{AB}$ (Point Q)

$$\left(\frac{11\ +\ 3}{2},\frac{3+(-11)}{2}\right)=(7,-4)$$

Midpoint of $\overline{BC}$ (Point R)

$$\left(\frac{3\ +(-13)}{2},\frac{-11+(-9)}{2}\right)=(-5,-10)$$

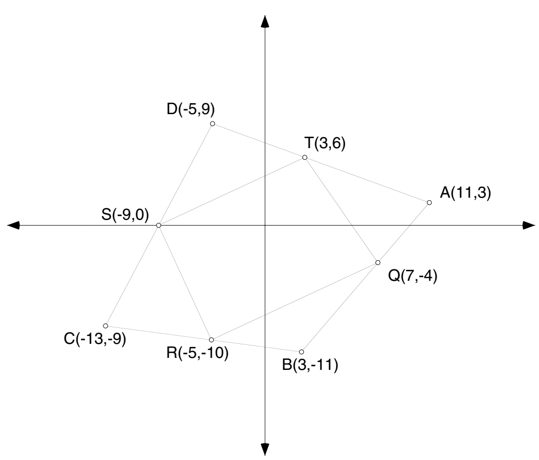
Midpoint of $\overline{CD}$ (Point S)

$$\left(\frac{-13\ +(-5)}{2},\frac{-9+9}{2}\right)=(-9,0)$$

Midpoint of $\overline{DA}$ (Point T)

$$\left(\frac{-5\ +11}{2},\frac{9+3}{2}\right)=(3,6)$$

Graph the midpoints and connect them in alphabetical order to form a new quadrilateral QRST.



Additional CCSSM in the Teacher Notes

(HS.F-LE)★ **A. Construct and compare linear, quadratic, and exponential models and solve problems.**

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

(HS.A-CED)★ **A. Create equations that describe numbers or relationships.**

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(HS.A-REI) **C. Solve systems of equations.**

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

(HS.G-GPE) **B. Use coordinates to prove simple geometric theorems algebraically.**

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Quadrilateral QRST would be the fence that encloses the archaeologists' dig site.

Quadrilateral QRST appears to be a parallelogram because the opposite sides of the newly formed quadrilateral appear to be parallel. One way to prove that a quadrilateral is a parallelogram is to prove that both pairs of opposite sides are parallel. Lines that have the same slope are parallel lines.

Use the slope formula:

The slope of the line through points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of $\overleftrightarrow{RS}$ is $\frac{0-(-10)}{-9-(-5)} = \frac{10}{-4} = -\frac{5}{2}$

The slope of $\overleftrightarrow{QT}$ is $\frac{6-(-4)}{3-7} = \frac{10}{-4} = -\frac{5}{2}$

$\overleftrightarrow{RS} \parallel \overleftrightarrow{QT}$ because both lines have the same slope.

The slope of $\overleftrightarrow{ST}$ is $\frac{6-0}{3-(-9)} = \frac{6}{12} = \frac{1}{2}$

The slope of $\overleftrightarrow{RQ} = \frac{-4-(-10)}{7-(-5)} = \frac{6}{12} = \frac{1}{2}$

$\overleftrightarrow{ST} \parallel \overleftrightarrow{RQ}$ because both lines have the same slope.

Quadrilateral QRST is a parallelogram by definition because both pairs of opposite sides are parallel.

Another way to show that QRST is a parallelogram is to prove that both sides of opposite sides are congruent (using the distance formula to find the lengths of each side).

$$QR = \sqrt{(7-(-5))^2 + (-4 - (-10))^2} = \sqrt{144+36} = \sqrt{180} = 6\sqrt{5}$$
$$ST = \sqrt{(-9-3)^2 + (0-6)^2} = \sqrt{144+36} = \sqrt{180} = 6\sqrt{5}$$
$$QR = ST = 6\sqrt{5}$$
$$RS = \sqrt{(-5-(-9))^2 + (-10-0)^2} = \sqrt{16+100} = \sqrt{116} = 2\sqrt{29}$$
$$QT = \sqrt{(7-3)^2 + (-4-6)^2} = \sqrt{16+100} = \sqrt{116} = 2\sqrt{29}$$
$$RS = QT = 2\sqrt{29}$$

Both pairs of opposite sides of quadrilateral QRST are congruent. Therefore, it is a parallelogram.

Extension Questions:

- Use algebra to find the point of intersection of the diagonals of quadrilateral QRST.

  To find the point where the diagonals intersect, the equations of lines $\overleftrightarrow{RT}$ and $\overleftrightarrow{SQ}$ must be identified and then used to find the point of intersection.

  The slope of $\overline{RT}$ *is* $\dfrac{6-(-10)}{3-(-5)} = \dfrac{16}{8} = 2$

  The slope of $\overline{SQ}$ *is* $\dfrac{0-(-4)}{-9-7} = \dfrac{4}{-16} = -\dfrac{1}{4}$

  The equation of $\overline{RT}$ *is* $(y-6) = 2(x-3)$ *or* $y = 2x$

  The equation of $\overline{SQ}$ *is*

  $$y = -\frac{1}{4}(x-(-9))$$
  $$y = -\frac{1}{4}x - \frac{9}{4}$$

The point where the diagonals intersect can be found by using linear combination.

$$y = 2x$$
$$y = -\frac{1}{4}x - \frac{9}{4}$$
$$2x = -\frac{1}{4}x - \frac{9}{4}$$
$$8x = -1x - 9$$
$$9x = -9$$
$$x = -1$$
$$y = 2x = 2(-1) = -2$$

The point of intersection is (-1,-2).

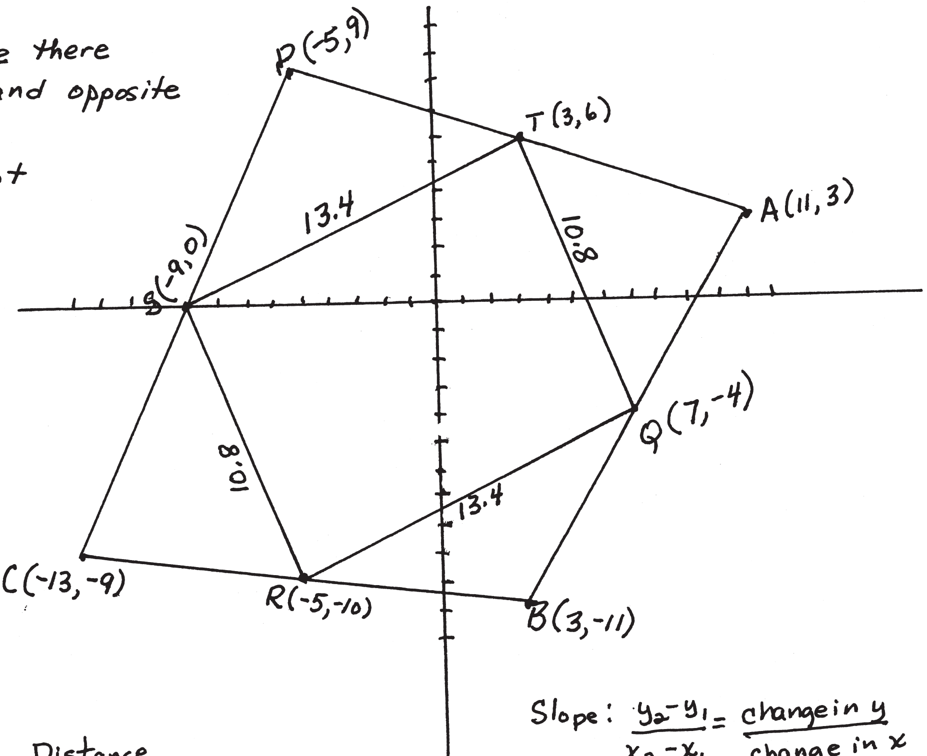
- Use coordinate geometry to prove the diagonals of quadrilateral QRST bisect each other.

The midpoint of $\overline{QS}$ *is* $\left(\frac{7+(-9)}{2}, \frac{-4+0}{2}\right) = (-1,-2)$

The midpoint of $\overline{RT}$ *is* $\left(\frac{-5+3}{2}, \frac{-10+6}{2}\right) = (-1,-2)$

The midpoints of the segment are the same point as the intersection point. The diagonals bisect each other.

A parallelogram - because there are two parallel sides and opposite sides are equal.
The diagonals are also not equal so it is not a rectangle or a square.



Midpoints on graph.

$RS = \sqrt{(-9 - {}^-5)^2 + (0 - {}^-10)^2}$

$= \sqrt{16 + 100} = \sqrt{116} \approx 10.8$

$TQ = \sqrt{(7-3)^2 + (-4-6)^2}$

$= \sqrt{16+100} = \sqrt{116} \approx 10.8$

Opposite sides are equal.
RS = TQ

$QR = \sqrt{(-5-7)^2 + (-10 - {}^-4)^2}$

$= \sqrt{144+36} = \sqrt{180} \approx 13.4$

$ST = \sqrt{(-9-3)^2 + (0-6)^2}$

$= \sqrt{144 + 36} = \sqrt{180} \approx 13.4$

Opposite sides are equal.
QR = ST

Distance

$RT = \sqrt{(-5-3)^2 + (-10-6)^2}$

$= \sqrt{64 + 256} = \sqrt{320} \approx 17.9$

$SQ = \sqrt{(-9-7)^2 + (0 - {}^-4)^2}$

$= \sqrt{256+16} = \sqrt{272} \approx 16.5$

The diagonals are not equal.

Slope: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$

QR: $\frac{{}^-10 - {}^-4}{-5-7} = \frac{-6}{-12} = \frac{1}{2}$
ST: $\frac{6-0}{3 - {}^-9} = \frac{6}{12} = \frac{1}{2}$
QR // ST because the slopes are the same.

RS: $\frac{0-10}{-9 - {}^-5} = \frac{10}{4} = \frac{5}{2}$
QT: $\frac{6 - {}^-4}{3-7} = \frac{10}{-4} = \frac{5}{2}$
RS // QT because the slopes are the same.