LINDA FISHER: We are here at Willard School and we are doing a lesson called "The Math Things Mingle." I'd like to hear from the planning team about some of the thinking that went into how you organized this lesson.

JACOB DISSTON: This is our first lesson this year, so we've spent four months meeting pretty regularly to flesh this out and to lay the ideas out that we think are important and that we want to talk about. This year the discussion has sort of focused on big ideas that connect-- that thread through the grade levels and that connect a lot of the mathematics that we teach. The big ideas we came up with were equality, variables, and sort of number patterns, how operations on numbers work. So trying to figure out where in our teaching, where in our lessons, where in the textbook do kids begin to develop broad understandings of these things that connect the skills that are taught. The procedures, you know, they spend a lot of time getting exposed to an equation that they have to solve. They get exposed to single-step equations and then multi-step equations and then inequalities and then... and so we're always, most of our lessons are "here's the thing you're going to see and here's what we want you to do with it. Show us that you can do it. Ok now, here's the thing that you're going to see and here's what we want you to do with it." And so, what I think we've come to sort of a question about is, does that lead kids to have a connected idea of how all these things relate to each other? Is there evidence, if we throw them a different sort of lesson, will we see evidence that they are starting to make connections between things, that they have developed language to talk about them? How things are similar and different and not just only able to perform when you say "do this."

JESSE RAGENT: What are some of the major problems kids have towards success in algebra? Where in sixth and seventh and eighth grade can we hit on those things that seem to be stumbling blocks? We listed, kids have trouble with variables. Having a facile-- an ease with variable, and with not really respecting the equal sign, not really understanding what equality is about. And then operations, and I think you said number patterns, I think we talked about different representations, different ways to represent numbers as well.

ALLISON KRASNOW: For kids to succeed in algebra, what do they really have to understand? We had a lot of discussions around that, so we haven't just sort of planned this lesson and planned this lesson, but we got derailed in terms of what are the big ideas that kids have to understand? To be able to succeed in algebra is related to these topics, variables, specifically about equality and operations, types of numbers. How do we avoid, like Jake mentioned, the textbook sort of takes you lock step through "we're going to do this type of problem and this type of problem" and we tend to start to teach like that if we're not careful. And how do we bring our teaching back to the big ideas around these topics without just teaching problem types. Because if you only know how to do types of problems then we're confronted with a problem that involves many of these concepts at once, and you don't have any steps anymore of how to solve it, or how to even start.

JESSE RAGENT: We also thought about the successful students in algebra, and what is it that they have? Allison brought up one time that they have certain intuitions. They see an inequality problem and it may not be that they have an exact sort of algorithmic understanding of the steps involved to solve it but they say "oh, oh, inequalities, there could be a lot of things that will make that true." "Oh, oh, I have this thing with no equal sign, it's an expression, so I'm not really trying to solve that." So kids have intuitions about these number strings, the successful ones, that some of our other kids don't have those intuitions.

JACOB DISSTON: The thing about this lesson that is different from lessons that students are used to is it's not about teaching them how to do something, it's trying to give an opportunity, it's presenting an opportunity for them to think about and discuss these math things which we had a hard time coming up with. You know we called them symbol strings but them as things are kind of ...that's why the title is what it is. That we're giving them an opportunity to talk about them, and we're going to see and it's really to expose their use of vocabulary, or that they don't use the right vocabulary -- to expose their ability to see connections or that they don't see connections. So it's more diagnostic, it's more a sort of formative assessment than a lesson designed to teach

LINDA FISHER: In listening to some of your planning discussion, there was a real indecision about whether to go with the narrow list of things for them to sort through or a wider list. Can you talk a little bit about your thinking behind the choices you made?

ALLISON KRASNOW: We thought about doing less different kinds of examples of things, but then got worried that we would get into like, "OK, these are all formulas, whenever you see formulas you should think about this, whenever you see inequalities you should think about that." And so we decided that the purpose is really to more see what they do and don't understand, what their instincts are or are not at this point in time. We decided to broaden it to more examples of things because it gives us, hopefully, a richer sense of what they do and don't understand.

JESSE RAGENT: A more simplified listing would have to do, I think, with how much disequilibrium, or how much chaos they could handle as seventh graders in order to make the lesson work, not to throw too much at them. But we opted, I think, to err on the side of more than less.

LINDA FISHER: You know you really thought about each expression or equation that you put up there very thoughtfully with the hope that it might bring up possibilities of some really interesting mathematical discussions. So I think there was a question about understanding the difference between, say, a less-than expression or a less-than-or-equal-to expression. As you were picking those what were some of the interesting side things that you were hoping the kids might end up discussing?

JACOB DISSTON: Part of this is asking them, "what do you see?" And then helping them. At first that's a strange question and they're like, "Well I see lots of letters and numbers, these are just letters and these have letters and numbers, these are plusses." It's helping them to distinguish between differences that

are maybe apparent but not mathematically significant or rich. And deciding what does that mean, mathematical richness or importance? And so, with all the inequalities, kids may notice that, may distinguish greater than or equal to, greater than, less than or equal to, less than, four categories. Other kids may distinguish greater than or equal to and less than or equal to, sort of like "or equal to" inequalities and then the strictly "greater than" and strictly "less than." And to get them to say, "So why is that important?" Like why do you think that is something that we want seventh graders to ponder over? Why is it important that we categorize them this way versus this way?

JESSE RAGENT: Right, and I think the prompts that Jake is going to be able to come up with to...

JACOB DISSTON: (laughter) Thanks, Jesse!

JESSE RAGENT: ... out of his magic bag of tricks in order to suss that out, or to get the kids to see that there are a lot of ways to distinguish these, and some of these are trivial distinctions and some are going to be more important, mathematically important.

LINDA FISHER: One of the things that interests me in the design of this lesson is the use of the cards. So, can anybody talk about the purpose of the cards?

JESSE RAGENT: Well, part of that is that it enables them to move around and meet up with people. It's used a number of different ways because they're each given an individual card and they have to find someone -- is that..?

JACOB DISSTON: Yeah.

JESSE RAGENT: So that's part of just getting them up and out of their seats and moving around and talking with people that are not necessarily part of their group, and they go to different groupings.

JACOB DISSTON: I think also it allows us to present them, if you'll look over on the board, it allows us to present them with twenty math things and say "what can we notice?" It doesn't rely on them having to transcribe them onto their page, solve something. It doesn't require them to produce the math in order to talk about the math. The risks in the type of lesson where kids create it is that you never get to the big ideas because they're stumbling over integer operations, or graphing, or, you know, doing things. So it just allows them to have the math in front of them, to move it around to -- it's very easy to read and reconfigure based as their ideas change.

LINDA FISHER: What kind of evidence would you like us to collect during the lesson?

JACOB DISSTON: Our research questions are, "What evidence is there that students are able to see these symbol strings as more than just collections of letters, numbers and operations? What evidence is there that students are developing an understanding of the attributes and features of the symbol strings which will be utilized in eighth grade algebra?" The bell is going to ring but I think, for the observers, look for the evolution of their thinking. How does their thinking about these symbol strings evolve?

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