ERIKA ISOMURA: We went back a little bit to multiplication as repeated addition with whole numbers just to remember that, "Have you done this?" "Oh, yes!" We have agreed that this is a method--it's not always efficient, but it works.

MIA BULJAN: Right.

ERIKA ISOMURA: So then I gave them problems like, um, "I have five pizza boxes, and each pizza box has half a pizza left. Am I going to stick all five pizza boxes in the fridge? Probably not." And they were very firm on the idea that's not going to fit.

MIA BULJAN: Yeah.

ERIKA ISOMURA: So then what would you do in real life? You want to take that half and put it with this half. So then what would be in this box? One whole pizza. And what would be here? So we did very basic putting together, and then I asked them...

MIA BULJAN: And there was no number equations, just story problems?

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: Okay.

ERIKA ISOMURA: Yeah, we didn't...

MIA BULJAN: So the first context was five half pizzas?

ERIKA ISOMURA: Yeah.

MIA BULJAN: And how many boxes of pizzas would go in the refrigerator?

ERIKA ISOMURA: How many boxes, yeah. Two. And they were able to get, you know, just real life, two and a half boxes would go in. Great. And then we talked about, "Well, we added them and we put them together, but since you're doing half, and then half again, and half again." And then we wrote that out as half, plus half, plus half. So if you did this, what's the short cut? And happily several of the kids, "Well, that's really just a multiplication problem." And they said, "Oh okay, so we could say half times five or five times half. Whichever order?" And they said yes. So then the next day, out of curiosity, I gave them five times half and I got five tenths.

MIA BULJAN: Yeah.

ERIKA ISOMURA: Because they don't know anything about operation, like, just kind of the standard way you operate that.

MIA BULJAN: So when you were kind of eliciting that conversation with them, where they were sort of talking about what they did with, like, the half and the half, you were recording that, like, on a chart?

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: So you recorded the one half plus one half and modeled that?

ERIKA ISOMURA: Yeah.

MIA BULJAN: And then the next day you just gave a five times one half?

ERIKA ISOMURA: Yup.

MIA BULJAN: Which they had generated the day before and they were not able to do the operation.

ERIKA ISOMURA: Nope.

MIA BULJAN: That sounds about right.

ERIKA ISOMURA: Yeah. And there was a lot of confusion because then I, you know, we went back and talked about the pizzas again, and there was this really, "Well, oh, five tenths must really be two and a half." So then we had to draw that out...

MIA BULJAN: We've got to prove that, too.

ERIKA ISOMURA: Right. And then we looked at it and we were like, "But that looks like a half."

MIA BULJAN: Yeah.

ERIKA ISOMURA: "So how did that happen because..." Again, they haven't worked enough with equivalence via forms of one to realize that they were multiplying five over five, which is one. So again, I kind of went, "Okay." So we set it aside and went back to our story problems without dealing with any kind of equation or writing it that way.

MIA BULJAN: So how did you handle that? So what did that sound like in the classroom? You did this and it kind of, like, you went down a path and you got down to a dead-end, basically.

ERIKA ISOMURA: Yes.

MIA BULJAN: So, so, like, what does that sound like? When you tell them, like, "Well, we're going to go back and work on some more steps, but, like, do you just leave that hanging or do you actually say to them, like, "Okay, we're still thinking about this," like, what is it?

ERIKA ISOMURA: So with the five tenths thing I said...and we drew it as a half and we said that doesn't make any sense and they said, "I think there's a method to multiplying numbers but I'm not sure if we're ready, because I'm not sure that we know what to expect yet.

So let's put it aside because unless we have..." We've been working a lot on having expectation or an estimation before you get something, and that way you can see if it makes sense. That's our check. So we put it aside. We went back to the story problems and I haven't really showed them an equation or an expression at all since then. It's all been via story problems. So.

MIA BULJAN: Got it. Okay.

ERIKA ISOMURA: Um, the two problems we've been working with for about a week now are Jesus and Camila “String Problems.” They're making these math games.

MIA BULJAN: Okay.

ERIKA ISOMURA: Jesus has this many pieces of string. Each one is this length long, and it's a unit fraction. How long would the piece of string be if you glued them together? I had a talk with a fifth-grade teacher yesterday and she said, you know, "Did you say tie it?" "No, because fifth graders will be picky about how, oh, that shortens it?"

MIA BULJAN: You lost it.

ERIKA ISOMURA: Yeah. So no, we glued it together, how long would be end to end.

MIA BULJAN: So is it the same numbers again? You said there were five pieces of string?

ERIKA ISOMURA: In this case... Well, no. It changes every day.

MIA BULJAN: I wasn't listening. Just kidding.

ERIKA ISOMURA: Sometimes it's seven and it's two thirds long. Sometimes it's ten...

MIA BULJAN: Okay, so there's blanks there and you can put in different...

ERIKA ISOMURA: Right.

MIA BULJAN: Okay, so there's "mmm" number of strings and each string...

ERIKA ISOMURA: And each string is this long. Yeah. And how long would the full thing be if you glued them all together.

MIA BULJAN: Got it.

ERIKA ISOMURA: And... So that's the one and then the other one that I did because I was curious. Because multiplications and fractions can be done as, you know, two thirds, and two thirds, and two thirds, and two thirds.

MIA BULJAN: Right.

ERIKA ISOMURA: But then there's also the "of" problems that's always the biggest problem in fifth grade when we multiply fractions. So two thirds "of" six, and five tenths "of" whatever. And, um...

MIA BULJAN: Is it always a whole number, like, a fraction of a whole number?

ERIKA ISOMURA: No.

MIA BULJAN: Like a group?

ERIKA ISOMURA: Well, that's how we started but now I've...

MIA BULJAN: Okay.

ERIKA ISOMURA: Lately I've been giving them, "What's one third of one and a half?"

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: So, um...and for the two types of problems...Camila's are the string problems where it's...I have... So I think the most recent one we did was "Jesus has seven pieces of string. Each is two thirds of a foot long. How long together?" And then Camila's...

MIA BULJAN: Okay, hold on. I have to picture it. Jesus has seven pieces of string, two thirds long. Okay.

ERIKA ISOMURA: Two thirds, two thirds, two thirds, and so on.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: Camila has a seven-foot-long piece of string, but she decides she only wants to use two thirds of that string.

MIA BULJAN: Got it.

ERIKA ISOMURA: And so they've been working them and finding that somehow these are coming out the same, which is really provoking because Dylan, for example, keep saying, "I don't understand why that's happening. I'm dividing."

MIA BULJAN: Oh, nice.

ERIKA ISOMURA: So...for the Camila problem.

MIA BULJAN: Yeah.

ERIKA ISOMURA: And he doesn't see that as a multiplication. So he's really been agitated by the fact that he feels like this is a divisions problem, and yet it's producing that same result with the same numbers as a multiplications problem. Like, in his view, he sees this problem of the repeated two thirds that's clearly multiplication.

MIA BULJAN: Yeah.

ERIKA ISOMURA: But the other one seems to be giving the same answer and yet it's clearly, in his brain, not a multiplication.

MIA BULJAN: How's he solving it? He's dividing seven...?

ERIKA ISOMURA: So he actually came up with seven times two. So the whole number times the numerator, and he's been dividing by three because in his words, "I have seven sets of these things that are two, like, two pieces but it takes three pieces to make a whole. So if I divide it by three then I'm making these little sets of three that are each wholes."

MIA BULJAN: So that's a totally legitimate explanation...

ERIKA ISOMURA: Absolutely!

MIA BULJAN: ...of the operation of multiplication, but in his mind, that dividing by three is...

ERIKA ISOMURA: Is driving him nuts.

MIA BULJAN: It's interesting because a lot of times we, um, describe or work with students on fractions as the denominator is a naming piece and, um, and the top is the counting piece. Like, in second and third grade that was a lot of the language. And, um, and what he's really doing is he's defining the denominator as the divisor and the numerator as a multiplier, which actually is the most powerful way to think about a fraction, and he's sort of backed into it by making sense of that.

ERIKA ISOMURA: And pretty much nobody else in the class agrees with him even though he's getting the same answers, but...

MIA BULJAN: That sounds right, too.

ERIKA ISOMURA: Yeah. But he's very convinced that it makes sense and I told him, "It does...what you showed me and the way you explained it, I understand that does make sense to me. So for the moment, go with it and we'll see what happens as more people keep playing with it, if they come to agreeing with yours."

MIA BULJAN: Yeah. So basically you convinced me, now you've got to convince somebody else, right?