MIA BULJAN: Another part of your lesson it seemed was this idea of, um...and you said this very explicitly to several groups, and then I think in your little mini-lesson that you pulled on the carpet it came out again, which is, "it's not so much content as it's a habit of mind."

Like, it's a way of thinking about making sense, which is, "are you matching the picture to the story or just to the numbers?" And so that bar diagram, a lot of them...where there was the choice, and I'm going to show you again. Sorry. So we're going to refer to two of these. Let's do Rosa Linda's because that's why we have her paper. So Rosa Linda is drawing a picture for her mom. She has a box of eight crayons but decides to use only half of the colors. How many crayons will she use? And when she's talking about...Erika, when you're talking about a partner...

ERIKA ISOMURA: It's Antonio's problem, which is right here.

MIA BULJAN: It's, Antonio found eight rocks and put them all in a bag. Each rock weighed half a pound. How heavy was the bag? And when we come over here and we look at those pictures, can you show which picture did you anticipate for Rosa Linda's eight crayons?

ERIKA ISOMURA: So this was Rosa Linda's eight crayons. Here's the box with eight crayons and then she chooses half of them. There's two sections.

MIA BULJAN: And then Antonio's rocks...

ERIKA ISOMURA: Versus Antonio's, here's eight rocks and each one is marked with half a pound, and they're going to hopefully put that together to...

MIA BULJAN: Mm-hm. So here is like the four...four pound...the four would be the, um...

ERIKA ISOMURA: Four crayons.

MIA BULJAN: Um, part and here the four is the ...?

ERIKA ISOMURA: Whole.

MIA BULJAN: The whole thing.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: Okay. And so, um, so you anticipated that they would, um, get these mixed up.

ERIKA ISOMURA: Yes.

MIA BULJAN: You said if they just read this and saw eight and a half that they could just say, "Well, this is eight halves."

ERIKA ISOMURA: Right.

MIA BULJAN: Like, just match those numbers basically. And this is eight rocks and then half is like implied there in the shapes.

ERIKA ISOMURA: Yes.

MIA BULJAN: So you thought that they would mix those up?

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: I felt like this one and this one was more problematic for them actually, um, because it's the same kind of combination.

ERIKA ISOMURA: Right.

MIA BULJAN: So you did that on purpose, hoping that they would go into [inaudible]...

ERIKA ISOMURA: Would stumble there and then, yeah. It was on purpose.

MIA BULJAN: And they did. A lot of them did.

ERIKA ISOMURA: Yes.

MIA BULJAN: Okay. So the first thing was that you structured your task in a way that had these pairs to sort of, like, surface this issue. Like, you wanted that to happen. And then you put the words on it. You said, "So in math, are you looking at the...are you matching that picture to the story or are you just matching it to the numbers?"

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: And you kind of kept...that was sort of, like, the thing that you kept starting with at each group.

ERIKA ISOMURA: Yes.

MIA BULJAN: So how successful was that in terms of when you ask them that, like, can you talk about Elijah maybe, or one of the other kids who was struggling?

ERIKA ISOMURA: Um, I know there are still some problems...

MIA BULJAN: Oh, yeah.

ERIKA ISOMURA: ...with the results because I didn't get to talk about everyone, but when I found one and we talked it through, I usually...I think I found a couple that were correct and I still made them talk me through it, but most of the ones that I did talk to were not the matches that should've happened. And what I asked them to do is, um...who did I work with last? I can't remember.

MIA BULJAN: Was it Elijah?

ERIKA ISOMURA: No, there was a group that I was actually working with, with, um... Oh, I think it was Jerry's group. They were working on the really hard question, Lizzie's elastic problem. That was a mess.

MIA BULJAN: Oh, that killed them.

ERIKA ISOMURA: Yeah. So...but what I kept asking them was, "So Lizzie's rubber band was four inches long and then she stretched it two and a half times the original length. And then there's another picture that's two and a half four times."

And so Adam was saying, "Well, I see there's four boxes and there's the two and a half." And I kept asking him, "Where in this picture do you see the four inches Lizzie started with? So where is that elastic?" "Well, there's four boxes." "But again, where is that one piece of elastic that's four inches long?" And we went back and forth several times.

MIA BULJAN: Yeah.

ERIKA ISOMURA: And Jerry kind of was watching and then at some point he pointed out that the picture right next to it had a box with four.

MIA BULJAN: So let's take a look.

ERIKA ISOMURA: So it was this here. So Adam was...here's the four inches of elastic.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: And here's the two and a half times because there's two and a half in the boxes. So Jerry said, "No. Here's four inches of elastic but then where's the two and a half times?" So we're really struggling with where...because this one clearly has both numbers.

MIA BULJAN: Yup.

ERIKA ISOMURA: This has the number, and to Jerry it kind of felt right but there was something off with the way numbers were.

MIA BULJAN: So, Federico and Alex.

ERIKA ISOMURA: Federico and Alex?

MIA BULJAN: You had the most interesting conversation with Federico about this one.

ERIKA ISOMURA: Yes. So Federico was talking about, he felt like this might be a Lizzie situation--but that two. Because here's the four and here's the four, and Lizzie was doing it two times, but there's that two. And how can you have a two...

MIA BULJAN: And he actually said, "How could this be a fraction? They're not equal parts."

ERIKA ISOMURA: Right. And how can you have a two sitting there when every single other bar model always seems to have equally proportioned parts?

MIA BULJAN: Right.

ERIKA ISOMURA: They all do and this one doesn't.

MIA BULJAN: Right.

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ERIKA ISOMURA: And he was very aggravated with that.

# MIA BULJAN: Yes.

ERIKA ISOMURA: So I'm hoping that on Monday I have Jerry's partnership work with Federico's partnership--specifically around Lizzie's question, because Jerry was able to say, "Wait a minute, two is half of four. So this is one rubber band, two, and that's the half." Adam was not convinced when I left. I'm not sure if Jerry did finally convince him, but I'd like him to talk to Federico because Federico has a good point of...and they can work it out. Can we have different size pieces on bar models?

MIA BULJAN: Adam may not have been hearing Jerry, but Federico is trying to hear this. Like, he needs an explanation for this, right? Like, it struck him that it's odd and he would like something reasonable to put there, right? Like, he's like...he totally, right?

ERIKA ISOMURA: Yeah. So I think that'll be a nice...

MIA BULJAN: Follow up.

ERIKA ISOMURA: And from there, those two groups or at least those two individuals can maybe start talking to the other groups who were irritated with the Lizzie problem.

MIA BULJAN: So on the other side of that, which is... So that was like a big part. Those paired things really surfaced this idea and you kept kind of pushing on that. It has to match. It has to match. It has to match. It has to match.

## ERIKA ISOMURA: Right.

MIA BULJAN: Not just the numbers. So Rosa Linda, one of our first friends, um, had this book of box of crayons.

## ERIKA ISOMURA: Yes.

MIA BULJAN: I don't know if you can pick this up, Mark [videographer], but I hope you can because it's the most beautiful thing ever. Um, so she has eight crayons and she's going to use half of them. So these are her eight crayons. 1, 2, 3, 4, 5, 6, 7, 8.

MARK: Mia, can you backtrack on that?

MIA BULJAN: Yeah. So she has eight crayons and she...in the box, but she's only going to use half of them.

ERIKA ISOMURA: So here's her notes to herself. I want to use eight crayons but I'm only going to...and she circled the half to remind herself she's only using half.

MIA BULJAN: Then she drew out eight crayons. So there's 1, 2, 3, 4, 5, 6, 7, 8 crayons. And over here what she did was she says, "I'm going to use half of this crayon, half of this crayon." She explains this beautifully, by the way. "Half of this crayon, half of this crayon." And when you count, there's 1, 2, 3, 4, 5,

6, 7, 8 times that she uses half of a crayon. And so her answer is eight halves of crayons, and she divides it out and gets four. I don't know why she's doing four halves. She didn't really ex...

ERIKA ISOMURA: No.

MIA BULJAN: She just kind of further work. We'll call that "further work" because she's very clearly making sense of this.

ERIKA ISOMURA: And on her card she said four crayons. So.

MIA BULJAN: Yeah so she has the four crayons answer. And so you want to tell us a little bit about the conversation you had with her regarding this particular work?

ERIKA ISOMURA: Mm-hm. So once she explained it to me I told her that...

MIA BULAN: Oh, have you already...did she already have this?

ERIKA ISOMURA: No. That was Camila's.

MIA BULJAN: Okay, so she had not matched it to a picture? Okay, so go ahead.

ERIKA ISOMURA: So that was just, um, after they had decided what kind of problem, they had the option of starting to just kind of playing with it on their own before they got into that. So I told her, you know, that really does, to me, show that she's really thinking about what's happening in the problem as far as the quantities. But what I asked her was, "Now, let's actually be in real life. If you had that box of eight crayons, it makes sense that you could take half of each crayon, but would you?"

MIA BULJAN: Right.

ERIKA ISOMURA: "Would you walk around snapping crayons in half and have, you know, all these broken crayons?" And her response was, "Well, no."

MIA BULJAN: Right.

ERIKA ISOMURA: So then we talked about, well, you know, in the real world, if you got this box and I said, "You're only going to take out half of the crayons, what would you do?" She said, "I'll take out four."

MIA BULJAN: Yeah.

ERIKA ISOMURA: And then we said, "Well, that's so interesting because the answer you got by snapping crayons in half was also four."

MIA BULJAN: Right.

ERIKA ISOMURA: But which one would actually be the real world? And she was very clear that, "No, it would be more normal to take out four whole crayons."

MIA BULJAN: So when you gave her this, was there any, like, confusion on her part? Like, which one?

ERIKA ISOMURA: So we looked at the two and, um, I feel like she was pretty clear that this was a box of eight, and here's the half and that would be four crayons in this half, and four crayons in that half. And, you know, we could do that. We could take those and snap them in half.

MIA BULJAN: Did she...did she recognize this as her original strategy or did you...?

ERIKA ISOMURA: I think I pointed out to her that...

MIA BULJAN: Okay.

ERIKA ISOMURA: ...or here's eight crayons where we snap them all in half.

MIA BULJAN: Got it.

ERIKA ISOMURA: And I told her specifically that there are two different ways that we could be thinking about those numbers, but when we go back to the actual problem, the story, maybe one of them makes a little more sense.

## MIA BULJAN: Right.

ERIKA ISOMURA: So, um, we looked at how these two both gave us the numbers, and they both gave us four but we could have a bunch of snapped crayons...

MIA BULJAN: You know what I'm thinking of? I'm thinking of their, um, who was it? Diego's strategy of, um...Diego and Ruchita.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: Where they talked about the wholes and the parts? Where, um...oh, sorry. On, um, on this one where they talked about the wholes and the parts, the four here, that would've meant that there were four crayons. Right?

## ERIKA ISOMURA: Right.

MIA BULJAN: And where as this one, like, there was eight crayons that was the whole thing, or whatever. So their interpretation of that question mark and how that affected, like, do I know the wholes or do I know the parts. Um, that could also be really helpful in that instance of, like, which one actually matches what I'm doing. Like, I didn't have four crayons, I had eight crayons. I mean that was the whole thing, so, um, I was just impressed that they were, like, deepening their understanding by trying to, like, match these up, which was what you had planned.

## ERIKA ISOMURA: Yeah.

MIA BULJAN: And then also, you were able to use that in the mini-lesson.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: So tell me a little bit. Do you do that a lot, where you, um, almost like a mid-workshop interruption, right?

ERIKA ISOMURA: Yeah, that's what it is.

MIA BULJAN: Like, yo, this is it.

ERIKA ISOMURA: Um, yeah, we actually *do* do that quite frequently.

MIA BULJAN: Yeah.

ERIKA ISOMURA: Sometimes it's...no, oftentimes it's because of behavior and we need to kind of settle back in.

MIA BULJAN: Yeah. You don't have to tell me that.

ERIKA ISOMURA: Yeah. But a lot of times it is the content where I notice that something is going just enough off track that if I let it keep going, it's going to turn into a major headache in the future.

MIA BULJAN: Yeah.

ERIKA ISOMURA: And so I want to pull them back. Sometimes it's to clarify the directions because they've been working and they've gotten slightly off, but what they're doing is valuable. So I might say, "Let's focus in. We need to finish this, but that's a really cool idea."

MIA BULJAN: Kind of bookmark it, sort of speak. Yeah.

ERIKA ISOMURA: "So once you do this, let's come back into that." And sometimes it's to address, "So here's what's happening. I'm not sure. Can we hear from everybody? Do we all think that this is working?"

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: "Um, or we don't, so let's go through it...let's try one together."

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: "Now that we have this new idea together, go off and see what happens with that."

MIA BULJAN: Yeah.

ERIKA ISOMURA: Yeah. And it actually, again with the combination, it works really nicely because I can alternate a little bit of back and forth.

MIA BULJAN: Who you pull in, yeah.

ERIKA ISOMURA: And I can also sometimes pull...even though they might be working on different tasks, something comes up that overlaps so I can pull three of these and then seven of these, and then we can do a quick little chat. So it's not even all whole groups all the time.

MIA BULJAN: So sometimes it's more like a conferencing situation where you might just conference with a little group?

ERIKA ISOMURA: Yeah. But if it's a big...like in this case, I really wanted them to know that those were pairs on purpose.

MIA BULJAN: Right. So they had this productive struggle where they were kind of talking and then you pulled them in and, like, kind of put a pin in that, right?

ERIKA ISOMURA: Yes.

MIA BULJAN: And that's when you said...

ERIKA ISOMURA: "That's supposed to be like that. It's supposed to be an issue. Now let's go work with it."

MIA BULJAN: And the big idea that you...that you named right then again was, you have to match the context, not just the numbers.

ERIKA ISOMURA: Right.

MIA BULJAN: And that was, um...I thought that was really motivating for them. They seemed to understand that there was a difference at that point.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: Probably because they had struggled with it.

ERIKA ISOMURA: Yes. And we've done a lot of work on the story matters. So.

MIA BULJAN: Nice. Context matters.

ERIKA ISOMURA: It does. Yeah.