MIA BULJAN: So along those lines, um, Ruchita and Diego there in the front, um, when they were still working on the parts and wholes, they had a great strategy. I saw a lot of great strategies. They were naming it, Jesus or Camila [inaudible], whatever it was. Um, Ruchita and Diego, they had written wholes and pieces on each one, and then they circled the one that they knew.

ERIKA ISOMURA: Right.

MIA BULJAN: "We know the whole, we know the pieces." And one of the things that you had worked into your lesson design was this idea that, um, some of this was still a lot of gray area for some of your students, but there would be something about putting those two pieces together--the context with the diagram--that would...you were hoping would deepen or strengthen their understanding where there was grayness.

And I think Ruchita and Diego definitely had that because they, um, they discovered that the bar models matched. So, um, so, like, if you look at the white one, what they discovered was that if they knew the whole thing, this, the whole would be, the number and the pieces would be the question mark. And, if the question mark was here then they were like, "Oh, we know the pieces but not the whole thing."

So the nature of the structure of this bar model was helping them, and it helped them where they had mismarked some of their papers. They thought that they knew the whole and then when they saw that the question mark wasn't the same, they were like, "Oh, that doesn't match. You know, my question mark is in the wrong place, if that's in fact how it works."

Now, in the first pass what they did was they said, "Oh, this doesn't match. Must be one of those extra ones she talked about."

ERIKA ISOMURA: Right.

MIA BULJAN: And so you came over and interacted with them a little bit, and when I stopped by later, they seemed to get [it]. They seemed to, like, realize they actually had to prove that. And I'm curious, do you remember what you talked to them about to help them work through that?

ERIKA ISOMURA: They had three that didn't match, and I knew there was an issue because there was only supposed to be two.

MIA BULJAN: Yeah.

ERIKA ISOMURA: So I looked and Antonio's was, um...so I had designed it to have these partner problems where the numbers are the same and the answer is the same, but the context is very different.

MIA BULJAN: Yes.

ERIKA ISOMURA: And I anticipated that would be a lot of the issue of mixing up and going just number, number, number, and not context.

MIA BULJAN: Yeah.

ERIKA ISOMURA: So what they had said was, "Antonio has eight rocks, so he must have eight total. And there's half, half, half, half, half." And somehow they ended up with one sixteenth as a response, which was...I asked them to explain it. They were like, "Well, obviously he has eight rocks. Each one is half a pound. So he must have one sixteenth." I knew at that point they weren't dealing with the problem at all--they were just dealing with the numbers.

MIA BULJAN: Manipulating those values.

ERIKA ISOMURA: Right. So I...there was a stickie pad, so I put eight stickies and I said, "Here's his rocks. Tell me about these rocks." And they told me, "Well, each one is half a pound," and I said, "Okay. So I'm going to grab them. How many pounds would that be? How heavy would it be?" Diego goes, boom, "Four pounds," because that was making sense of the problem. And he's very...both of them are very capable with numbers.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: Which is why I think they jumped to just playing with the numbers. So then we looked at their card, [on] which they had written one sixteenth, and I said, "But you just told me four, and you wrote one sixteenth. Which one makes sense? Like, just the rocks."

And they both were pretty immediate, "No, it's four pounds." So then we had a little discussion about, "So do you think when you did this originally, you were actually working with the problem or the numbers?"

MIA BULJAN: Right.

ERIKA ISOMURA: And they both felt that, yeah, they were just working with the numbers. So then I asked them to go back and stop working with just numbers and trying to match numbers, and think about how the stories work.

And then, um, shortly after that I pulled the whole class because I knew if those two were struggling then undoubtedly, a large portion of the rest were going to be doing something similar. At some point they were going to, "Oh, I see a short cut. I can do this faster and more efficiently."

MIA BULJAN: Right. I was impressed, actually, with the...with the lack of, um, uh...you know, they had their notebooks there and, um, I guess I could imagine a class that would grab the notebooks and just sort of start cranking through calculations of whether it makes sense or not. I was impressed with how long they were willing to talk to each other about where they thought those, um, you know, did they think they were wholes or pieces. Did they understand the...I mean...

ERIKA ISOMURA: Yeah, yeah.

MIA BULJAN: Yeah, I know.

ERIKA ISOMURA: That's what I mean work in progress with this class because they are definitely number people.

MIA BULJAN: Yeah.

Inside Mathematics