MALLORY WILLIAMSON: This particular lesson is in the form of a Three-Act Task. A Three-Act Task is where a particular picture, video, or something is shown to the students to bring about questioning, and then bits of information is released a little bit at a time. So the whole part of a Three-Act Task is to see if they're able to reason through certain things or ask certain questions that help them solve bits of information that are needed for a problem. And it's not necessarily the focus on the solution, it's just having kids basically struggle with what they're trying to solve in order to find a solution, which is something that students aren't necessarily used to, they want to get the entirety of the problem and then quickly solve it and get it over with or they want to know right away if they're right or wrong and within a Three-Act Task, that doesn't lend itself to be true. So we're focusing on what kind of questions our students are asking and how we're providing feedback on their particular strategies to find a solution.

Within this lesson of finding volume, I think that based on prior knowledge, it's students look at a formula and can solve and plug in certain amounts with that formula. So the reasoning behind using a Three-Act Task for finding volume is to actually use a form of estimating with volume, reasoning through with spatial recognition and not just a particular formula. I'm taking a look at what they're doing that's not related to necessarily formula or an easy way to calculate volume, how they're getting their estimates, where are those estimates coming from, what connections are they making with what they see, with whether a picture or in this case a video clip in order to help them find an estimate of volume. A lot of the kids understand the concept that volume is the space inside of an object or how do we find that total amount, but they definitely don't understand the concept that volume is also base times height in which there are layers within that volume taking place so it's repeated addition.

Having them try to see if they can come up with different strategies to find volume instead of just plugging in a formula. I anticipate with these students when they're given an act to the face of the volume, it represents a layer. I don't know if a lot of those students understand or will understand what that means that there is a layer and that if you were to take away that layer, what would be behind that? I really think the students are going to understand that something is behind it because there are rows and columns, but oftentimes they're given a face or a layer and they're given the total volume and then they're going to have to find a missing piece. So it'll be interesting to see if they can take the information they've given and reason through to find a total volume. So a lot of the students are just going to throw out a number. And what I want them to do is kind of use what they have and go from there.

MALLORY WILLIAMSON: So, the first thing I'm going to ask you to do is there is a handout on your rollie cart, instead of doing this in your journals, we're going to do it on the handout so that we have some tangible data. All right, so the first thing I'm going to do is I'm going to actually show you a video. It's a short clip, and in this clip, I just want you to watch. Can you guys do that for me? You're not going to write anything down. I want you tracking the video. I want you observing what's in the video. After it's done playing one time, I'm going to then play it again. And the reason why I'm having to play it again, is so that during that time, Caden, you guys can write down any questions that you might want to ask as a mathematician or anything that you wonder. Okay? All right. So. here's the first clip.

[video plays silently]

MALLORY WILLIAMSON: Okay, now that you have a few questions, what I'd like you to do is I'd like you to turn to someone who's, like, your shoulder partner, side by side, with a group of three, you guys can talk to each other, and with one person speaking at a time, share at least one observation or one question that you have. If you as that shoulder partner really like that question, go ahead and record that question, okay? So, go ahead and share one question with your groups.

STUDENT: How many sugar cubes are there? That's my main question.

STUDENT: Same. That's what I, I said how many sugar cubes are in all?

STUDENT: Mm-hmm (affirmative).

STUDENT: I did that and I did what, what is like volume of the box.

STUDENT: That's a good one as an option.

STUDENT: I also put how much does each one cost out of, like, the whole thing?

STUDENT: You guys wanna add, like, the second one, pick one?

STUDENT: Um, I made an observation sugar just put into a cube.

STUDENT: I also asked, um, why he showed the side of the box, 'cause you know how, like, he showed the side of it, that was, like, weird for me. I'm like, what? So that confused me.

STUDENT: I had the same one. Another one I had was how much does the box of sugar cubes cost?

STUDENT: We were on the second one, like, the second question that we had. I put, why did he show the side of the box?

MALLORY WILLIAMSON: Okay, so maybe we're going to do something with that side of the box in the future, 'cause if he definitely puts it in that particular clip, it might refer to the next clip that we're going to see or something that we're going to do with that clip.

STUDENT: ... the width and the height.

MALLORY WILLIAMSON: Okay, so the width of the box and the height of the box, and I, I like where your train of thought is going because he's showing you different sides of the box and then he's also showing you this small cube, so the first question that you guys all kind of really went to is how many cubes are going to go into the box. And then when you bring up the width of the box or the height of the box, well, then maybe they relate together. Maybe that's going to be one of our focus questions, okay?

STUDENT: Or we can also figure out the width and the height of the sugar cubes—

MALLORY WILLIAMSON: Yeah.

STUDENT: ... estimate how much is in the box.

MALLORY WILLIAMSON: And why would that be important to figure out the width and height of the sugar cube as well as the box?

STUDENT: Because it would help us understand how much could fit in the box.

MALLORY WILLIAMSON: Yeah, how much could fit in that box. 'Cause if the volume of the cube is large, we may not be able to have as many cubes. If the volume of the box is smaller, we might be able to fit more cubes. So, that's a great relationship that you guys have discovered early on, okay?

MALLORY WILLIAMSON: Drake, can you share one question that you guys had about this clip?

STUDENT: How many sugar cubes are there in the box?

MALLORY WILLIAMSON: Okay. All right. Ciara.

STUDENT: Um, how much sugar does it contain?

MALLORY WILLIAMSON: Okay. So, when we talk about sugar cubes, I'm going to add on to this a little bit if that's okay with you. When we talk about how much sugar is it in the box versus how much sugar cubes, well, maybe we want to figure out how many, like, sugar cubes there are, however, how much sugar fits into a sugar cube or is made up of a sugar cube. So, that also can help you to see if there's a relationship there, okay? So, how much sugar is the cube made up of, okay? Roberto.

STUDENT: How many, how much of the cube dots will there be if you put it in fourths?

MALLORY WILLIAMSON: Okay. So, maybe how many groups of fourths are there? Angel.

STUDENT: Um, how many more sugar cubes are in the box?

MALLORY WILLIAMSON: Okay. So, that kind of relates to how many more sugar cubes are in the box.

STUDENT: How many calories are in—

MALLORY WILLIAMSON: Okay. So, maybe we could talk about, he did show us that side nutritional label, so maybe he's presenting something on that nutritional level, label, excuse me (laughs). So, maybe how many calories. And Izzy. When you say each one, are you talking about each box of sugar cubes?

STUDENT: Yeah.

MALLORY WILLIAMSON: Okay.

MALLORY WILLIAMSON: So, we have some good starter questions just to kind of get us rolling on maybe what this task is going to be all about, or maybe what kind of a problem we're going to be solving. So, a lot of you, right on the dot, have got the main question, okay? So, I'm going to reveal to you guys the main question. And what I want you to do is I want you from there, estimate what that is. Okay? So, the main question is actually the first question that was asked. And that was how many sugar cubes are in the box? So, if you would go ahead and write down and record that main focus question. How many sugar cubes are in that box?

MALLORY WILLIAMSON: Okay, to help us kind of then start thinking about how to answer this focus question, what I'd like you to do is we're going to go to the next section which is a part of act two. And I'd like you to brainstorm with your group. Okay, if we're trying to figure out how many sugar cubes are in this box, what is some information we need to know? So, I'd like you to record everything you can think of, whether it be a question, a concept, an idea and write it down. What are all of the pieces of information you're going need to know in order to find out how many sugar cubes are in that box, okay? All right, you can begin.

STUDENT: Multiply and we can find how much it is.

STUDENT: Well, let me add on to a bit of that. How... 'Cause we're not do... 'Cause a sugar cube as you can see is 3D, so we want the length, the width, and the height. So, it should not just flat on here, and then we can't go up.

STUDENT: Mm (affirmative).

STUDENT: So, I think I'd like to know the length, the width, and the height of the sugar cubes in the box so like, if it's an inch tall, an inch wide—

STUDENT: Then we can multiply to find how much cubes are in the thing.

STUDENT: Exactly, so, if we... We could then add it up until it fills the box. So, I would...

MALLORY WILLIAMSON: ...grams, it might give you an estimate on when you're dividing the total amount of grams per gram, but that's going to necessarily... That doesn't know if it's like, loose sugar or if it's the packed-in sugar in the cube. That's not going to give us necessarily an accurate measurement on the length of something or the width of something.

STUDENT: I would like to know the length, the width, and the height...

MALLORY WILLIAMSON: What might we need to know in order to find out how many cubes are in the box?

STUDENT: Are the sizes of the sugar cubes can... the same and do the sugar fill—cubes fill the height and the length...or the width?

MALLORY WILLIAMSON: So, we can write down...And I know Elizabeth did that as well. We can write down we're going to need to find out the size of the sugar cube. And what's the second thing you're going to write down, Elizabeth?

STUDENT: The size of the box.

MALLORY WILLIAMSON: The size of the box? Okay.

STUDENT: That would make sense.

STUDENT: Yeah.

STUDENT: I think you should go with that.

STUDENT: So, we're gonna have to estimate the measurements.

STUDENT: Yeah, it also depends on... because not every time, it's not always gonna be filled to the top. Sometimes it'd only be halfway.

STUDENT: Yeah.

STUDENT: So, it also depends on if it's gonna be filled halfway, a quarter, three quarters...

STUDENT: Yeah, how like, how.

STUDENT: How full is it gonna be? The capacity.

STUDENT: I get that. So, uh, any other things that you guys are thinking of maybe? 'Cause I have (laughs)…

STUDENT: Well, it also depends on like... 'Cause you can also break down the cubes, and you could also put it in there. And can't you just measur—measure that by, like, grams?

STUDENT: 'Cause some of them could have broken while they were in the box, you know?

STUDENT: Yeah, some of them could have broken.

STUDENT: So...

STUDENT: So, we could also see if we can try to do that and see if it... All the sugar that's like leftover could equal into, like, one sugar cube.

STUDENT: Yeah, cause I... Yeah, that's what I was thinking in my head. Like the sugar cu... I'm, I feel like sugar would be like falling off already.

STUDENT: Yeah, ‘cause it can trip off.

STUDENT: Yeah.

STUDENT: Okay, so we can put that down.

MALLORY WILLIAMSON: So, both of you have the same concept. Both of them related together. Okay? Is there anything else we might need to know?

STUDENT: Um...

MALLORY WILLIAMSON: 'Cause you said something really important that once you find these two, what's maybe a direction or strategy we can use?

STUDENT: Multiply...

STUDENT: Multiply and dividing.

MALLORY WILLIAMSON: We can use multiplication, or I know that your strategy was taking and dividing the measurements.

STUDENT: The box by...

STUDENT: One cubed.

MALLORY WILLIAMSON: Okay? So, there could be multiple ways of finding that. We just don't know yet. Okay?

STUDENT: Okay.

MALLORY WILLIAMSON: All right, what do you guys have? I would like to know the length, width, and height cause you need to know the height of the cube to figure out how tall it is. What do you mean by, "it is?" The box?

STUDENT: No.

MALLORY WILLIAMSON: How tall the...

STUDENT: Cube is.

MALLORY WILLIAMSON: Cube is? Okay. To figure... To use in order to find the length, width, and height of what? So, how can we use the length of the cube...

STUDENT: To find how much cubes you have, like, within—

MALLORY WILLIAMSON: To find the length of the box. How many cubes would equal that length of the box? So, I would add on the height of the cube in order to figure out how tall, or how wide, or how long that box is, okay?

STUDENT: So...

MALLORY WILLIAMSON: So, you are going to need to know the length, width, and height of that cube in order to help you figure out...

STUDENT: What about also the box?

MALLORY WILLIAMSON: Mm-hmm (affirmative).

STUDENT: So, we know how many sugar cubes can fit in that area. So, like, say the sugar cube is in an inch on all sides. I don't think it really is...

MALLORY WILLIAMSON: Okay.

STUDENT: And then the sugar cube... Um, then the box on the sides is three inches. So, then we know, oh, these sugar cubes can fit three on this side. And it's six inches this way. Oh, it can fit six on this side.

MALLORY WILLIAMSON: Right, because the cube, if the dimension is one inch by one inch by one inch... That can help us find the height, or length, or width of a box. So, both of you have the same concept.

STUDENT: And then, like, if it is, like, a foot tall... So then, 3 times 6 is 18. You would just need to multiply that by 12.

MALLORY WILLIAMSON: If you're converting from feet to inches, yup. So, I like where you're going. We're going to see if your strategy works.

STUDENT: I found one.

STUDENT: Wait, we found another one.

MALLORY WILLIAMSON: You have a good one?

STUDENT: How did they put the sugar cubes in the box?

STUDENT: Did they stack them or did they just...

MALLORY WILLIAMSON: Oh, hm. That's a good idea. If you just throw all the sugar cubes in—

STUDENT: Then, you can fit more if you stack them.

MALLORY WILLIAMSON: Yeah, compared to if you were going to compact them or stack them together.

MALLORY WILLIAMSON: So, how do you think they would stack them?

STUDENT: They would in rows.

MALLORY WILLIAMSON: Rows?

STUDENT: Yeah, like five...

MALLORY WILLIAMSON: Like, columns?

STUDENT: Yeah.

MALLORY WILLIAMSON: Okay, so it's a good point. If I just throw all the cubes in, I may not have as many as if I were actually organizing them into rows or columns.

STUDENT: Yeah.

MALLORY WILLIAMSON: That's a great idea. Great concept.

STUDENT: Don't they [inaudible].

MALLORY WILLIAMSON: Huh?

STUDENT: What are those for?

MALLORY WILLIAMSON: For, like, coffee. You can drop a sugar cube in. It dissolves.

STUDENT: Oh. Are they like marshmallows?

MALLORY WILLIAMSON: No. (laughs) They're sugar packed into a cube. So, if you put it in your tea, or if you put it in your drink it dissolves and that adds sugar to it.

MALLORY WILLIAMSON: The next step I'd like you to do is, I'm not necessarily going to reveal any information to you just yet, but I'd like you to now think about what would be a low estimate of how many sugar cubes would fit in this sugar cube container. What would be a high estimate on how many sugars could fit into this container? And then what would be a reasonable estimate, okay? So, on your handout you're going to see, right below the word estimate, there's a box where you can record and number for low estimate, a box where you can record a number for high estimate, and then you can place your estimate on the number line according to where you think it's going to go between the two, okay? All right, so go ahead and have that discussion with your group, and I'm interested to see what you guys come up with.

STUDENT: 25 is—25 cubes. Have you seen 25 sugar cubes?

STUDENT: But think of the width. They could probably pack like 10 of them, boom boom, that's already 20.

STUDENT: I was thinking of 100 for that.

STUDENT: 100?

STUDENT: 100 is definitely higher than my estimate. All right, let's go with 20 then. Maybe a reasonable one would be about 50?

STUDENT: Yeah, around 50 would be. So, about, like—

STUDENT: Right here.

STUDENT: 50 is closer to 20 than 100. Wait, yeah. 30 and then—so this should be a little bit more, like, over here. Instead of, like, right in the center it should be like a little bit over here like this.

MALLORY WILLIAMSON: ...looking at this image to reason through that.

STUDENT: Like, I, I'll pick us 50 for here.

MALLORY WILLIAMSON: So, 20 is a low, so you're saying if you double that, it's going to be a high estimate.

STUDENT: I would say fif—500 is more.

STUDENT: I feel like—I feel like there's gonna be like [inaudible].

MALLORY WILLIAMSON: Well, looking at that image, how tall do you think that box is going to fit with sugar cubes?

STUDENT: Well, compared to—If you look at the sugar cubes they're pretty tiny against the box so—

MALLORY WILLIAMSON: Right, so how many do you think—If we were to stack them up, how tall do you think that would be?

STUDENT: 453.

MALLORY WILLIAMSON: You think it would be 453 tall?

STUDENT: Yeah, because one thing [inaudible 00:02:23].

MALLORY WILLIAMSON: I think 453 stacked up is going to be a lot larger than the size of that box.

STUDENT: Yeah, but once you—Wait.

MALLORY WILLIAMSON: How many cubes would we have?

STUDENT: Like 200 and—

STUDENT: No, half is like, like—

MALLORY WILLIAMSON: 'Cause, Angel, look at the cube. [inaudible 00:02:39] here.

STUDENT: That's not gonna fit—

MALLORY WILLIAMSON: So how much would be about half?

STUDENT: 30.

STUDENT: 60 or 76.

STUDENT: 35.

MALLORY WILLIAMSON: For half?

STUDENT: Like 35?

STUDENT: No, 30—30, 35 would be like, less than a quarter.

MALLORY WILLIAMSON: I think you guys are not reasoning through what you see, 'cause if you notice one cube goes about here, so much would be about half?

STUDENT: Five cubes.

MALLORY WILLIAMSON: Okay, that's a better estimate than 30. So, if you were to then double it --

STUDENT: That would be 15?

MALLORY WILLIAMSON: That would be about...

STUDENT: Ten.

MALLORY WILLIAMSON: Ten, 'cause five times two is ten, as far as the height goes. So, reasoning through your different measurements, you can kinda get something that's high. Now, I agree, 500 would be a high estimate.

STUDENT: That's too high.

MALLORY WILLIAMSON: Right, it would be okay 'cause it's an extreme estimate. So, now what do you think would be a reasonable number?

STUDENT: 50.

MALLORY WILLIAMSON: Okay.

STUDENT: I think 50.

MALLORY WILLIAMSON: 50? Okay, if we're all—

STUDENT: Because, um, that's probably gonna be the lowest.

MALLORY WILLIAMSON: Why?

STUDENT: 'Cause the cubes are small and they could, like, fit in them.

MALLORY WILLIAMSON: Okay, so you think that you can fit more cubes than 10?

STUDENT: I was thinking, like, there's four going this way for how—

MALLORY WILLIAMSON: The width, is that what you're describing? Like going back?

STUDENT: Yeah.

MALLORY WILLIAMSON: Okay, so there's at least four of the width.

STUDENT: And then going up is at least 8 or 10.

MALLORY WILLIAMSON: 8 or 10? So, if your height is at least 8 or 10, you know that's a pretty low estimate, okay? And you guys have 90 as your high estimate, so wh—how many cubes do you think are actually in the box?

STUDENT: 30.

MALLORY WILLIAMSON: 30?

STUDENT: Around, around 30.

STUDENT: About 32, yeah.

MALLORY WILLIAMSON: Around 30, okay. So, 30's half of 60, so I like that you put yours in between on that number line accurately.

MALLORY WILLIAMSON: So where do you think 30 would go on this number line, since you used 90 as your high?

STUDENT: About like, right here?

MALLORY WILLIAMSON: Right, 'cause 30 is a third of 90, so I agree with you on that.

MALLORY WILLIAMSON: You have 60? 30 is half of 60.

MALLORY WILLIAMSON: Group 5, can you guys give me the estimate that you decided to go with on how many cubes would go into the box?

STUDENT: The reasonable one, low, or high?

MALLORY WILLIAMSON: The reasonable one.

STUDENT: Oh, 60.

MALLORY WILLIAMSON: 60? Okay. Is there a reason, like a strategy you guys used for why you decided 60? 'Cause I know we can say, "Oh I eyeballed it." Well, what on there kind of helped you decide that 60 would be kind of reasonable.

STUDENT: How small the cube is.

MALLORY WILLIAMSON: Okay, so maybe how small the cube is?

STUDENT: And we don't know how wide it is.

MALLORY WILLIAMSON: Okay. So, we're—How many cubes do you think creates the width? Just based on the short snippet I showed you?

STUDENT: About two, three.

MALLORY WILLIAMSON: Two or three, okay. So, taking two or three and starting from there, with your height or your width, okay. All right, group 4, what was your reasonable estimate?

STUDENT: 65—

MALLORY WILLIAMSON: 65?

STUDENT: ...for me and, and then he had 50, right?

STUDENT: Yeah.

MALLORY WILLIAMSON: So, 50 to 65?

STUDENT: Yeah.

MALLORY WILLIAMSON: Okay, group 3?

STUDENT: Um...

MALLORY WILLIAMSON: What would your reasonable estimate be?

STUDENT: Well, first we said 60 but now we put 100.

MALLORY WILLIAMSON: Okay, so maybe between 60 to 100? All right, group 2?

STUDENT: Um, 20s to 90s.

MALLORY WILLIAMSON: What was your middle estimate, your reasonable estimate?

STUDENT: 55.

MALLORY WILLIAMSON: 55 or 50. And group 1?

STUDENT: 30.

MALLORY WILLIAMSON: 30?

MALLORY WILLIAMSON: The second thing I'm now going to show you is more information. And from this, what I want you to do, is I want you to use this piece of information to then create another estimate. The first estimate is based on just on looking at something and reasoning through it. The second estimate I give you is actually something where you're going to go a little bit deeper. Okay? So, you're going to have to take a look at it, and I'm going to show it to you twice so that you can record any information as you go. Okay?

MALLORY WILLIAMSON: Okay (laugh). I'll play it again and this time I'll pause it briefly because I know it happens fast [inaudible]. Okay, write down any information you need.

STUDENT: 6, yes it is. So maybe we can [inaudible 00:01:18], which is 18. So that's one row [inaudible] 3 times 6. Well that's just one row. Imagine how much can fit in like 5 rows.

MALLORY WILLIAMSON: Well instead of calling it row because it’s at the front, when you call it top, we call it a layer. So, if that's in the front layer, which is three times 16, which is what?

STUDENT: 18.

MALLORY WILLIAMSON: 18, so if we say there is 18 in a layer, then you have to start thinking about well how many layers does it create in order to fit equally inside that box. [inaudible] So that's where your brain’s got to start going. So how many layers do you think there are to fit inside that box? Six or seven? What do you guys think?

STUDENT: I wanna say maybe eight. Like eight or seven or six.

MALLORY WILLIAMSON: And in this particular session, it's okay to be in agreement. Like it's okay, if you say six or seven or eight, or if he says six or seven, go with the seven. Like the middle number because either way it's still going to be an estimate. Like that way you guys all can reason through what you think that final volume's going to be. Okay?

STUDENT: It's like three sugar cubes, um, like, wide.

MALLORY WILLIAMSON: Okay.

STUDENT: And six sugar cubes, um, long ways.

MALLORY WILLIAMSON: Okay, so how many cubes do you see facing us?

STUDENT: 18.

STUDENT: 18.

MALLORY WILLIAMSON: 18. So we're going to use that amount and then we have to decide, what next? So, what do we need to discover now in order to figure out the rest of the volume?

STUDENT: How many sugar cubes tall.

STUDENT: How many sets of 18.

MALLORY WILLIAMSON: How many sets of 18?

STUDENT: Or how, how many sugar cubes high it is.

MALLORY WILLIAMSON: Yeah, and when we're looking at it this way, it would probably be how tall the height, how high it is, or the height of the box. Okay? So, you could do sets of 18, but also trying to figure out, okay, what would be a reasonable estimate for my height, and then apply it to the 18 that we see.

STUDENT: 90 to 108?

MALLORY WILLIAMSON: So how many sets do you think there might be?

STUDENT: About like five or six.

MALLORY WILLIAMSON: Five or six. Want to agree on one number, we'll say six?

STUDENT: Yeah.

MALLORY WILLIAMSON: Okay.

STUDENT: I agree on five.

MALLORY WILLIAMSON: Five? You want to do five cubes?

STUDENT: Sounds more reasonable than [inaudible].

MALLORY WILLIAMSON: You're pretty close. (laugh) 'Cause either way it's not going to be an accurate answer because we don't know for sure. So, either way, whether you go with five or six, it's still an estimate. Okay?

MALLORY WILLIAMSON: What do you mean by the bottom?

STUDENT: We think it's 100.

STUDENT: Oh, that's a top.

MALLORY WILLIAMSON: Right that is a top, we're seeing only the front top. So, when you say bottom, are you saying what's behind it?

STUDENT: Yeah, I think there's 18.

STUDENT: What if there's nothing behind it?

STUDENT: But 18 makes no sense.

MALLORY WILLIAMSON: Well, you saw the box. So there has to be, the top, the sugar cubes can't necessarily float on top, there's got to be something there.

STUDENT: I know.

MALLORY WILLIAMSON: It's not a trick question.

STUDENT: It's 180 because, like we said before, our estimate would be like 10 that going up so and, and since, since three times six is 18 and then you add the zero from the 10, would be 180...

MALLORY WILLIAMSON: So, from here what you could say is your next estimate is about 180.

STUDENT: Yes.

MALLORY WILLIAMSON: And I haven't given you any like guaranteed measurements. Okay, it's just an estimate. Okay? But if that's what you guys think you can go ahead and write that down. That you think the height, that the height is, about 10? So, there might be around 180 cubes?

MALLORY WILLIAMSON: What are you thinking, Ayla?

STUDENT: I don't know.

MALLORY WILLIAMSON: That's because you don't see it in real life. It's a picture that you're trying to reason through a space.

STUDENT: The height is 10.

MALLORY WILLIAMSON: Possibly, about. All right, anything from you guys [inaudible].

STUDENT: I think it’s—I think it's gonna be, I don't think it's gonna...

MALLORY WILLIAMSON: Let's let, let's see if your groups teammates can figure out how you get 18.

STUDENT: It's 18 cubes because you can multiply six by three.

MALLORY WILLIAMSON: Okay, so the height that we're looking at is three and the length on that is six so that's where he's getting 18. And why did you call it a layer?

STUDENT: Um, because I have a feeling that there, that is not the only place where there's sugar cubes, there's more layers. Like, this is a layer of sugar cubes then on top of it is another layer of three by six, and another layer of three by six.

MALLORY WILLIAMSON: Right, so when we see the front view, we know that when we buy the box, that front layer of sugar cubes, there's got to be something underneath it. Or else those sugar cubes would fall to the bottom. So, we have to understand, maybe how many sections or layers of 18 there are behind that front view to, in order to figure out how many total sugar cubes there are. Okay? So now, how can you use this information to come up with a new estimate? If there's 18 cubes in a layer, how many total sugar cubes do you think there are?

STUDENT: If one cube is one inch and if the box could be like a foot tall, and you could just multiply. Because, 12 inches equals a foot, 12 times 18...

MALLORY WILLIAMSON: We're not doing feet, we're still doing, we're, I haven't given you any measurements so we're just using sugar cubes as a unit. So, if you're saying, do you think the box is four tall? Four sugar cubes tall?

STUDENT: No, 12.

MALLORY WILLIAMSON: 12? Okay, so you could do 18, which is what we counted in the layer, times it by 12 and see maybe that's our next estimate that we're going to. Okay?

STUDENT: These two are saying it's like 80 or 90.

STUDENT: I'm saying it too.

STUDENT: No, you said 180.

STUDENT: You said 180.

MALLORY WILLIAMSON: Okay, so let's go back. We have 18 on the front layer, right?

STUDENT: Mm-hmm (affirmative).

STUDENT: Yeah.

MALLORY WILLIAMSON: We need to all agree on what's a reasonable estimate for the height of the box. So how many sugar cubes will equal the height of the box?

STUDENT: 10, we're saying about 10.

MALLORY WILLIAMSON: We can say just the general 10. Which is okay.

STUDENT: We don't know what's behind it though.

STUDENT: But I feel like 180 is just...

MALLORY WILLIAMSON: There is more behind it, I promise.

STUDENT: But like we don't know if it's all the way to the top.

STUDENT: It is.

MALLORY WILLIAMSON: It is, that's the front layer of the box. So, these are on top. If we opened up a box and this is what was being shown in the front. This is the top layer. So, if you say the height's about 10, you can then say 18 times 10 is 180.

STUDENT: But I don't think there's like it's 180.

MALLORY WILLIAMSON: It's a first estimate so it doesn't have to be right or wrong. You have to remember that. It's okay if it's 180 and it's an estimate and then if I give you more information you can alter that answer. Okay? I think you guys are afraid of being wrong and that's okay.

STUDENT: No.

MALLORY WILLIAMSON: You can have another estimate and when I give you more information you can change or edit it as you go. Okay?

STUDENT: Uh-huh (affirmative).

MALLORY WILLIAMSON: All right.

MALLORY WILLIAMSON: I'm curious to see for some of you guys, what was some observations that we made? Ciara, can you share with me what was an observation you noticed?

STUDENT: The three by six sides. The three by six sides.

MALLORY WILLIAMSON: Okay, so we noticed that there are three cubes by six cubes, okay? So that would be an array that would equal 18. So thumbs up if a lot of your group noticed that particular observation. And we had some good conversations about, okay, well, is this the top of the box? Is there anything behind it? The answer to that is yes and yes. It's not a trick question because if I actually turned the box right side up, which is the original place we saw it, those cubes would fall to the bottom, if there was nothing there, okay? It's not a trick question, I promise. All right. It's a brand new box. Okay. So if you would flip your paper over.

And one thing I want to show you guys is another illustration on the screen. And before I do that, we used our one for assessment and that first estimate is just based off of sight. We saw something on a screen and we came up with an estimate. We don't really know if it's close or not. But now that you guys have 18 as a number to work with, I want you to once more, make a second estimate on how many sugar cubes are in the box. And a lot of you have already gone in that direction. So I'm going to give you a couple more seconds and then report each group and see how that's changed. It's okay for this round. It's just an estimate, okay? So use the data that I just gave you and create a second estimate of how many cubes are in the box. Go. Okay. What was you gu—what's going to be your estimate—second estimate?

STUDENT: We estimated 90.

MALLORY WILLIAMSON: So using 18, how are we getting—going from 18 to 90?

STUDENT: We multiplied it by five.

MALLORY WILLIAMSON: So you think that the height of the box is five cubes? Okay. So that would be your second estimate, okay? You don't necessarily have to do anything right now. I just wanted an estimate. What I hear you saying is, we think there's 12 layers of 18 or the height is 12. Okay. So what was you-all's estimate?

STUDENT: 160 to 200.

MALLORY WILLIAMSON: So how are we going from 18 to 160?

STUDENT: We thought that instead of five because that would be shorter than, um, the length—

STUDENT: Width of the box.

STUDENT: The width.

MALLORY WILLIAMSON: Okay.

STUDENT: We were thinking—so we made it into nine.

MALLORY WILLIAMSON: Like the nine would be the height? So what's 18 times 9?

STUDENT: 162.

STUDENT: 162.

MALLORY WILLIAMSON: So go with 162 as your estimate, okay?

STUDENT: Okay.

MALLORY WILLIAMSON: This, this is the range. I just want you guys to go with the 162, okay?

STUDENT: Okay.

MALLORY WILLIAMSON: All right. Group 1, what was your second estimate now that we have new information? Group 2, what was your second estimate?

STUDENT: 180.

MALLORY WILLIAMSON: Group 3?

STUDENT: 216.

MALLORY WILLIAMSON: Group 4?

STUDENT: 216.

MALLORY WILLIAMSON: Group 5?

STUDENT: 162.

MALLORY WILLIAMSON: All right.

MALLORY WILLIAMSON: All right, here's the last image that I'm going to share with you. So, on the back, this is where you're starting on the back.

MALLORY WILLIAMSON: So I already see some people recording the information, which is important.

MALLORY WILLIAMSON: Interesting, so I think we're still going back to the fact that we feel like we're right or wrong. And those are estimates. The important part now is to understand the first—the first estimate that we had, is how—Why do we think that? So, you guys got 10 in height; was that close?

STUDENT: Yeah.

MALLORY WILLIAMSON: Yes, it was because you had 10 cubes in height and the actual dimensions were 11.

STUDENT: So do we copy this 12? [inaudible]

STUDENT: We do 10 over 12?

MALLORY WILLIAMSON: You all are talking about the other group?

STUDENT: Yeah.

MALLORY WILLIAMSON: Okay, then that's not necessarily something that should bother you, it's not about other groups, it's about how are you getting to the answer with your team. Okay.

MALLORY WILLIAMSON: Okay, this is an important part of this step. What we're going to do is, I'm going to give you chart paper. And I'm going to release one photograph at a time, so normally we do a lot of this pre-planning, pre-collaboration, pre-brainstorming before we put together our final presentation, okay? So what I want you to do is I want you to, just like we did in class, on our final presentation, is break apart your presentation into three acts. So if I showed you the first clip, I'm going to then give you a picture of what I showed you on your presentation paper, and I want you to draw strategies, show some kind of work as to how you got your first estimate, okay? There's some sort of reasoning that takes place. You can't just say, "I looked at it and I guessed," right? And then I'm going to release the second act to you that you can put on your presentation. How did you come up with that second estimate? And then the final act, the third act, will be the third thing that I provide for you for your presentation chart paper, okay? You may use marker but I ask that before you use marker, you use—?

STUDENTS: Pencil.

MALLORY WILLIAMSON: Pencil so that way we be careful about our mistakes, okay? So if one team member can go get the chart paper on the back table and you can go ahead and start putting together your presentation for act one.

STUDENT: Don't put it all the way up in the corner, because we need some space for our names, so like.

STUDENT: Like right there.

STUDENT: Yes.

STUDENT: What about you? Or you could—Or you could just do it all on one.

STUDENT: No, she said we have to do act one, two, and three on one chart paper.

MALLORY WILLIAMSON: So if I were to use this on our paper, what kind of strategies can I show somebody to kind of connect the dots between what I see and your answer of 65? Because I can't just say 65 sounds reasonable by looking at it. There's something that you think about, whether it's the width of the box, the height of the box, shape of the cube, to help you get 65, okay? So that's something that you guys can draw on, beside, or next to it.

STUDENT: I thought, like, 65-ish, 'cause, like, I thought, like, this was going to be, like, 4 wide by 3 wide.

MALLORY WILLIAMSON: Okay, so that's something that you can talk about by drawing an arrow or just underneath saying, "I believe that the width was going to be about 4 wide."

STUDENT: And we talked also, it would be, like, 48 or, like, 2, like backwards if that—It would be like two so—

MALLORY WILLIAMSON: That's exactly what I want you to write down, okay? So make sure some of those thoughts come out on your chart paper, okay? So if y'all want to glue this down, you can put your thoughts beside it, if you don't want to necessarily draw on it.

STUDENT: Mm-hmm [affirmative].

STUDENT: If the last unit was 10 [inaudible].

STUDENT: So, you can glue it underneath this.

STUDENT: We shouldn't trace around it. Look, like, bam, marker. [inaudible]

STUDENT: I can't see it with pencil.

MALLORY WILLIAMSON: You're not going to be able to see much with marker. But if you put a line here, like a red dash there. Okay. So, that's where 1 sugar cube is.

STUDENT: The number 10 should be like...

STUDENT: Five—

STUDENT: Five should be right here.

STUDENT: Five sugar cubes would be like, right here.

MALLORY WILLIAMSON: Well, we're going from 1 sugar cube to 5 sugar cubes. So, mathematically, what I would is—if we mark 1, where would 2 be?

STUDENT: Like, right there.

STUDENT: Right here.

MALLORY WILLIAMSON: Okay. At that space? Anna, you want to go ahead and mark. Make sure there is, like, equal as possible. I know it's still an estimate. Be careful ‘cause I think these are a little bit wider. So, the third one would be, about here. And keep going on until you get to the height.

STUDENT: Number 4 should be right here. Number 5 should be right there.

MALLORY WILLIAMSON: Keep going. I'll go ahead—

STUDENT: Number 6 right there.

STUDENT: How many are there? 1-2-3-4-5-6-7-8-9-10-11-12.

MALLORY WILLIAMSON: Okay. So then, we could put down, the height is 12 sugar cubes. And then go from there.

STUDENT: Oh, we should write it with pencil first.

MALLORY WILLIAMSON: Okay. So, how are we going from, the image that we see, to 60. So, how did we get that?

STUDENT: We wrote 4 up here, like, 4 sugar cube by 5 sugar cubes.

MALLORY WILLIAMSON: So, we can, down here, to kind of see a little—the length of these 4 sugar cubes. No. Mm-mm (negative). Because I... I know. I'm a math teacher, and I know what I mean. But if you have somebody who doesn't necessarily know what I mean, they're not going to make that connection. So, we can say the length—

STUDENT: Equals 4 sugar cubes.

MALLORY WILLIAMSON: Sugar cubes. G-T-H. (silence) 4 sugar cubes. Okay. And then we thought the height—

STUDENT: And width.

MALLORY WILLIAMSON: And width.

STUDENT: And width.

STUDENT: Width were 3 sugar cubes.

MALLORY WILLIAMSON: 3 sugar cubes. You're writing really small, dude, and we have this entire chart paper. (laughs) Just want to let you know that, okay? But you guys have recorded here, so just... you can put it off to the side or down below.

STUDENT: All right.

MALLORY WILLIAMSON: Just kind of nice and large so people understand how you maybe got that 60.

STUDENT: Do you want to put it up here? Caden do you want to put it up here? So, it'll be like, that. Like, act 1, act 2, act 3.

MALLORY WILLIAMSON: Between looking at this image and how we got 60 or a hundred.

STUDENT: Well, we could write how many sugar cubes there are on each side, or how much we thought there were.

MALLORY WILLIAMSON: Okay. So, let's do that, and we're going to do a little bit larger than what we're writing ‘cause we have a whole chart paper for all three acts, okay? You don't have to reset, just write that larger when you... Okay. Let's talk about the length. How many sugar cubes did you guys originally think the length—

STUDENT: We thought it was 6 [inaudible].

MALLORY WILLIAMSON: Exactly how it was on the screen? Okay. All right. So, then record that. The length is 6 sugar cubes. And Jaadiay, is there something you can do on this to help us understand what 6 looks like when we're talking about length?

STUDENT: Um, maybe put lines on something that shows, like... and then put the numbers.

MALLORY WILLIAMSON: I-N.

STUDENT: I-N. (laughs)

STUDENT: Will this show?

MALLORY WILLIAMSON: I think you could use marker.

STUDENT: Thought... [inaudible] length... was... 6 sugar... cubes. And the width was 3... 2 cubes.

MALLORY WILLIAMSON: So, the width is going to be hard to do because that's the side of the box. But we can at least do something to this, to show that you guys thought the length was six. So, how can we do that?

STUDENT: And...

STUDENT: Put a line and then put 6. But if you put edges on—

MALLORY WILLIAMSON: Well, a cube kind of looks like grid paper, right? So, if we kind of make it look like a grid on top of the front of the box. That can help people look at the face of the cube and make that connection.

STUDENT: So, put like—

MALLORY WILLIAMSON: So, I will start off with... If you have... if you're not sure where 6 goes, that's ‘cause we want to be as accurate as possible. And we can draw a line straight down the middle.

STUDENT: Mm-hmm (affirmative).

STUDENT: Do we estimate how much do we have, like—

MALLORY WILLIAMSON: It's okay. And then, these each side ‘cause you have... you need 3 and 3 to make 6. Subdivide these up into thirds.

STUDENT: Into thirds?

MALLORY WILLIAMSON: Okay. So, from here, we can go into act 2. So, I'm about to go into act 2. So, when she's done with that, we should be ready for that.

STUDENT: So, can we write—

MALLORY WILLIAMSON: You're still going to go straight down this way, ‘cause we're still going, the length.

STUDENT: So, I have to make thirds.

MALLORY WILLIAMSON: Yep. You can actually take a marker and draw it on straight across the box, this way. So, you can just use the line that you guys created then go straight across. On the top, so, the front face of the box. Okay?

STUDENT: We thought there was [inaudible].

STUDENT: You mean going that way, and then—

MALLORY WILLIAMSON: What did you guys originally think?

STUDENT: 30.

STUDENT: Um, our first [inaudible] is 30.

STUDENT: But now we think it's 198 or something.

STUDENT: That was ou—that was our third one.

STUDENT: Yeah.

MALLORY WILLIAMSON: Well, we're still on the first one. So, if you think the width is 3... Is that where you... the 3 came from?

STUDENT: Yeah.

MALLORY WILLIAMSON: Okay. So

STUDENT: No. We...

MALLORY WILLIAMSON: And that means—

MALLORY WILLIAMSON:
Well, if you're trying to get 10, ‘cause you have 2 numbers right now.

STUDENT: Mm-hmm (affirmative).

MALLORY WILLIAMSON: So, we've got... Either break this into two separate numbers and then use this as a separate length, or we've got... ‘Cause we have three sides of the box that we need to measure. So, what are some ways to get 30?

STUDENT: 5 times 6?

MALLORY WILLIAMSON: ‘Cause if you use this as 10... The height is 10 and the width is 3, we're kind of missing out on the length.

STUDENT: We could do 5 times 6.

MALLORY WILLIAMSON: Which is okay ‘cause we're learning through it. It's okay if we're editing as we go, okay?

STUDENT: Or...

MALLORY WILLIAMSON: So, we could do 6-5-3, okay? So, that's 5 times 6 is 30.

STUDENT: Yeah.

MALLORY WILLIAMSON: 5 times 2 is?

STUDENT: Wai—wai—wait, what? Did you say 5 times 6 is 30, and then what, what did you say?

MALLORY WILLIAMSON: Like as you said—

STUDENT: 5 times 2 is 10.

MALLORY WILLIAMSON: Okay.

STUDENT: So, we could do 5 times 2 and—

MALLORY WILLIAMSON: So, maybe the height being 5.

STUDENT: Yeah.

MALLORY WILLIAMSON: The width being 3 or the length, excuse me, being 3, and the width 2.

STUDENT: Yeah.

MALLORY WILLIAMSON: That will get you what you want. So, if you draw 5 going straight across, that can give you the height.

STUDENT: Okay.

MALLORY WILLIAMSON: Okay. Now I'm going to give you the second act. So, you can use that information to then share your thoughts.

STUDENT: I'm gonna glue it.

STUDENT: 5 times 6 is 30.

STUDENT: But, will she do—

STUDENT: This is where—

STUDENT: This shows where we got 3 and 6, which we got 18 from.

STUDENT: Yeah. So, you guys can work on that while we work on this. Cause we still need to work on this. So—

STUDENT: 3. 6. Oh, we kinda [inaudible]. This isn't—

STUDENT: How you told me the measurement if you don't know that yet?

STUDENT: ‘Cause this is 3.

STUDENT: Oh, that is.

STUDENT: So, that means they're 18 all.

STUDENT: Yeah.

STUDENT: I don't know how to explain [inaudible].

STUDENT: Put that right here. [inaudible]

MALLORY WILLIAMSON: —on each cube. And you guys wrote in your notes. Remember that you wrote, there's 3 columns—

STUDENT: And 6... and 3 rows.

MALLORY WILLIAMSON: And 3 rows. So, we can show that as well. So, you can write this information underneath, Angel. Y'all can write this information underneath and draw arrows with markers, something to indicate where the rows are, and where the columns are. Okay? So, there's multiple ways to show that you got 18 and then after that, we can go further and explain what...why we think our height is now ten. Okay? Christopher? Christopher?

MALLORY WILLIAMSON: Cause, this shows, like, a front layer. This is a front layer, there's 18 in the front. But then when you also talk about, you can show that's a second strategy. So, we can also record that below as well. Okay? There's more than one way to record what we see.

STUDENT: What?

STUDENT: Write 6 columns and 1, 2, and 3.

MALLORY WILLIAMSON: Okay, what are we going to do for our second image.

STUDENT: To show that there are 3 and 6.

STUDENT: You can draw one more.

MALLORY WILLIAMSON: Yep. You want to do that?

STUDENT: So, you can circle 3 and then you can circle 6.

MALLORY WILLIAMSON: Because 6 times 3 means you did repeated addition. 6 plus 6 plus 6, okay? So, we can circle our groups of 6.

STUDENT: I'm just circle like that.

MALLORY WILLIAMSON: Well, that would be circling groups of 3s. So, if you want to do... You can either circle the groups of 6, or circle the groups of 3. Who's got the marker? You want to do that?

STUDENT: Right here.

MALLORY WILLIAMSON: You want to do that?

STUDENT: And now I'll do this one cause—

STUDENT: Wait, so, like that? Or, circle like that?

MALLORY WILLIAMSON: Well, she wrote.... It's up to you. Because she wrote 6 times 3 is 18. So, how would you like to show that on your image?

STUDENT: Like that.

MALLORY WILLIAMSON: Okay. So, go ahead and do that.

STUDENT: Try not to... I'll try not—

MALLORY WILLIAMSON: That's okay. It's a picture. As long as people understand what you're trying to get.

STUDENT: What was that? That was loud.

STUDENT: It's my marker.

MALLORY WILLIAMSON: Okay. So, instead of taking a look at this and saying, "Oh, 1-2-3-4-5-6. What can we record in the groups, so someone automatically knows what's—

STUDENT: You could put 6.

STUDENT: You could the number 6.

MALLORY WILLIAMSON: Okay. Let's see that.

STUDENT: I'm just gonna put—

MALLORY WILLIAMSON: That's fine. Either way you want to do it. Okay. And so, this is repeated addition. So, we can put the addition symbol in between, and then Joyce below can do the "6 times 3 is 18," which is she had on her paper.

STUDENT: That's the line for the plus.

MALLORY WILLIAMSON: That's not going to look like it. (laughing) You can go put the line—

STUDENT: There.

MALLORY WILLIAMSON: Okay.

STUDENT: That will work.

STUDENT: Okay.

MALLORY WILLIAMSON: Okay. So, how do we use this information now to figure out how many cubes are in the box? So, that's the second step that we need to try to record.

STUDENT: Um, oh man—

MALLORY WILLIAMSON: So, we go back to what we wrote down. We wrote down 18 times 6 is a 108, or 18 times 5 is 90. So—

STUDENT: We thought there were like 5.

STUDENT: We thought there was 5.

MALLORY WILLIAMSON: Okay.

STUDENT: Cause, we didn't know how—

MALLORY WILLIAMSON: So, that's something that we can now record.

STUDENT: Cause, we didn't know how big each cube was.

STUDENT: Yeah, so we thought there was, like, 5 going upward. 3 going this way?

STUDENT: Uh,

MALLORY WILLIAMSON: Yes.

STUDENT: Yeah.

STUDENT: And then—

MALLORY WILLIAMSON: [inaudible] So, then we took the information and then.... So, how can we show 5 layers of 18? What does 5 layers or 5 sets of 18 look like?

STUDENT: 18 times 5.

MALLORY WILLIAMSON: With the picture.

STUDENT: The picture?

MALLORY WILLIAMSON: Yeah. And you can use the grids on your paper to show you. So, 5 sets of 18. Um, we did 6 sets of 18, right?

STUDENT: Mm-hmm (affirmative).

MALLORY WILLIAMSON: ‘Cause we thought our height was 6? So, how can I draw a picture to show what 6 sets of 18 looks like? ‘Cause that's what layers are, the layers are like slices. Okay? So, using... ‘Cause we have plenty of grid paper, how can we draw 6 sets of 18?

STUDENT: We could, um, like—

STUDENT: It'll be, like, 6 times [inaudible].

MALLORY WILLIAMSON: Yeah. Yeah. 18, 6 times. And when we draw 18, 6 times though, we still have to follow this pattern of 3 by 6. So, we can go 3 by 6. 8... 6 times? 6 times.

STUDENT: Can we use that as one? So, if our second, it will be two. [inaudible 00:06:28]

MALLORY WILLIAMSON: Yeah, you can. As long as you number it. So, if I were to come back and see it, you could say, "Oh, that's the top layer." And then, do the 5... other 5. (silence) And I'm going to actually switch spots with you, so then you could start drawing the second one.

STUDENT: We'll each draw one. That'd make more sense if we each do it.

MALLORY WILLIAMSON: So, this was the top layer. How can we start drawing the second layer, which is right behind that front layer?

STUDENT: We can use the grid paper squares [inaudible].

MALLORY WILLIAMSON: Sorry. (laughing) Okay.

STUDENT: Do we have [inaudible]?

MALLORY WILLIAMSON: Well, you can just put 18 in the middle.

STUDENT: All right.

MALLORY WILLIAMSON: For the sake of time, I'm not going to make you number all those. Okay.

STUDENT: Uh, [inaudible].

MALLORY WILLIAMSON: Yeah, go for it.

STUDENT: Okay. Okay. [inaudible]

STUDENT: We have 18, um—

MALLORY WILLIAMSON: Well, we did do a height. You guys said that there are 10—will be nine, was your estimated height. So how can we use this information and the information of nine to draw a picture to show how many cubes are in the box?

STUDENT: We could multiply it and do, um, I think show 6 times 3 is 18 right here and then draw a different arrow right here, and then do 18 times nine to get it.

MALLORY WILLIAMSON: So we could do that in number expression. So let's go ahead and do that, and then I want to challenge you guys with a picture.

STUDENT: Numeri—

MALLORY WILLIAMSON: Numerical.

STUDENT: Numerical.

MALLORY WILLIAMSON: So 18 times 9? Or 6 times 3 is 18, times that by nine.

STUDENT: Here, I'll move. Do it from here.

MALLORY WILLIAMSON: Thank you. And the other two of you, what I'm trying to get you guys to do is, it's really hard to see, like, when we see the front layer of a box, it's really hard to see what's behind it, obviously, because it's being blocked. So this is what we put in numbers, and we, we can connect 18 to the front layer, but drawing a picture, because we have grid paper, drawing a picture, how can I then use 18 to show somebody what 162 sugar cubes looks like. And I'm not going to count on 162 and then draw a rectangle around that, so how can I use this layer to help us make a connection between 162?

STUDENT: Just like, right here we draw 18 of these—

MALLORY WILLIAMSON: Okay.

STUDENT: Down here, and then draw the height that we think it is.

MALLORY WILLIAMSON: Which is?

STUDENT: Nine.

STUDENT: Nine.

MALLORY WILLIAMSON: Okay. So what that states is we can take 18 and draw it nine times. And that will show you the nine different layers or nine different groups of 18.

STUDENT: So like, draw like this 18 groups right here, and then draw it nine times?

MALLORY WILLIAMSON: Uh-huh.

STUDENT: I was thinking of, like, do it, do it, and then go like that, put an arrow.

MALLORY WILLIAMSON: Oh, create a more three-dimensional? Yes, you can do that. If you want to try to attempt that, go for it and if you need a ruler, they're in the drawer.

STUDENT: You wanna go grab a ruler?

STUDENT: Okay.

STUDENT: Excuse me.

MALLORY WILLIAMSON: In—we use numbers, so I know how we got 18.

STUDENT: Yeah, I'm confused. Because in this—

STUDENT: You guys, I think we should have put it in half first and then drawn this because—

STUDENT: No, not like that. Because then we could have gotten 10 how we actually do, and [inaudible].

MALLORY WILLIAMSON: Okay, I'm going to focus on Act 2, because that's what I'm asking us to do right now, okay? I'm not worried about Act 1 as of now. So with Act 2, I can see where we get 18 from. So how do I go from 18 to 180? How can I show somebody who's not seeing what we're doing? I'm right here. Where did you get 30 from? Eighteen to 180.

STUDENT: Because I thought there were—

STUDENT: About 10, there's about 10.

STUDENT: Because before we counted this, how many layers there were, we thought there were 10.

MALLORY WILLIAMSON: Okay?

STUDENT: And then we multiply 18—no, we multiplied—

MALLORY WILLIAMSON: And it's okay that you changed your height from this amount to 10, okay? That's okay. So we need to now show somebody who's never seen what we're doing before, how we're connecting 18 to 180. So I can easily say, "Oh, I multiplied 18 by 10 to get 180." However, you guys haven't showed me that work yet. The second thing is how can we draw a visual picture to show them how we're getting 180?

STUDENT: We can do—

MALLORY WILLIAMSON: So any thoughts on how we can draw a visual picture?

STUDENT: 30 times 6.

MALLORY WILLIAMSON: Where are you getting 30 times six? We're doing 18 and 10.

STUDENT: We could draw, um, we could do, um, check in the box, and then just like, break the number up into, like, two digits.

MALLORY WILLIAMSON: So an array model?

STUDENT: Like, you draw a box, and then 10, and then eight, and then 10 and then one.

STUDENT: I did this.

STUDENT: No, because then it'd be 10 and then—

MALLORY WILLIAMSON: But this is the same thing as what you did multiplying it by 10, that's the same thing. Taking this, which I'm still on this part, you guys are still jumping ahead. If you did 18 times 10 numerically and got 180, that's the same thing as what he's doing with this, which is distributive property. What I'm asking is, how can we present that with a visual model? So this is what 18 looks like. How can I show them what 180 looks like using this information?

STUDENT: Use a place value.

MALLORY WILLIAMSON: Place values with numbers.

STUDENT: Draw this 10 times.

MALLORY WILLIAMSON: Yes, that's what I'm getting at, okay? So you can do 18 times 10 numerically, and then you can draw 10 layers of 18 to show how many sugar cubes are packed in, one behind the other, okay?

STUDENT: You want me to draw with a pencil first?

STUDENT: So we have to—

MALLORY WILLIAMSON: Are you guys ready for Act 3?

STUDENT: Yes, ma'am.

MALLORY WILLIAMSON: Which—One of the things I'm talking to the other group about is that volume is not just the formula, okay? We're used to plugging in a formula, length times width times height and being done, all right? But what's harder is when you see the front of this box, what's behind it?

STUDENT: [inaudible]

MALLORY WILLIAMSON: Yes, you do.

STUDENT: It's 11.

STUDENT: Other sugar cubes.

MALLORY WILLIAMSON: Other sugar cubes. Thank you. There's other sugar cubes behind it, right? And before he put in that last one, you could kind of see all the other sugar cubes. All right, so when you take away this first layer, how many is going to be behind that first layer?

STUDENT: More.

MALLORY WILLIAMSON: Eighteen more. And when you take away that second layer, how many is going to be behind it?

STUDENT: Eighteen more.

MALLORY WILLIAMSON: And when you take away that third layer, how many is going to be behind it?

STUDENT: Eighteen more.

MALLORY WILLIAMSON: Okay, so, how could I show that to somebody who's looking at our picture?

STUDENT: You have to multiply it by a number.

MALLORY WILLIAMSON: Which we did, right? You guys said that the height was what?

STUDENT: Two hundred and sixteen. I mean 12.

MALLORY WILLIAMSON: Twelve. So, how can we show them 12 layers of 18?

STUDENT: We multiply 18 times 12.

MALLORY WILLIAMSON: We did already, which is 216. How can we visually draw a picture to show that we think there's 12 layers of 18?

STUDENT: Why can't we do that in Act 1?

MALLORY WILLIAMSON: I don't want Act 1. I want Act 2. How can I show 12 layers of 18?

STUDENT: You can draw in a summary.

MALLORY WILLIAMSON: I'll be right back.

STUDENT: How can you show that?

STUDENT: Right there. You can just draw, like, 18 more, 18 more,18 more.

MALLORY WILLIAMSON: Yeah. Yeah. And if you want, you can use the side space, you can use the grid, or if you want, you can create your own array of 18. Okay? So, what I'd like to show is in our minds, what we're thinking about is 18 layers 12 times, or eight—excuse me, 12 layers of 18. And so, when we do that, we want to draw a visual to represent that. Okay? So go ahead and work on that before I give you Act 3. All right? And like I said, you can use my grid paper or you can decide to draw your own visual of 18, okay?

STUDENT: How do you want to do that? We're using red and blue.

MALLORY WILLIAMSON: To a degree. Because, was it 18 by 1, or not?

STUDENT: Here, you wanna help?

STUDENT: Come down here.

STUDENT: Thank you.

MALLORY WILLIAMSON: It was six by three.

STUDENT: Oh, six by three.

MALLORY WILLIAMSON: Six by three.

STUDENT: I was thinking of that.

MALLORY WILLIAMSON: Can you explain this part?

STUDENT: Um, what we are doing is, we were taking three for the—

STUDENT: We're technically drawing this, but like 3D to, like, show all of the sugar cubes that are in here, but, like, it's laying flat, is it, over here. Like, you took off the cover and you just make it flat. You show like the nine sugars it looks right now. It's the nine sugar cubes tall and then the, the three right here and then the six for like up here.

MALLORY WILLIAMSON: Around the whole cube. Nope. That's only six by two. One, two, three, four, five, six. We have to go as a height of three. Yeah. So it'd be around here. There you go. So we got—You guys thought the height was five, correct?

STUDENT: Yes.

STUDENT: Yeah.

MALLORY WILLIAMSON: So, we've got to do it one more time. So, this is a more accurate model than doing 18 by 1, because we didn't fit 18 by 1 sugar cubes in that box.

STUDENT: No.

MALLORY WILLIAMSON: Okay. That would be a very tall, skinny box. So, when we draw a model, we want to try to draw in layers or as accurate as possible. Okay? All right. Let me give you the last act.

MALLORY WILLIAMSON: There's the last act.

STUDENT: That's our last act.

STUDENT: I don't know how to explain this.

STUDENT: Do you want me to glue it, or do you want to glue it?

STUDENT: Okay, so you—Wait. Wait. Wait.

MALLORY WILLIAMSON: Doing this down here and working on this part, okay?

STUDENT: Okay. Um, um, how did you know I didn't wanna glue? I haven't glued yet.

STUDENT: I haven't glued yet too.

STUDENT: I haven't wrote yet.

STUDENT: How do you know I don't wanna do it? I never root it.

STUDENT: I will root it.

STUDENT: You said you wanted to work.

MALLORY WILLIAMSON: Okay. Christopher, can you do me a favor, please, and sit up all the way to your desk? Thank you. Okay. Glue that down while the girls are working on that. Okay. So, what are we putting in here? Just the number?

STUDENT: No, the drawing.

MALLORY WILLIAMSON: Well, I will tell you, it's going to be a little bit harder because when we talk about six times three and then the height, that's a three-dimensional model. So, to make it easier, I would just write 18 so that the person can see that you have 10 groups of 18. Okay? All right, Christopher, so you and Angel can work on this one together. So do me a favor and sit up, okay? So now—Now, on your bottom piece, we were given this new information, how can we take this information and find the solution?

STUDENT: We sum up each number.

MALLORY WILLIAMSON: Sit on your bottom please, because you're just talking to me, okay?

STUDENT: Put 11 down, 6 across.

MALLORY WILLIAMSON: What does down mean? Like what—Is there a mathematical term we can use?

STUDENT: Three—I mean, 11 up and down.

MALLORY WILLIAMSON: What do we call that, when we're measuring up and down?

STUDENT: Rows and columns.

MALLORY WILLIAMSON: It could be 11 columns, but we're applying it to volume, we call—We call it height.

STUDENT: Yeah.

MALLORY WILLIAMSON: So height is 11.

STUDENT: The height is 11.

MALLORY WILLIAMSON: Or in this case it could be the length because of how long it is. Okay. At least you two can work on the rest of this and then we'll finish up on this, okay?

STUDENT: Three.

MALLORY WILLIAMSON: Okay. When—As soon as we saw that image, what do we start doing?

STUDENT: Oh, we got, we got—

MALLORY WILLIAMSON: We did 18 times 11.

STUDENT: Because there's 11 right here and then there's 18 right there.

STUDENT: Six times three is 18.

MALLORY WILLIAMSON: So, what vocabulary word, vocabulary term, do we call this 11?

STUDENT: Length.

MALLORY WILLIAMSON: We could be calling it the length. Okay. It just depends on how you look at the box, because in this perspective, it is the width, like how wide it is going backwards. But you can call it the length. Okay? You can also call it the height because if I were to stand the box upright, that would be how tall it would be. Okay? So we did 18 times 11 or 3 times 6 times 11. So let's do that first. [OVERLAPPING] Guys, please speak to each other properly, don't just—

STUDENT: So we do 18 times 11 or—

STUDENT: Yeah.

MALLORY WILLIAMSON: Right. So this will be 198.

STUDENT: The ratio is six by three, so 18 times 11.

MALLORY WILLIAMSON: 198.

STUDENT: 198.

MALLORY WILLIAMSON: But something that's important to understand is that you're actually finding, also, the layers of something that can fill up that space. We want to understand how we calculate volume instead of just following a simple formula and plugging it in and multiplying it, okay? So when you find a space, you could do a lot of repeated addition. So you could take 3 and times it by 11. So what's 3 times 11?

STUDENT: Three times 11 will be 33.

MALLORY WILLIAMSON: It would be 33. So I could technically take this box and say 33 plus 33 plus 33 plus 33 plus 33 plus 33. Or I could do what you guys did when we just see the front of it, 18 plus 18 plus 18 plus 18 plus 18, and keep going 11 times. So there's several different ways to calculate a volume instead of just following the formula. Okay? So I think you guys did a great job communicating with each other figuring out that. So in the long run, do you think your estimate was close the first time around compared to the end?

STUDENT: No.

MALLORY WILLIAMSON: So, sometimes it helps to actually go through it and say, "Hey, oh, I was not close, but that's okay. I now know that, that box has about 200 cubes in it. So when I go shopping, if your mom needs sugar for whatever reason, you can say, hey, that has about 200 cubes in it. You'll impress her with that. All right. Thank you, guys. And you can just put your final recording, 198 sugar cubes. Because your original estimate was 60.

STUDENT: Oh, I thought—

MALLORY WILLIAMSON: Yeah. This one was close. And I think that's important to understand what—it's hard to estimate when you just look at something. But the more information we find out about it, it's easier to understand, okay, we can actually edit our estimate and get closer to the actual answer. Okay? One thing I want to point out is there are several ways to find volume. So instead of plugging in a formula, was that the very first thing that we did was just multiply length times width times height?

STUDENT: No.

MALLORY WILLIAMSON: No, we didn't. If we did that, we'd be over with this task like that. So a lot of it has to do with layers that take up space. And it all depends on what you see and what you don't see. Because a lot of these layers, sometimes you're going to notice, are going to be hidden. Okay? What do you guys—I want you guys to talk about, is there another way you could have found the volume of this instead of doing length times width times height?

STUDENT: We could add—we could divide 198.
STUDENT: That would be checking our answer; that wouldn't, like, give us the answer.

STUDENT: That's because we already know the answer. We have to know the answer in order to do that.

STUDENT: We could draw models.

STUDENT: Of what?

STUDENT: That's true.

STUDENT: We could have drawn models. We could draw eleven model, and then—Or six models, and then do 13.

MALLORY WILLIAMSON: Okay. If you would return to your seats, please.

MALLORY WILLIAMSON: Make sure that you have nothing in your hands, and that you are tracking me, that you are sitting upright. So one final fact as we're going through this, is that volume is not just a formula. I've kind of talked to a couple of groups about this, but we say sometimes that, volume is easy to calculate because I just have to plug in length times width times height, and I'm done. But a lot of you guys can tell right away that the very first estimate that you have was kind of really far off from the, the very last solution. So finding volume by looking at something is very hard to do. We've got to get more information in order to find an accurate solution. But something I want to bring to your attention is this. In act two, I had you guys think about layers, like a cake, okay? Slicing layers, because we only see the front. You guys are going to come across volume and they're only going to give you a portion of the volume, and then you have to figure out what's hidden behind it. Well, how can I figure out what's hidden behind it? But those layers that you guys used when you calculated that length times the width, and then times it by the height, you actually did a portion of a volume called base times height. Everyone say base.

STUDENTS: Base.

MALLORY WILLIAMSON: Layer is another word for base, okay? So you can actually use a base amount and repeatedly add it or multiply it to get your total volume, okay? So remember the volume is not just a simple formula we're going to plug and chug and get it done with. It's going to take a little bit more reasoning than just looking at a problem and solving it, okay? So I appreciate your discussions today. I think we've got some really great models, really great visual representations of what was going on here. So I am grateful that you guys decided to do a little bit more than just go length times width times height, okay? All right? How many final cubes were in that box?

STUDENTS: 198.

MALLORY WILLIAMSON: Hundred and ninety eight sugar cubes. How many of you guys did not expect that? I actually didn't even know the answer until after today. All right. So there's a lot more cubes than what meets the eye, all right? Hey, do me a favor, if one person from each group would bring me their handout? Another person—you guys can actually keep the chart paper on your group's desk, another person to make sure all the supplies are put back.

MALLORY WILLIAMSON: I was surprised that a lot of the students didn't choose to actually interact with the visual when they were doing their final presentation. A lot of the students didn't interact with the picture or visual that they were given. They just kind of put an estimate down on the paper, based on what they just see or what they notice with their eyes, and instead of actually using more accurate reasoning to kind of struggle through that, a lot of the students then, after—through several conversations or really trying to them, not necessarily force them, but require them to interact, they were able then to come up with some strategies that were accurate or on on line. Once we got past the second act and we started to show them the face or the layer, first layer of the sugar cubes, students then—some groups were able to reason through that there's multiple layers and so they could create an estimate. If I think the height is 10, that means I'm going to have 10 layers.

But oftentimes a group would just use the 18, which was the total of the layer, but we had to go back and remind them that, that layer was made up of a three-by-six array. So when they drew their strategy to find the total amount of sugar cubes, they actually drew the 18 by one by 18. So going back to that group and explaining that if you're drawing one by 18, you're then changing the shape of the box. So it'll get you the same solution because you're trying to make the connection between how do I go from 18 to my final answer of 180 or 216. However, they then change the dimensions of the box when they change the array.

So making that connection that you cannot necessarily change the row by column because that is the layer, the first layer that takes up the shape of that box, a lot of the students decided to do the repeated addition and go layer by layer when drawing a model. One group attempted to do the actual three-dimensional model, that one—so I really thought that was a great challenge for them, that was great with reasoning and spatial. However, they would need more time. So I think giving them more time to continue to try to attempt that would have been great.

But I think the really hard part today wasn't necessarily that there was a major misconception that I need to reteach. It was more, so how can I start off with the reasoning that volume is—appears in several different ways, like layers or a base times the height, and having them draw that out so they can reason through; maybe this is the stuff I don't see and this is what was given to me, versus just plugging in a formula, length times width times height, and then getting it done. I think when you follow a formula, you're not able to actually understand what's happening within the space that's provided.