MALLORY WILLIAMSON: The task I chose for this particular standard is with adding and subtracting fractions, and the additional standard on there was with line plots. And so, with this particular task, they're analyzing a line plot and with that data that's put on a line plot, the students are then going to be required to add and subtract fractions. Which, when we're adding and subtracting fractions, they may be multiplying within that because there's a relationship that's evolved. So, the students, first and foremost, are taking it through several steps by recreating the line plot and then using that data to kind of find a solution.

I hope to see the relationship they use between adding and subtracting with the multiplying of fractions. I think at this grade level, that's very common for them to make that relationship and also believe that they should be able to take data from a line plot and answer questions. It's very hard for them to just start creating a line plot from scratch, but once they take that step forward, I think they could use that data to then answer any kind of question.

I know that the students are going to do a lot of repeated addition. Students who don't necessarily feel comfortable with multiplying fractions will use the repeated addition, so that they can keep track of their data. Another strategy that they're probably comfortable with is just using any kind of numerical expression to solve. One of the parts on the task that would be interesting to see is asking them to model their work. So, when you implement a model, they then want to take and model the solution instead of actually using a model to find a solution. So, I'm excited to see what comes up with that, what they come up with, but I'm also nervous because I know that that's a more challenging area for students.

Before adding or subtracting fractions or multiplying them, a major misconception is that students often want to multiply the denominator. So, within this line plot, there's a lot of fractional amounts that have fourths or halves or even eighths. So, they have to rename equivalent fractions into eighths before analyzing that data. And so, I'm, I'm probably anticipate seeing some of those misconceptions of trying to repeatedly add denominators that are different amounts or multiplying the denominator instead of just keeping the denominator the same because that whole stays the same, as well as struggling with the models. I think the major misconception I'm looking for is, are they able to create a model to find a solution or are they just going to model the solution.

This task is really lengthy. So, often times, our tasks, because we go so deeply, cover so many different standards, we're looking at probably a 60-minute task. So, it's important for students to constantly shift their perspective on what they're actually solving. So, the way we have it is that they're going to start off with a task themselves and try to look through it, just kind of using what they know, what they observe, and then collaborate with one another. Once they have a concept or an idea, they're then going to take those ideas and place it on one final chart paper for a presentation. So, that way they're using that one presentation or chart paper to then answer their questions. So, it's more collaboration, more willingness to speak to one another instead of just individually working it out on their paper and answering questions by themselves. And breaking it up into parts of—after they solve a question, discussing it.

MALLORY WILLIAMSON: So, if I see any misconceptions, I can go over those solutions or misconceptions before they move on to the next part, which that's important because they build off of one another. So, if they do the whole assignment wrong, I'm not able to catch maybe a minor misconception and have them recorrect it before then moving on. So, having those discussion so they can actually be successful in a solution, even though they made a minor mistake.

Pulling them to carpet is one of the strategies I found out often students, if you talk to them as a whole, for the whole class, it's easy for them to see their concentration and look elsewhere and not really be engaged in the conversation because of proximity. So, if I move them closer, they're more willing to pay attention to something that's right in front of them or listen to somebody who's beside them, instead of being further away from one another.

MALLORY WILLIAMSON: All right, the first thing I'm going to make you guys do is go ahead and—there's some—the handouts that we're working on, go ahead and take a copy, and put your name and date on it. Also, what you're going to notice is that when we get started you have half-sized chart paper. This is going to be what you're going to be focusing on in a second. All right, so you can spread it out on your desk if you'd like. If you have an empty desk, it's easier for you guys. Okay, today's task is going to be focusing on something that deals with fractions. Okay? So, when you're taking this in mind, the first thing we always want to talk about is what do you notice, what do you observe before I give it to you. So, take a couple seconds by yourself to take a look at the assignment. See what do you notice, what do you observe about what's in front of you.

And that also includes reading the task—the paragraph at the top. Okay, if you go to have a conversation with your group. Ones, you can share first. Just something simple that you'd like to share with your group. What do you notice, what do you observe, before we get started?

STUDENT: Um, some are—that they are—that all of them are in pounds.

STUDENT: I was gonna say the pounds one, but I agree.

STUDENT: Then there's that some have whole numbers in that. That they have one, which is a whole. They have whole numbers in some fractions.

STUDENT: Most of the fractions are eights.

MALLORY WILLIAMSON: Okay, so some of the fractions are listed repeatedly. Anything else you notice about the fractions, Ayla? The fourths and eights? Are those related to each other? Okay, what is something that you noticed or observed, Isabella? So, some of the denominators are fourths and eights. Did anybody else share that commonality? Okay, a couple groups. All right, what else did you notice, Christopher?

STUDENT: I noticed that there's all—there's some of them and mixed numbers.

MALLORY WILLIAMSON: Okay, so we see some mixed numbers. I know that Angel also shared that there's some whole numbers there. Jaadiay, what did you observe with your group?

STUDENT: Well, I said that—I said some of the fractions that are the same, and that I knew that we're going to be like either x-ing something out or whatever we're doing, um, to put them in order.

MALLORY WILLIAMSON: Okay, so we might be placing those fractions in order. There's more than one type of fraction. And then, if we see a certain fraction, we might see it multiple times. Okay? All right, so this morning, you're going to actually create a line plot with the fractions that you see. Okay? So, I'm going to give you some time with your group to collaborate, and the way

I want you to do this is, first, on your own sheet of paper, you're going to create a line plot on how you think it should be created. Okay? So, you're going to take some time and process it, and then after we've had some time to process it, then together, you're going to create one large one. So, it's almost like we put together a final presentation of our line plot, okay? So, right now, by yourself—Or you can even talk to each other, but on your own handout, I want you to go ahead and try to create a line plot with the data that you notice. Okay? All right. Go ahead and get started.

STUDENT: So, one-eighth is the smallest number we have. But we don't have a one-eighth.

STUDENT: We have three-eighths.

STUDENT: We have a one-and-one-eighth.

STUDENT: We have the three-eighths.

STUDENT: Mhmm [affirmative].

MALLORY WILLIAMSON: And how many tick marks are you placing in between the zero and the one? How many tick marks do you have here?

STUDENT: One, two, three, four, five, six, seven.

MALLORY WILLIAMSON: Okay. So, why did you decide to use eight tick marks?

STUDENT: Because that's—Oh, because it's the highest, um, number in the denominators.

MALLORY WILLIAMSON: Okay, so highest number in the denominator. Okay. So, why did you decide to mark your intervals with eighths?

STUDENT: Because, um, all of the numbers over here—up here are multiples of eight.

MALLORY WILLIAMSON: Multiples of eight. Okay. That's a great explanation.

Student: The eight is [inaudible 00:05:04].

MALLORY WILLIAMSON: And eight is also the greatest denominator that’s being used. Okay. Why is it important for us to all use the same denominator, because I know I see some fourths, I see some halves. So, why is it important for us to all rename them into eighths?

STUDENT: [inaudible 00:05:19] renamed them so we have one whole—one number that we can just go with. We don't have to keep switching through.

MALLORY WILLIAMSON: Yeah. It's easier, right?

STUDENT: Yeah. It makes it quicker.

MALLORY WILLIAMSON: You don't have to keep switching back and forth. Okay, so now that you've created your line plot, or your number line, excuse me, how are we going to create our line plot with this data?

Student: Um, we are going to, um, put the numbers that are—what we're going to, like, on one-fourth, we're going to put in two-eighths and one-eighth, and then put in—

MALLORY WILLIAMSON: So, why is one-fourth going to be marked on two-eighths?

STUDENT: Because, um, if you—

STUDENT: It's half.

MALLORY WILLIAMSON: How do you know it’s half?

STUDENT: Because half of two is one, and half of eight is four.

MALLORY WILLIAMSON: Okay.

STUDENT: Yeah.

MALLORY WILLIAMSON: All right. I'm going to come back and see your line plot. How did you guys decide to divide your number line into two parts?

STUDENT: Um, that one-eighth equals one, and we're going to convert all of them into eighths.

MALLORY WILLIAMSON: So, why didn't we just stop at one if we know that's equivalent to eight-eighths? Why did we go all the way to two?

STUDENT: Because there's one-eighths right here.

MALLORY WILLIAMSON: Okay, so there's a number listed that's greater than one.

STUDENT: Yes.

MALLORY WILLIAMSON: All right. How are you guys doing?

STUDENT: Once we'd be—We started with one-fourth.

MALLORY WILLIAMSON: One-fourth. So, why did you decide to start with one-fourth?

STUDENT: Because it was the lowest fraction.

MALLORY WILLIAMSON: Okay, so what is one-fourth equivalent to? So, I notice that you have one-fourth equals two-eighths. So, two-eighths is our least amount?

STUDENT: Yes.

MALLORY WILLIAMSON: Okay. All right. How can we, um, go from there? So, if we know that we're working with eighths, how can we create a number line from there?

STUDENT: Two-eighths.

STUDENT: We can try to find all the line that also have fours and try to convert them into eighths.

MALLORY WILLIAMSON: Okay. So, renaming your fourths into eighths? Okay. And one thing that I see Elizabeth doing is, if we start off with two-eighths, we listed three-eights, so is there a four-eighths anywhere? So, I'm going to take a step back, and we're going to go with what Izzy suggested, and we're going to rename our fractions into eighths, okay, and then when we're creating a number line, we also ended with—Elizabeth ended with one-and-one-half, so Elizabeth, why did you decide to end your number line with one-and-one-half?

STUDENT: [inaudible 00:07:52].

MALLORY WILLIAMSON: The largest number. Okay. So, I'm going to come back, give you guys some time to rename your fractions into eighths since you already have told me that four-eighths—You know, the two—one-fourth is equivalent to two-eighths, and then that can maybe help you create your number line. Okay? All right.

MALLORY WILLIAMSON: All right, so now that you guys have your number line, what are you guys going to go from, go from here?

STUDENT: We're going to convert all of them into eighths.

MALLORY WILLIAMSON: Okay. So, can you show me how you're going to do the first one?

STUDENT: One-fourth to one-eighth equals—

MALLORY WILLIAMSON: And you guys can jump in anytime. How do you think we convert one-fourth to eighths?

STUDENT: If you have a denominator of four, and you multiply four by two to get eight.

MALLORY WILLIAMSON: Okay.

STUDENT: Which means we would multiply both the numerator and the denominator by two to get two-eighths.

MALLORY WILLIAMSON: Okay, so one-fourth is equivalent to two-eighths.

STUDENT: Two-eighths.

STUDENT: Okay, so...

MALLORY WILLIAMSON: All right, show me how you renamed three-fourths.

STUDENT: Times two is eight. Six.

MALLORY WILLIAMSON: To keep your fractions organized, what I would suggest is recording your new equivalent fraction right beside it, so that way it can help you organize your data. Okay. All right. I'll come back.

MALLORY WILLIAMSON: All right, what are some things that we've noticed so far?

STUDENT: That [inaudible] are—wait, you can speak.

STUDENT: Um, we need, we need all the denominators that are fourths. We gotta—what's the word? Convert, convert them into eights, and then we need to put one-eights, two-eights, three-eights.

MALLORY WILLIAMSON: Which is what I see Roberto started.

STUDENT: Yeah.

MALLORY WILLIAMSON: So, why is it important for us to number our number line one-eight, two-eight, three-eights?

STUDENT: Because then we can just put x's that we used last grade. And because how many fractions there are.

MALLORY WILLIAMSON: Okay, so to keep a list of how many fractions that we use. It's easier for us to see. Are we going to just stop at one?

STUDENT: No. We will keep going.

MALLORY WILLIAMSON: Why not?

STUDENT: Because I noticed that some of them—it goes to one one-fourth, which is equal to one pound and two-eights, so that is greater than one—

MALLORY WILLIAMSON: Okay, excellent.

STUDENT: And therefore, need to go further than two.

MALLORY WILLIAMSON: All right, so I'm going to give you some time—more time to kind of mark your data, and then we'll come back, okay?

STUDENT: Yes ma'am.

MALLORY WILLIAMSON: All right, so how are—do you guys have a direction of where we're going from here?

STUDENT: Yeah.

MALLORY WILLIAMSON: Okay.

STUDENT: So, we converted all of them and now we're gonna see what [inaudible] and put them in order.

MALLORY WILLIAMSON: But why is it important for us to put that data in order? So—

STUDENT: Because that's how much amount that we're gonna have, is it gonna be biggest or is gonna be smallest.

MALLORY WILLIAMSON: So, it's easier to compare least to greatest? Okay.

STUDENT: Yes, it's how much we have but we can—do we have more of one or do we have more of another?

MALLORY WILLIAMSON: Okay. All right. Couple questions. I noticed that you guys created the number line from one-eight to two. So, why did you guys decide to create your number line from one-eight to two?

STUDENT: Because there's one—

STUDENT: Because it's mixed numbers.

MALLORY WILLIAMSON: Okay, so we have some—what does that mixed-number tell me when you're comparing it to a whole number?

STUDENT: That there's gonna get greater.

MALLORY WILLIAMSON: That it's greater, okay. And you guys have noticed you've already started plotting your data.

STUDENT: Mhmm.

MALLORY WILLIAMSON: So, can you explain to me how you decided to plot the three-fourths?

STUDENT: Convert them all into eights.

STUDENT: We're converting them all into eights because in the big paragraph up here it says nearest eight.

MALLORY WILLIAMSON: Okay. And then explain to me how you converted into eights.

STUDENT: You do two times three, is six. And two times four is eight, so we checked that off.

STUDENT: We're times-ing it by two for—

MALLORY WILLIAMSON: And why do you have three x's above that six-eights?

STUDENT: Because there's one, two, three, three-fourths.

MALLORY WILLIAMSON: Okay, so there's three three-fourths. That's [inaudible]. Okay. Sounds great. Sounds like you guys are on the right track.

STUDENT: I got three-fourths for that one.

STUDENT: One one-fourths is five-eights.

STUDENT: I must have put two.

MALLORY WILLIAMSON: Were there any disagreements about your line plots before we put it on a final presentation?

STUDENT: Um, yes.

MALLORY WILLIAMSON: Okay, so what was the disagreement or something that we need help through?

STUDENT: Um, Angel, said that—before he said that this equal three-eights, and now it didn't equal three-eights, it equals six-eights, and I showed him why. And then he—

MALLORY WILLIAMSON: So, it's important when we—when we rename our numerator to a six. Mathematically what's actually happening is that we're renaming the whole fraction, not just the numerator. Okay? So that was really a great suggestion that you brought out.

STUDENT: I forgot to—we can multiply this by two, and I multiple this by one. I forgot.

MALLORY WILLIAMSON: So that would—in reality be three-fourths times one-half, and we don't want to do that. We want to rename it to an equivalent fraction. Okay? Great suggestion. So you guys are going to put together your final presentation? All right, you can go ahead and get started. I would suggest that you guys can do it with a marker so that it pops a little bit more.

STUDENT: I think I'm just gonna do it with a pencil so—

MALLORY WILLIAMSON: Sure, absolutely.

STUDENT: Christopher, I think like the [inaudible] will be like right here.

STUDENT: We should write a little bit bigger so—

STUDENT: Let Joyce write—and then let Joyce write normally, and you draw the numbers. You draw the line, and he draws the—it's like right there. Right here.

MALLORY WILLIAMSON: Looks like you guys have the same amount. Which is important to check because before you get started, if you do have a different amount plotted than she does, then it makes it harder to start answering the questions that go with it. Okay?

MALLORY WILLIAMSON: All right. What I would like us to do before we get started on our final presentations, I'd like you to take your line plots and come sit on the carpet with me.

MALLORY WILLIAMSON: So, we have a lot of our data that you guys started with and we had a couple groups kind of figure out a certain — things — a little bit later than others, and that's okay. I think what's important right now is to make sure that we answer some questions before you start on your whole presentation projects because if you create the wrong line plot and start answering questions, then it changes our data completely. Okay? So, I want to give you some review of what we just did really quick before you work on your final presentation with your group and then we can start answering questions. Okay?

All right, Ciara, what is the first thing that your group decided to do?

STUDENT: Um, put down one-eighth.

MALLORY WILLIAMSON: So, the first thing you decided to do is put down one-eighth. Okay. Is there any — anything somebody else would like to add on too?

STUDENT: I see mixed fractions, so I know that there is going to be a one in the [inaudible].

MALLORY WILLIAMSON: Okay, so what was one example of a mixed fraction that you observed.

STUDENT: One-and-one-eighth.

MALLORY WILLIAMSON: Okay. So, when you're placing a one in the center of your line plot, where do you think one-and-one-eighth would go?

STUDENT: Right by the one.

MALLORY WILLIAMSON: And why would we put it that close to the one?

STUDENT: Because the one is one and it's just like counting to ten, so it's going to be, like, close to one.

MALLORY WILLIAMSON: So, one-and-one-eighth is closer to one, and one question I want to ask is, how did we decide to divide up our line plot? How did we decide all of a sudden to start dividing up into eighths? Roberto?

STUDENT: We decide to — because we can — you can turn one-eighth into fourths because they will be equivalent.

MALLORY WILLIAMSON: Now—

STUDENT: You can turn [inaudible].

MALLORY WILLIAMSON: So, we have one-fourth and then we're looking at, I heard you say one-eighth.

STUDENT: I mean two-eighths.

MALLORY WILLIAMSON: Okay, so, it helps to write it down. Why is one-fourth equivalent to two-eighths? Anna?

STUDENT: Because there are, like, four—eight divided by four is two, and you multiply the denominator by two and then the numerator by two.

MALLORY WILLIAMSON: Okay, so when dividing or reducing, they are equivalent. Okay. That's also like taking one-fourth and when we want to create eighths, instead of dividing by two, I notice that some other groups also multiplied by two. I had that conversation with Omar. So, if we were multiplying these by two, I now have two out of the eight. Okay. Yes, Angel?

STUDENT: The eight is the greatest denominator?

MALLORY WILLIAMSON: Eight is also your greatest denominator. I noticed that talking to Tanner, also in the directions says eighths. You're working with eighths on your line plot. Okay? All right. So, go ahead. I'm going to give you some time with your group. You can use a pencil first or a marker. Create your final presentation for your line plots and then you can start answering number one.

STUDENT: So, we both have three and two and one. Then, five-eighths—How'd you get one?

STUDENT: What do you want?

STUDENT: Hey, is there two ways anywhere to one—where did you get one?

STUDENT: One, two. There's two. There's three five-eighths.

STUDENT: What do you mean? There's one.

STUDENT: There's one, two, three.

STUDENT: That's a five-eight?

STUDENT: Yeah, 'cause look. Two—'cause look. If you, you have to put that one, which is four—

STUDENT: Because there's no like—but there's no like.

STUDENT: Which is four—

STUDENT: Wait, I thought, I thought this was one, two-eighths.

STUDENT: Yeah, one. One, [inaudible].

STUDENT: So, it's not one two-eighths—

STUDENT: And there's three one, one-eighths.

STUDENT: So, it's—three five-eighths?

STUDENT: There's three, three—there's three five-eighths. There's three five-eighths. Because that one would four. Would be five-fourths and then times four by two, which is eight.

STUDENT: Did you get this?

STUDENT: [inaudible]

STUDENT: No, did get this?

STUDENT: Oh, yeah. I went all the way up to two.

STUDENT: Me too. I just cut my line short. I didn't need up to three.

STUDENT: Wait. Hmm. Wait, so, one plus would be here?

STUDENT: Yeah. Which there is a one—right there.

MALLORY WILLIAMSON: Are you ready?

STUDENT: [inaudible]

MALLORY WILLIAMSON: Okay. All right, so, when you get started on your line plot, what are some things that are important to keep in mind when you're creating it?

STUDENT: That you need to start with zero.

MALLORY WILLIAMSON: All right, we're going to start on number zero. What else?

STUDENT: Umm—make sure that you end with two.

MALLORY WILLIAMSON: Okay, so, we can go from zero to two. All right, anything else?

STUDENT: That—we need to, like, fill all the number to two because, like, not just the numbers that are, like—yeah.

MALLORY WILLIAMSON: Listed?

STUDENT: Listed. Yeah.

MALLORY WILLIAMSON: Yeah, I think it's very important to understand where these values are set, so, to list all of these fractions that you listed and so, I noticed that yours are improper fractions. They're fractions greater than one and you did mixed numbers. So, you can combine those and list both on top of each other just to show that they're equivalent and I also love how all three of you guys decided to create your number line with really accurate spacing, okay. So, when we're trying to draw it on here, compared to this chart, what's—we could probably use finger spacing to make sure that your intervals are the same, okay. But before we answer those, we want to make sure we have our final plot.

STUDENT: [inaudible] No.

STUDENT: Oh, okay.

MALLORY WILLIAMSON: Do you want a ruler?

STUDENT: Yes.

MALLORY WILLIAMSON: Okay. Go get one. (laughter) I know that you guys wanted to be as straight as possible. Did you have any questions, Jaadiay, that you're finishing up?

STUDENT: Um—

STUDENT: Do you think that'll help?

MALLORY WILLIAMSON: All right.

STUDENT: And then, we can cut this in half—

STUDENT: [inaudible]

STUDENT: I don't think that'd be half. Smaller.

MALLORY WILLIAMSON: And I, I noticed that on your guys' number lines, you guys did a great job at your spacing. So, I know that it's important when having a data like this or a line plot that these spaces are the same as these spaces because that's more accurate. You should have the same space in between. So, the way to do that is maybe we can use finger spacing or anything like that to keep the same space between each fraction, okay. So, that way we know we get a more accurate answer when we're creating our line plot. Okay? So, you guys can go ahead and get started on drawing your line plot. Okay? Do you think you're going to need a ruler or anything.

STUDENT: Uh, I'm fine.

MALLORY WILLIAMSON: You guys can go get one. And I also will tell you, I've noticed that you guys did not start at zero, and that's okay. And I think some groups started at zero, and I think it's okay not to start at zero because you explained earlier that your least amount is two-eights, so it's okay to start your number line or your line plot at two-eights, okay? All right, show me what you've got.

STUDENT: Um, do you want to do, like, arrows or just a line at the end?

STUDENT: I think arrows would be more accurate, to show where the plot is.

MALLORY WILLIAMSON: And we use arrows to know that the value keeps going. It doesn't necessarily stop at that amount. Okay. All right. I'm going to give you guys some time. So, go ahead and create your line plot, and then you guys can start answering these questions.

STUDENT: And we'll start here.

STUDENT: Yeah, okay. You're on your last one? Okay.

STUDENT: Did I turn my five [inaudible 00:06:17]?

STUDENT: Yeah.

STUDENT: The [inaudible].

STUDENT: Thank you.

STUDENT: You're welcome.

STUDENT: There you go, and um—oh, group six, sorry.

STUDENT: I think we're finished with our line plot. Oh wait. Maybe, to write the two—there.

STUDENT: You want to erase—

STUDENT: Yeah, try to erase the numbers—the pencil lines.

STUDENT: Just leave it.

STUDENT: There you go. There we go. We can start answering question one on the back.

STUDENT: Suppose you took all the best candy that—

STUDENT: Because if it's in the middle, the relatively middle, I can then go and put these in the middle, then put it in the middle.

MALLORY WILLIAMSON: So, what would we put in between one and one-and-a-half here?

STUDENT: I'd put two-eighths, because—because that—because the—I need to put—Oh, I need to put one-and-two-eighths, because one-and-one-eighth would go here, and three-and-one-eighth would go here. So, I'm doing this to give a relatively even spacing between them.

MALLORY WILLIAMSON: Spacing. And when you—One—One-and-one-half is half of two, so one-and-one-fourth is half of one-and-one-half. Okay. So, it is nicer to kind of divide these up in halves so that you do get a more accurate interval. Okay.

STUDENT: Okay.

MALLORY WILLIAMSON: So, what—What fraction would be equivalent to three-fourths?

STUDENT: Three-fourths would equal six-eighths.

MALLORY WILLIAMSON: How do you know that?

STUDENT: Because three times two is six and four times two is eight.

MALLORY WILLIAMSON: Okay, so we're looking at six-eighths. So, how many amounts of candy are on six-eighths?

STUDENT: Six-eighths. We have four-eighths, which means we have four—we have three-fourths.

MALLORY WILLIAMSON: Okay. So four-and-three-fourths. And what is question number one asking us to do?

STUDENT: How many—write a numeral equation and draw a picture/model to show how you could find the overall weight of the bag.

MALLORY WILLIAMSON: So, Grace, how did you figure out that we need to add those four sixes?

STUDENT: Because, um, and combine them into one large bag, so you could do three-fourths plus three-fourths, or you could do three-fourths times four.

MALLORY WILLIAMSON: So, why can we do both of those?

STUDENT: Because three-fourths times four is just, um, repeated addition.

MALLORY WILLIAMSON: Repeated addition. Okay. All right.

MALLORY WILLIAMSON: So, talk about, Grace, what you just shared with your group so that you guys can all be on the same page.

STUDENT: So, three-fourths times four, because there are four of the six-eighths, or you could do three-fourths times three-fourths—I mean, plus three-fourths, plus, and so on, until you get to four.

STUDENT: I think we should do the adding instead of multiplying. Because this is a multiple-step problem, so.

MALLORY WILLIAMSON: Sometimes we can just list the numbers we are using on our line plot, like your team talked about how they need to be in eighths, so it's easier to notice [inaudible]. So, finish up on that number line before you start answering those questions and when—Even if you're not done here, you're just going to use that one.

STUDENT: When you're done with that, make sure you put, like, these, so they don't, like, [inaudible].

MALLORY WILLIAMSON: Even if these other fractions that you [inaudible], how many two-eighths did you notice there are?

STUDENT: Yeah. Make sure that they, make sure that they make [inaudible], you know?

MALLORY WILLIAMSON: Two-eighths is equal to one-fourth, so how many one-fourth or two-eighths do you have?

STUDENT: One.

MALLORY WILLIAMSON: So, that's one. Two. Do you notice any other ones? And those are mixed numbers. So, because those are mixed numbers, those are going to be greater than one, on this side. But there's also one over here. So that's three altogether, so what can we mark on top of that to show that there's three one-fourths. Good. Okay. So, go to the next one. We've got three-fourths. So, find all your three-fourths and mark those.

STUDENT: Three-fourths.

STUDENT: I notice that we're only doing three-fourths. Wait. The three-fourths—That would be six-eighths. Six-eighths pounds, which we have four of those bags.

STUDENT: Would we have to times it?

STUDENT: We'd have to multiply it by four, seeing as there's four of those bags. So, we have to write like this. Six-eighth times—We'd do four. But because of how it is, it's four over one.

STUDENT: So, isn't it one-fourth.

STUDENT: No, it's six-eighths times four over one. So, we then do six times four equals twenty-four, and then there's eight below it, which that then equals, um, that then equals three pounds.

STUDENT: But we have to show why.

STUDENT: Yeah. We do.

STUDENT: So, I think a good model that could be shown is we do—

MALLORY WILLIAMSON: Talk to me about number one, because I notice that you guys have completed number one.

STUDENT: For number one, we did three-fourths times four. It equals twelve-eighths. Wait. Twelve-eighths. Oh, wait, no, twelve-fourths, which is equal to—is equal to three.

MALLORY WILLIAMSON: Okay, so you did three-fourths times four, and where do we get that four from?

STUDENT: We got the four from all four three-fourths, and so we times that, the three-fourths by four, because there's four of them.

MALLORY WILLIAMSON: Okay. So, and I also notice that you used repeated addition as well. So, can, um, one of you guys talk to me about how that addition is equivalent to multiplication? Why can we use both strategies?

STUDENT: Because—

MALLORY WILLIAMSON: Hold on. I'm going to let one of them explain.

STUDENT: If we do three-fourths, then we could just add three-fourths four times.

MALLORY WILLIAMSON: Okay, and how do I—How do I know that that's okay to do?

STUDENT: Um. Three-fourths times four is equal—

MALLORY WILLIAMSON: Three-fourths times four, you got what?

STUDENT: Twelve.

MALLORY WILLIAMSON: Twelve. Just twelve?

STUDENT: No, which equals three.

MALLORY WILLIAMSON: Okay, so I'm going to have a step back for a second. Can you guys draw me a model of what three-fourths plus three-fourths plus three-fourths plus three-fourths looks like? Like a picture or a model?

MALLORY WILLIAMSON: That's one three-fourths. Okay. What have you got? Joyce, you've got the same thing. That's very similar to what Christopher drew. So, Joyce, how many total shaded fourths do you have right now?

STUDENT: Um, three. Three shaded.

MALLORY WILLIAMSON: Three in this one. What about in this one?

STUDENT: Three.

MALLORY WILLIAMSON: So, what does that add up to be?

STUDENT: Three. No, twelve.

MALLORY WILLIAMSON: Twelve-fourths. And the reason being is because I notice that you see twelve over four is—How do we get twelve-fourths and three? Why are those equivalent?

STUDENT: Three times four.

MALLORY WILLIAMSON: So, three times four is twelve? Is there another way we can decide?

STUDENT: We could divide twelve by four.

MALLORY WILLIAMSON: Okay, and we could divide twelve by four to get three; a fraction is a form of division. Okay? So, when you're shading in these fourths, you get twelve-fourths, which does equal three, okay, and then when you multiply them, it is repeated addition. You have three-fourths listed four times. Okay? All right. Do you guys want to talk to me about how you're answering number two?

STUDENT: How I'm answering number two is that I'm adding these up, adding, like, on my line plot, on our line plot, have the three-and-one-eighth, so I'm adding those.

MALLORY WILLIAMSON: What does the question ask us to do for number two?

STUDENT: It asks us to—

STUDENT: Find all the bags of candy that weigh one pound into a single bag.

MALLORY WILLIAMSON: So, how on Earth, number line or line plot, are we able to see which one is greater than one pound?

STUDENT: We see that it's one pound and then we have them—We have to get the—We put the numbers down that are great—

MALLORY WILLIAMSON: Greater than one.

STUDENT: —up here in this area.

MALLORY WILLIAMSON: Okay, and what does it mean to combine them?

STUDENT: It means to combine them by adding them.

MALLORY WILLIAMSON: Adding them. Okay. Make sure you're listening to your teammates before you're the first one to talk, okay? They're wanting to share, but as soon as they hear you, they stop talking. Okay?

STUDENT: It would be three-and-three-eighths.

MALLORY WILLIAMSON: So, do you think you're going to get a large number? Or a pretty small one?

STUDENT: Large one.

MALLORY WILLIAMSON: How do you know it's going to be a larger value? Do you guys know how it's going to be a larger value?

STUDENT: Because it's one-and-one-eighths.

MALLORY WILLIAMSON: Tell me a little bit more.

STUDENT: Because there's—That's—There's going to be one, two, three, four, five, six, seven, about eight whole numbers.

MALLORY WILLIAMSON: About eight whole numbers. So, you are going to look for a number that's larger than normal. Okay. All right.

STUDENT: Next number is four-and-two-eighths.

STUDENT: Number one means one pound.

STUDENT: So that'd be two. And then since the next one is one and one-eighth, so it'd be three, but then it'd be three and three-eighths plus 2.

STUDENT: Wait. So, we're going to take these and multiply them by how many are up here, and then we're gonna add them all together?

STUDENT: So, we're adding all of them. So, this is one, two, three, four, five, six, seven, eight.

STUDENT: So, nine-eighths would be multiplied by 3. It's twenty-seven-eighths.

STUDENT: It's 16, (inaudible)?

STUDENT: Mhm, yeah. (background noise)

STUDENT: You guys are on one, right?

STUDENT: Yeah.

STUDENT: Yeah.

MALLORY WILLIAMSON: So, adding something four times is the same thing as multiplying it four times.

STUDENT: So, like repeated addition?

MALLORY WILLIAMSON: Repeated addition, okay. (laughs) (inaudible) I didn't want to tell you. Okay, you guys, show me how do you do that. How you solve that.

(Inaudible speech by kids)

MALLORY WILLIAMSON: Okay, so let's try that strategy first. And not just any whole number, it's four, it's a value of four. So, if we just put—What happens if we put a four underneath the four?

STUDENT: You would...

STUDENT: That would equal one.

MALLORY WILLIAMSON: Just—That would equal just one and we wanted to make sure it stays as a four. Okay?

STUDENT: It would also change your denominator.

MALLORY WILLIAMSON: How did you get it? ‘Cause I noticed she has twelve-fourths. How did you get three out of twelve-fourths?

STUDENT: It's twelve divided by four.

MALLORY WILLIAMSON: Okay. Now let's try the other strategy. What was the other strategy we can try to do?

STUDENT: Um, add three-fourths four times.

MALLORY WILLIAMSON: Okay. (background noise)

MALLORY WILLIAMSON: Isabella, can you read—restate what you’re doing?

STUDENT: Um, candy.

MALLORY WILLIAMSON: Can you talk to him? He’s not looking at you.

STUDENT: We have to find the overall value of three-fourths pounds of candy.

MALLORY WILLIAMSON: And what does three-fourths equal to on our (inaudible) plot?

STUDENT: Six-eighths. (background noise)

MALLORY WILLIAMSON: So, Izzy, what did you discover?

STUDENT: I discovered that you could take two of each since we’re doing it by four and we could add them both together, and then we could add those two sums that we have together to make it.

MALLORY WILLIAMSON: And what was your sum?

STUDENT: Two-fourths, which is equivalent to what we got by—

MALLORY WILLIAMSON: No matter whether you’re multiplying, or you’re doing repeated addition, they both have the same amount of twelve-fourths, okay? The next thing I want you to try to do is see if you can draw a picture to justify that. So, we did it numerically, with numbers, but how can we draw a pic—a picture to reflect what you just did? Okay? And Dylan, if you’re stuck, I need you to talk to your team, okay? You’re a little bit behind. Okay, so right now we’re going to try to take and draw a picture, so you can do three-fourths plus three-fourths plus three-fourths plus three-fourths, but we need to try to use a picture model.

STUDENT: I think that for our total we should do like a big—like, you know, how candy comes in either small packages or a small packet and then there's the big size that come with like multiple? We could do like that, like you have one big little bag, which is the total, right? So that we have like one with three-fourths—

STUDENT: And then four little bags and then split them into four with some kind of shading degree.

STUDENT: Yeah, we can, like, put them in there and then just, yeah. So, we do, like, our little packs. So, these would be, like, our total, then we would convert them and put them all in here like in the large bag, the larger bag. So, we would have three-fourths, another three-fourths, another three-fourths, another three-fourths, which as our model here would all equal to twelve-fourths, which would be our total, which is also our large bag totally.

MALLORY WILLIAMSON: So, how can we use this to reflect what we did here?

STUDENT: We could put three more lines to make it—

STUDENT: That'd be multiplying, we're adding.

MALLORY WILLIAMSON: Okay.

STUDENT: So, we'd have to draw another box and then shade this one and then the other one.

MALLORY WILLIAMSON: Okay, so you draw another box and shaded three more out of that, what would that equal?

STUDENT: That would equal six-fourths.

MALLORY WILLIAMSON: Okay, six-fourths. (background noise)

STUDENT: Ugh.

MALLORY WILLIAMSON: And what, what are you trying to get to? What's the total number that you're trying to get to?

STUDENT: Twelve-fourths.

MALLORY WILLIAMSON: Twelve-fourths.

STUDENT: Uh-huh.

MALLORY WILLIAMSON: And your math is going to look a little bit different than Tanner's and that's okay. Tanner's adding onto his model until he finds a solution and you're showing me

repeated addition, so three-fourths plus three-fourths, okay? So, just to let you know, they're the same, okay? All right?

STUDENT: And then we gotta add—

MALLORY WILLIAMSON: Shark bait!

STUDENTS: Ooh ha ha!

MALLORY WILLIAMSON: Come meet me on the carpet with your paper. Call on some of you guys to share what you're doing so I'm not the one who's going to be showing you a lot of this so if you happen to get up and come to the board just be careful about who's around, fingers and toes, okay?

All right, so number one, what—try not to reread the problem to me, just kind of summarize it in your own words. What were we asked to look for in number one, Grace?

STUDENT: For number one we have, um, to see how many three-fourths we can put in one bag.

MALLORY WILLIAMSON: Okay so—and how many three-fourths were there?

STUDENT: Four.

MALLORY WILLIAMSON: Okay, so can you come up and show me one way that you guys solved that? And are you going to use model or numerical?

STUDENT: A model.

MALLORY WILLIAMSON: Okay.

STUDENT: And it would be twelve-fourths divided by twelve.

MALLORY WILLIAMSON: Okay this is one—thank you, Grace—one model I want to share and just kind of show with you guys. Before we get started is there anything that we would like to observe, agree, disagree, add on, Drake?

STUDENT: I would like to observe that she didn't use the—you had told us we could've used two different colors to mark so it's not all the same one, it's different bags and not just one big bag because I observed she could've used two different colors to mark that as not all the same bag.

MALLORY WILLIAMSON: So, it could've been easier if we did use different colors to kind of see how many different types of free force we used, okay. One thing I like to notice is that Grace's group decided to use one giant model. And so, Grace, why did you decide to shade, for instance, all of this model and all of this model because I do know that some groups decided to use a model that might look like this. And these aren't as accurate as I'd like them to be but it's okay. So why did you guys decide to use this model versus this type of model?

STUDENT: Because if you do three, that would be one times and then you would do three again.

MALLORY WILLIAMSON: So, three.

STUDENT: That would be one bag.

MALLORY WILLIAMSON: Okay.

STUDENT: And then another three would be another bag and another three would be another bag. And then another three.

MALLORY WILLIAMSON: Okay, so you decided to add as you went along to find your solution? Okay. So, you added the three-fourths as you went along. And since we only have three, we don't necessarily need this one, but that's okay. Okay, because it also shows me that it's less than four, and another thing is, is when we place our three in this division statement, make sure that we place that three in the correct place value.

So, notice it's above the one. That kind of shows that four goes into one three times. Okay? Which we've talked about before, so that's okay. Real quick fix, no biggie. Three pounds, sorry. All right, anything else someone would like to share, of either numerically or with a model, Sydney? Thanks. I'll rewrite that when you leave.

MALLORY WILLIAMSON: So, talk to us about what you did there.

STUDENT: So, what we did for our group was we did three-fourths plus three-fourths plus three-fourths plus three-fourths. We got to four three-fourths because there are three—or four in each part that we made, so we added all those together and we got twenty-fourths, which is equal to three pounds.

MALLORY WILLIAMSON: Okay, so we label that is addition. Thank you.

STUDENT: (to another student) Sorry.

STUDENT: It's okay.

MALLORY WILLIAMSON: And the connection with this is very similar to what Grace and Sydney showed us. Okay, so it is—here's the three-fourths, here's another three-fourths. These are modeling three-fourths but what Grace did was connect the two. But she actually decided to add them and it shows the total, okay? All right, what's a one last strategy that we could have used as well as adding? Isabella, you want to come show me right below? And then I'll let you guys accomplish or tackle number two.

All right, one important conversation, thank you, Isabella, is I want you to talk to someone near you on the carpet. How are these two equivalent strategies, why are these two, two strategies that we can use? So, talk to someone near you on the carpet.

STUDENT: I think instead of putting one-fourths, she flipped it and put four over one. And so, she times-ed that three-fourths times the four over one and got twelve-fourths because four times one is four. And then if you simplify it—

MALLORY WILLIAMSON: (sings McDonald's jingle) Ba da ba ba ba!

STUDENTS: I'm lovin' it.

MALLORY WILLIAMSON: All right, Isabella, can you share with me, because I know that you wrote down this fraction, how these two strategies are related, how we can use both of them to find our solution?

STUDENT: Adding three-fourths four times and multiplying three-fourths four times is the same because you're both doing it four times.

MALLORY WILLIAMSON: Okay, so you're repeatedly adding—our whole number is four, so you're repeatedly adding something four times, okay. All right, you may go work on number two.

STUDENT: I wanted to stay within because we're already doing in eighths. And it was six-eighths marked here, so we decided to in six-eighths.

MALLORY WILLIAMSON: And did you get the same solution?

STUDENT: We got the same solution.

MALLORY WILLIAMSON: Yeah, so—and that shows you also that they're equivalent fractions, six-eighths and three-fourths. When you get that same solution, they are equivalent. That's a great observation. All right, how are you guys tackling number two?

STUDENT: Um, um—

STUDENT: And—but does this count [inaudible]?

MALLORY WILLIAMSON: It can, yeah. A number line is a strategy that you could use—

STUDENT: I started from one to two because I didn't want to do the whole thing.

MALLORY WILLIAMSON: So, you just redrew your line plot, and you—when you redrew it, you only used the fractions that were greater than one. Okay. This isn't necessarily a model because the model will show you how you find the solution, but this still could be used to kind of show me what fractions are greater than one, okay? When you do a number line, I want to see the actual adding that's happening. So, you would start a number line off and show me the total of, like, one and one-eighth.

STUDENT: So, I just write the numbers, like these—

MALLORY WILLIAMSON: Yeah, you can write it like this. That's a strategy.

STUDENT: So, I'm done?

MALLORY WILLIAMSON: I don't know. Did your team get the same solution as you? They may be still working. Are you guys still working?

STUDENT: Yeah, I'm still working. I am now on this [inaudible].

MALLORY WILLIAMSON: Okay, so you're almost done? So, you guys are adding the fractional amounts that you saw?

STUDENT: Yes.

STUDENT: Is there another way to—that you put it into a model?

MALLORY WILLIAMSON: Yeah, there's a lot of different ways. You can draw a picture, which is very similar to what we did with this one.

STUDENT: Okay.

MALLORY WILLIAMSON: Or you can create a number line to show that you're adding those amounts. Is there one you would prefer to start over the other?

STUDENT: Well, so, could I just—

STUDENT: I would do a box—I would do another box model.

MALLORY WILLIAMSON: You're going to try to do a box model.

STUDENT: Then make it into eighths, and then—

MALLORY WILLIAMSON: Yeah, you could start a number line off into eighths, and then show me how you're adding those parts together.

STUDENT: Wait, can you not [inaudible]? Like, one times two, and I got two whole—

MALLORY WILLIAMSON: Yes. So sometimes you can multiply amounts. Why would you multiply some amounts and add the others?

STUDENT: Because adding may be different [inaudible], because one plus one equals two and one times one equals one.

MALLORY WILLIAMSON: So once—you might get different parts, okay? And then sometimes it's faster with—when you have these amounts, and there's more than one. If you want a more efficient way, you could multiply them, then just repeatedly add them, okay? All right, I'm going to give you some time to think about the models you're using, and I definitely want to come back and see what you guys came up with, because I know that you guys are about to use different models, which is okay.

Why did you decide to multiply one and one-eighth by three?

STUDENT: Because there's three one-and-one-eighth in the line plot.

MALLORY WILLIAMSON: Okay, which I can see. Is there another way we can find the total of—

STUDENT: We could add them—

MALLORY WILLIAMSON: Which I see that you did. And what did you get when you added them?

STUDENT: Oh, added this one? I got three and three-eighths.

MALLORY WILLIAMSON: So, when you're done, I want you to double check with Tanner to see if that's what you got, as well.

STUDENT: And then I'm adding the two because of the two [inaudible].

MALLORY WILLIAMSON: [inaudible] So what is question number two asking you to find?

STUDENT: To combine all the bags of candy that weigh one pound or more into a single bag.

MALLORY WILLIAMSON: Okay, so how do I know on my line plot which amounts are more than one?

STUDENT: Um, because in the middle, we have a one and then you'd add the ones that have the [inaudible] all together, or you could add them separately and, like, add ones that, like—add one that has, like, two X's, and add that to—add those two together to times—wait, one time.

MALLORY WILLIAMSON: So, you're not only adding the amounts that are listed above that fraction, but what I also hear you saying is you're adding the amounts greater than one together? So, you got a lot of addition going on. Or what did you end up getting when you multiplied by three?

STUDENT 4: Three and three-eighths.

MALLORY WILLIAMSON: Which—is that equivalent to him? Repeatedly addition—

STUDENT: Yep.

MALLORY WILLIAMSON: Yeah, so you can go either way, okay?

STUDENT: [inaudible] your total? I got six and seven-eighths.

STUDENT: And one plus one equals two—

STUDENT: Um, um, I'm not—

MALLORY WILLIAMSON: So, I'm going to—while you're waiting, what's something that you can do while you're waiting for them to finish?

STUDENT: Um—

STUDENT: Read the [inaudible].

MALLORY WILLIAMSON: Aw, he's not done with number two.

STUDENT: Oh, you gotta draw a model.

MALLORY WILLIAMSON: So, think about a model that you could possibly draw. So which one makes you feel comfortable? There's several that you could probably use for this one.

STUDENT: Wait, isn't this like an [inaudible] equation?

MALLORY WILLIAMSON: Yeah.

STUDENT: Oh, so I don't have to draw a model?

MALLORY WILLIAMSON: It's got "and." You've got to do both.

STUDENT: Aw.

MALLORY WILLIAMSON: [laughs] You've got to do both. [laughs]

STUDENT: We needed to add these—this one, this one, this one, and that one.

MALLORY WILLIAMSON: Why is it important to add all of these amounts?

STUDENT: Because—

STUDENT: Because it's more—it's one pound and more.

STUDENT: It's one pound and more.

MALLORY WILLIAMSON: Okay, so it's greater than one. All right.

STUDENT: And then we added them by doing, like, nine-eighths is equivalent to one and one-eighth, and there's—it's three times. So, we did one and one-eighth three times, and then we did ten-eighths—one and one-eighth—one and two-eighths—and then two times. And then one-fourth once. And then we add those altogether, and then we added the two pounds up from here, and got nine and three-eighths.

MALLORY WILLIAMSON: So, just to kind of—what I'm hearing you saying is that you added your mixed numbers—the amounts for each mixed number, and then once you found those amounts, you added them all together?

STUDENT: Yeah.

STUDENT: I didn't get that.

MALLORY WILLIAMSON: What did you do?

STUDENT: I just added them all at the same time.

MALLORY WILLIAMSON: At the same time?

STUDENT: I added them separately.

MALLORY WILLIAMSON: Separately? Okay. So, what did you guys end up finding in the solution?

STUDENTS: Nine and three-eights.

STUDENT: Yeah.

MALLORY WILLIAMSON: Uh, my question to you is how could we show this with a model or a picture?

STUDENT: Oh yeah. Forgot to do that.

MALLORY WILLIAMSON: [laughs] So what model or what strategy would you guys like to try in order to find the solution?

STUDENT: We could draw, like, a circle and write one and three—I mean, one and one-eighth—well, or three and three-eights, and then we could do another one, and [inaudible]—

MALLORY WILLIAMSON: So, what, what's a model that would represent one and one-eighth? What would that look like if I drew that?

STUDENT: One whole, and then—

MALLORY WILLIAMSON: Can you show me?

STUDENT: Oh, I know now.

MALLORY WILLIAMSON: Okay, so that's one.

STUDENT: And then [inaudible].

MALLORY WILLIAMSON: So, you just said that one and one-eighth is equal to how many quarts?

STUDENT: I'll do it next.

MALLORY WILLIAMSON: Okay. [laughs] I can come back. You're fine. Okay, so you're doing it, dividing it into eighths. Okay, so that is—is that 1 and—what, that's one and two-eighths?

STUDENT: No.

MALLORY WILLIAMSON: [laughs] [inaudible] Okay, so you can use either one. She's doing a circle, which is fine. So that's one and one-eighth, and that's also one and one-eighth. So, how many times are we going to do this to model what we did [inaudible] three times?

STUDENT: That's a lot.

MALLORY WILLIAMSON: It is a lot, but it's going to also show you the value that you're adding together. And when you shade this whole thing in, what do you want us to also know—that this is in eighths as well?

STUDENT: Oh yeah.

MALLORY WILLIAMSON: Especially since I noticed that you guys added up all your eighths.

STUDENT: Okay.

MALLORY WILLIAMSON: All right, so I'm going to give you some time to, to do that, 'cause I know that you guys have a lot of work ahead of you.

MALLORY WILLIAMSON: I have a question for you. If—I noticed that we just did it numerically, we also want to try to show how we find our solution with a model. So, how would you guys want to try to show with using a model how to answer number two?

STUDENT: Oh.

MALLORY WILLIAMSON: Talk to me about what you're doing since you said "oh."

STUDENT: We have to draw one whole, and then we have to draw another one, and then we draw eighths, and then we draw a color one.

MALLORY WILLIAMSON: So, you're modeling the one and one-eighths? Okay. Are you guys okay wanting to try to draw some models?

STUDENT: Yeah.

MALLORY WILLIAMSON. Okay. Is that the strategy you want to use or do you want to try a different strategy?

STUDENT: I think I want to try a different strategy.

MALLORY WILLIAMSON: So, what would be another model that you could use to show how you're finding your solutions?

STUDENT: I could show like this. It can be having one as three-eighths—three and three-eighths, plus the two.

MALLORY WILLIAMSON: But that's the same thing as this. Repeatedly adding. So, if someone were to look at this, how would they know it'd be equal to three and three-eighths if you took off that label? So, if you want to model your whole numbers that's fine, like these, these partial add-ins equal three and three-eighths. How could we model three and three-eighths?

STUDENT: One whole because there's one whole, and then you can start adding in the other one.

STUDENT: So, I would do one whole thing that's—

MALLORY WILLIAMSON: An eighth. Okay.

STUDENT: One-eighth. Here we go. And then—

MALLORY WILLIAMSON: So why did you draw four of them and only shade in one-eighth?

STUDENT: Oh, I meant to draw three not four.

MALLORY WILLIAMSON: (laughing) I was asking. Okay, so you drew three of them and you only shaded in one-eighth out of them, why?

STUDENT: Because there's three one-eighths, and then I'm gonna add one whole to all of them. And then—

MALLORY WILLIAMSON: So once again, when I go back, how do I know what one whole looks like using a model other than just writing "one" there?

STUDENT: I would shade this.

MALLORY WILLIAMSON: Okay, so you shaded in one whole.

STUDENT: And then, and then I would, like, add all of them, which equals three and three-eighths.

MALLORY WILLIAMSON: Okay, and I would recommend when you're doing your models make sure they're as neat and organized as possible, because you see how small this one is compared to this whole? And I know you shaded in the whole thing, but when I look at it, how many parts is that whole divided into? Talk to me about what you just did.

STUDENT: I divided them by eight.

MALLORY WILLIAMSON: So why is it important for us to have this divided into eighths and this model divided into eighths?

STUDENT: Because it shows that both of them are eighths.

MALLORY WILLIAMSON: Because your—the whole part that you're looking at is in eighths. Okay. What does this represent?

STUDENT: Two.

MALLORY WILLIAMSON: Two?

STUDENT: Two.

MALLORY WILLIAMSON: From—when I look at this it looks like one.

STUDENT: Oh.

MALLORY WILLIAMSON: So, let me talk to you about this. It's important for us not to attach them together, to have some space. So, then it looks like it's two parts to a whole, not just one giant one, okay?

MALLORY WILLIAMSON: So, we're going to take a look at this—number two, and I'm going to actually change the smart board a little bit because we might have some different models coming up. So number two asks us to take the amounts that are greater than one and combine them. So, can someone talk to me about how they figured out what fractional parts were greater than one? Omar. Take your time. How did you know what to combine with those amounts greater than one?

STUDENT: I added two and one pounds, and the ones greater from it.

MALLORY WILLIAMSON: Okay, so let's take a look at our line plot. I know mine's not quite finished. So, where did your eyes tend to go when you're looking at fractions that are greater than one? Because I heard you say one and one-eighth was one of them. So, how did you know that one and one-eighth was greater than one?

STUDENT: Because one one-eighth is higher?

MALLORY WILLIAMSON: So, it's one-eighth greater than one. Okay? So, when we subtract the two amounts, you do get a difference of one-eighth, okay? That's [inaudible]. So, all of the fractions to the right of that one is going to be greater than one. Good, I like how you explain that. That it's one eighth higher or one-eighth greater than one. Okay. That's a great observation. Okay, so when we found the combined solution, I want to know if you had the same solution as someone near you. So, share with someone beside you on the carpet what was the combined total of all the fractions greater than one.

STUDENT: [inaudible]

STUDENT: How come?

STUDENT: [inaudible]

STUDENT: And I did mine—I probably have this—

STUDENT: Where did you get the two and the fourths?

STUDENT: Um.

STUDENT: Oh, I got it. I got it. Yeah.

STUDENT: I did two and—

STUDENT: Well, it's the same thing we all did.

STUDENT: Yeah.

STUDENT: I did two of the—

STUDENT: Yeah.

STUDENT: And I had to multiply to get the [inaudible]. It's easier and faster.

STUDENT: I used the [inaudible]. I did it like [inaudible]. So, is it three and then three and eighths? So, three and the eighths.

STUDENT: Did you—those are—how did you—

STUDENT: How did you do that? That doesn't make any sense.

STUDENT: [inaudible]

MALLORY WILLIAMSON: Okay, so I really want to talk about the model part. So, for the sake of time, I might add these differently than you, but as I'm adding these up, can you do me a favor with your pencil and kind of check off the fractional amounts that you added. Some may have added the ones last, some first, so as we go, just kind of check off what you are adding up. Okay. How did you guys do?

STUDENT: Good.

MALLORY WILLIAMSON: Now I noticed group two, can you guys share with me? I know that Caden, you recognized something, an error or mistake that you made, which I'd like you to share, which is okay. So, what error or mistake that you noticed that you guys made as a group?

STUDENT: We made the mistake of thinking those three—three [inaudible] pounds—

MALLORY WILLIAMSON: Instead of the two?

STUDENT: Instead of the two.

MALLORY WILLIAMSON: Okay. Which is not too far off. You have the three-eights. Okay. So, I was telling to one group, as you add these off, one strategy I would suggest with you guys is to check off the amounts as you add them, just to kind of keep your thoughts organized because there's a lot of fractions going on. Okay? All right, so I want to take this. How can we use a strategy to find this solution without using any numbers? Okay. So, Angel, would you show one way on the smart board. Angel's way was actually the most popular. So as Angel shows you his way, is it okay if I show you my way?

STUDENT: Yes.

MALLORY WILLIAMSON: Now, whose way is correct?

STUDENTS: Both.

MALLORY WILLIAMSON: Both. So, don't start judging Angel because he doesn't look like mine. And Elizabeth, you might want to scoot back a little bit, so he's got some space. Take up as much space as you need, what you need, okay?

STUDENT: How did we say this?

MALLORY WILLIAMSON: Huh? See, even your teacher makes mistakes. Let's do this. Okay, so while Angel is working on his, my number line is not complete but I'm going to show you guys this. If we had this like giant chart paper, it would probably be incredibly long or take up the length, but I want to show you how I started off with. I'm a number line type of person, so with me, what we could start off with is all of the whole numbers where you're starting off with one and one plus one, is two. Okay?

MALLORY WILLIAMSON: So, when you're talking about going from zero to one, that's one. And one—from one to two, that's adding another one, so that's two whole numbers, and so from there, you can add one-and-one-eighth. So, two plus that one-and-one-eighth is three, okay? So, I can keep going on my number line until I get to nine-and-three-eighths. That's a really long strategy, but I can easily see all these whole numbers that I'm adding up. If I'm looking at my whole numbers, I've got two ones that were plotted on the line plot. So, one plus another one gives me that two. Okay? And two plus one-and-one-eighth is three-and-one-eighth, okay? So, I can keep going until I get to the nine-and-three-eighths. But like I said, that's probably a really long number line, okay?

So, let's take a look at what Angel's got so far. I just want to at least recognize it for a quick second. So, I'm going to go to this side, Angel, if you don't mind. All right, so—You want to step off to the side for a second? All right, Angel, what amount did you start with?

STUDENT: Um, one-and-one-eighth.

MALLORY WILLIAMSON: Okay, so one-and-one-eighth. Where did you draw one-and-one-eighth? Oh, yeah, one-and-one-eighth plus one-and-one-eighth plus one-and-one-eighth plus one-and-one-eighth equals what? There you go.

STUDENT: There's three-and-three-eighths.

MALLORY WILLIAMSON: Okay, so, just to kind of show you where Angel's going. This is the three-and-three-eighths that he started off with, okay? Now, let's take it—Look at Angel's whole numbers. I know it's easy to take a box and shade it in, but when we look at parts-to-whole, with our whole number, to get a more accurate answer, what could our one whole be divided into? Sydney?

STUDENT: Eighths.

MALLORY WILLIAMSON: Eighths. You want to divide it up? Divide your whole number into eighths. So, these that you shaded in, we want to divide them into eighths. So, what Angel is doing is, he's modeling all of the amounts that he added together. So, one-and-one-eighth plus one-and-one-eighth plus one-and-one-eighth is three-and-three-eighths. So, he modeled three-and-three-eighths. Be careful about this because a lot of you guys want to add it and then model how your solution looks. So, like, when you add one-and-one-eighth plus one-and-one-eighth plus one-and-one-eighth three times—you can sit—you get three-and-three-eighths. He modeled his solution, but when you're going through this, and you have your eighth, okay, I have one-and-one-eighth. I want to shade in the one-and-one-eighth. And then from here, we need to add another one-and-one-eighth, yes?

STUDENTS [collectively]: Yes.

MALLORY WILLIAMSON: So, how many do I have to shade in to equal one-and-one-eighth? Instead of drawing all over again one-and-one-eighth, how much do I need to add on? So, here's another way of wording it. One-and-one-eighth is a mixed number. What's another equivalent number that's equal to one-and-one-eighth? Roberto?

STUDENT: One-and-one-eighth is equivalent to one-and-two-sixteenths.

MALLORY WILLIAMSON: So, one-and-two-sixteenths. Izzy?

STUDENT: Nine-eighths.

MALLORY WILLIAMSON: Nine-eighths. Okay. So, from here, I can shade in nine more. So, here's the one-and-one-eighth. That's eight-eighths, which what is eight-eighths equal to?

STUDENTS [collectively]: One.

MALLORY WILLIAMSON: One. And then I'm going to put another one-eighth there. Okay. So, now what do I need to do?

STUDENT: Divide.

STUDENT: Add another one-and-one-eighth.

MALLORY WILLIAMSON: Add another one-and-one-eighth. Okay, so, I need to add on nine more. One, two, three, four—. There's only room—There's only room for six.

STUDENT: Add another.

MALLORY WILLIAMSON: Say it.

STUDENT: You have to add another box.

MALLORY WILLIAMSON: I have to recreate another array. So, six, seven, eight, nine. So that's the total of my three-eighths. Be careful of not just modeling your solution but using a model to find that solution. Okay? All right. Go ahead and try number three.

STUDENT: One-and-one-half times—What could be some additional fraction that allows you to [inaudible]?

STUDENT: Is this all the [inaudible]?

STUDENT: I guess. Of the total, which means, yeah.

STUDENT: Yeah. I think it is. What could be some additional fractions—fractional amounts that students might have?

STUDENT: They all have their things in halves.

STUDENT: I don't know. Hm.

STUDENT: So, um, so an additional fraction that they combined all their candy together? And got—

STUDENT: It says fractional amounts—

STUDENT: Additional fractions.

STUDENT: Oh, yeah.

STUDENT: So, it's their—Addition. They're adding they're fractional amounts the students may have. So, I think we're adding.

STUDENT: Adding what? All the amounts together?

STUDENT: I think so. That's what it says.

STUDENT: That's going to take up a lot of room.

STUDENT: Or we could just say, like, that, um—So, they've got sixteen-and-one-half, but so, if we make that one-half, it would be sixteen-and-four-eighths.

STUDENT: Mhmm [affirmative].

STUDENT: So, we'd have to do something to the number to make it something, and if we add it—What's sixteen times four? And then add that to—

STUDENT: Sixteen times four.

STUDENT: Yeah. And then add that to the numerator and then we—That'd be our fraction. And then if we simplify it, that'd be our answer. It's 64.

STUDENT: That's 64.

STUDENT: Yeah. So, sixty-four-eighths.

STUDENT: Sixty?

STUDENTS: Four-eights.

STUDENT: And then if we simplify it.

STUDENT: Wait. Did we say 60?

STUDENT: Because sixteen—Sixty-four, four, because of them, all their candy together. Then if we simplified that, we'd get sixteen-and-four-eighths. Like that.

STUDENT: Hold on. I'm going to try to simply this. Simplify.

STUDENT: Same. If that doesn't work, then whoop. So, how much—Eight and [inaudible].

STUDENT: Sixty-four times—Or, eight times eight is sixty-four.

STUDENT: Yeah. So, divide by eight?

STUDENT: Yeah. That's—

STUDENT: That's eight.

STUDENT: Yeah.

STUDENT: Eight over one. That didn't work. That didn't work. All right.

STUDENT: I think we messed up.

STUDEN: That didn't work. [inaudible]. All right. Now, I thought that would work, but it did not work out, so. Um.

STUDENT: Hey, it's worth trying.

STUDENT: I have the total amount of pounds, um, of the candy. And we need to get the sixteen-and-a-half-pounds. Well, but I'm—

STUDENT: We don't have to put the sixteen-and-a-half pounds.

STUDENT: It says the total of candy when [inaudible] is sixteen-and-a-half pounds, but what could be some additional fractional amounts the students may have?

STUDENT: It says some additional.

STUDENT: One of them could be, um—

STUDENT: Fifteen-and-seven-eighths.

STUDENT: They could add another five-eights of candy because the five and seven—the five-eighths and seven-eighths, if you add it together, equals one-and-four-eighths. Four-eighths is half of eight, so that would be one-half. And it also, do a one, so that would be sixteen-and-one-half. Or we could do two-eights and add, put another x on the two-eighths and three-eighths,

'cause that equals five-eighths. So that'd be some. Or we could do two checks here, um, and a check here.

STUDENT: Where did you get five-eighths?

STUDENT: Well, with five-eighths, I did five-eighths because I was thinking about it.

STUDENT: But two times three and two times three is...

STUDENT: And 'cause five. 'Cause, let's see, I'm going to do seven-eighths plus five-eighths, and if you do the math, that is twelve-eighths, which if you then divide would equal one pound and four—one-and-four-eighths of a pound. So, and if we add one to fifteen, that's sixteen and get the four-eighths.

MALLORY WILLIAMSON: You're trying to figure out the table of all the data.

STUDENT: We'll add all the fractions.

MALLORY WILLIAMSON: Okay. Have we already added some of those fractions? So, how can we take what we've already done to save us some time.

STUDENT: Well, we know from question two, we already got nine-and-three-eighths, and it was asking for the one or more, so we can automatically not look at that.

MALLORY WILLIAMSON: So, let me kind of write this—Can I see this? Can I use your pencil? So, you're saying all the amounts from the one here to here is how much?

STUDENT: Nine-and-three-eighths.

MALLORY WILLIAMSON: Nine-and-three-eighths. Okay. So, this is nine-and-three-eighths, and so what do we need to figure out now?

STUDENT: What the part comes below one.

STUDENT: What those equal.

MALLORY WILLIAMSON: Okay, so what are we going to do with all these corrections?

STUDENT: Um, multiply those by three because there's [inaudible].

STUDENT: Also, look at the ones that already have one out of them. So we could just write that down.

MALLORY WILLIAMSON: So, that would be—So, once you find the total of this amount, what are you then going to do?

STUDENT: Going to add [inaudible].

STUDENT: Add that.

MALLORY WILLIAMSON: Okay. So I'll give you some time to do that. We've already found a total. What does this total represent?

STUDENT: Nine-and-three-eighths.

MALLORY WILLIAMSON: Yeah. What does nine-and-three-eighths represent? How did we find that?

STUDENT: The weight of the bags. We found it by looking at how many were in one and above.

MALLORY WILLIAMSON: Okay, so if we know this half of the number line or line plot is nine-and-three-eighths, how can we figure out the total of this part of the line plot?

STUDENT: By doing the same thing that we did.

MALLORY WILLIAMSON: Which was what?

STUDENT: Add them.

MALLORY WILLIAMSON: Okay. So, yeah, sometimes you can take work that you've already started and just continue off of that and jump off of that.

STUDENT: And then once we get this one, do we add them both together?

MALLORY WILLIAMSON: What do you think?

STUDENT: Yes.

MALLORY WILLIAMSON: Did you ask your team? Caden? Ask them?

STUDENT: Once we add all these, will we add both of these together to get...

MALLORY WILLIAMSON: Because you missed that conversation before, while you were finding your eraser.

STUDENT: Actually—

MALLORY WILLIAMSON: She said, beforehand, we've already found the total of this part of the line plot, which was nine-and-three-eighths. So, she's asking, once we find the total of this part of the line plot, do we add both totals together to find the sum.

STUDENT: Then how do you find this one? So, I guess we would just add this one, this one, this one, this one, and this one?

MALLORY WILLIAMSON: To find the total. And then she's saying, once you find that total, do we just add it to this total?

STUDENT: Yeah.

MALLORY WILLIAMSON: What will that give us?

STUDENT: The overall total.

MALLORY WILLIAMSON: Okay.

STUDENT: Once we get the overall total, would we subtract it from sixteen-and-one-half?

MALLORY WILLIAMSON: That's a conversation that you guys can have together. I'm not going to say yes or no.

MALLORY WILLIAMSON: I know that you added up your amounts and you added it to nine and three-eighths and go fifteen and seven-eighths. So how did you go from the sixteen and one-half to five-eighths? What did we do with those amounts?

STUDENT: We subtracted them. We subtracted sixteen and one-half from fifteen and seven-eighths.

MALLORY WILLIAMSON: And what did you guys discover when you subtracted fifteen and seven-eighths from sixteen and four-eighths?

STUDENT: That—Well, we subtracted the fifteen from the sixteen.

MALLORY WILLIAMSON: Okay.

STUDENT: Yeah, what we discovered was that we had to, like, um, since four-eighths was less than seven-eighths we had to borrow from the sixteen.

MALLORY WILLIAMSON: Okay.

STUDENT: And then we had to use that whole number that we got and turn it into an improper fraction to make it a—like, so we get—

MALLORY WILLIAMSON: Able to subtract.

STUDENT: Yeah.

MALLORY WILLIAMSON: Okay, so you have five-eighths. So where could you place that five—How could we get a total amount of five-eighths on our line plot? So where could we add a plot on our line plot to get the five-eighths? (student points to paper) So, we could either put a plot there, what else could we do?

STUDENT: We could put it—

STUDENT: We could put two and three-eighths.

MALLORY WILLIAMSON: Okay, so which one do you guys want to do, two and three-eighths or just five-eighths?

STUDENT: Wait, this one or that one? Or just that one?

MALLORY WILLIAMSON: Either one. It just has to add up, an additional fractional amount just has to add up to five-eighths.

STUDENT: I'll say two and three-eighths because it's creative.

STUDENT: Or you could do four-eighths and one-eighth.

MALLORY WILLIAMSON: One-eighth, mm-hmm. So, you have three options. So, all you guys have to do is choose one.

STUDENT: This is hard.

MALLORY WILLIAMSON: But you already told us, right? (laughter)

STUDENT: But I don't know which one to do.

MALLORY WILLIAMSON: So T, what do you guys want to do?

STUDENT: Um, I guess we could do two-eighths and three-eighths.

MALLORY WILLIAMSON: Three-eighths? Okay, so let's plot two-eighths and three-eighths. Okay.

STUDENT: Okay.

MALLORY WILLIAMSON: All right the last question I want you guys to work on is, we've been practicing drawing models or pictures so how could we draw either a picture or draw a number line that reflects what you guys did with the sixteen and one-half and the fifteen and seven-eighths? Okay so, I want you guys to work on that.

STUDENT: We already did it.

MALLORY WILLIAMSON: You already did it?

STUDENT: Yeah.

MALLORY WILLIAMSON: Where?

STUDENT: Um, here.

MALLORY WILLIAMSON: Okay, so If I were to take a look at this, you did all these individual amounts, how do I know what six and six-eighths looks like?

STUDENT: We have to draw every single one?

MALLORY WILLIAMSON: If you want to do a picture. But you could also do a number line. So, with the number line how could we maybe draw a number line if you don't want to draw a picture?

STUDENT: We could—I don't know.

STUDENT: So, we could just label it from zero all the way to twelve and put five-eighths—

STUDENT: You'd have to put it all the way to—

STUDENT: Twenty-five. No we have to put it—

STUDENT : No, 17.

STUDENT : No, because we have 24. So, we have to put it all the way to like 3.

STUDENT: No, that's twenty-four-eighths.

STUDENT: I know but that's equal to 3.

STUDENT: That's 3 pounds, we have to—

STUDENT: But the last one would be a whole number. That's not a whole number.

MALLORY WILLIAMSON: Does the number line always have to start at zero?

STUDENT: No.

STUDENT: No, it could start at the lowest number.

MALLORY WILLIAMSON: Which is what?

STUDENT: Four-eighths.

STUDENT: Four-eighths. So, we could start here.

MALLORY WILLIAMSON: Okay and I—what you could do is, we could focus on just modeling this part, so we could start a number line at fifteen and seven-eighths—

STUDENT: And then go to sixteen and four-eighths?

MALLORY WILLIAMSON: And show me how you can go from fifteen and seven-eighths to sixteen and four-eighths.

STUDENT: Okay, that's easier.

MALLORY WILLIAMSON: Okay, so try that.

STUDENT: Okay, so we're gonna go from 15—

MALLORY WILLIAMSON: All right. So, can you come on to the carpet, please? Okay. The last part I'm going to go through, 'cause we're wrapping this up, but I want you to turn to someone who's not in your group. Make sure you're working with someone not in your group. Make eye contact with them, and share what was the total that you discovered for all of the plots for number three.

STUDENT: And then I made it into a [inaudible] and then I got sixty-fourths, and then I added that to the one I got up here because this is just half of the number line. I added this part of the number line, and then I took sixteen and a half and I made one-half of it four-eighths, so I subtracted fifteen and seven-eighths from sixteen and one-fourth and I got five-eighths.

STUDENT: I agree.

MALLORY WILLIAMSON: For the sake of time, 'cause once again I want to focus on the modeling part, the total for all of the data is fifteen and seven-eighths. Okay. One important thing to know is that when we subtracted from sixteen and a half, or sixteen and four-eighths, there's something that should stand out to you right away. So, Christopher, what do you recognize before we even subtract?

STUDENT: Um, that you've got to be—You'll have to regroup.

MALLORY WILLIAMSON: Why do we have to borrow and regroup or rename our fractions in order to subtract?

STUDENT: Because if we subtract four-eighths minus seven-eighths, that would be negative-three-eighths.

MALLORY WILLIAMSON: So, right now, this would be a negative-three-eighths, and we're not in middle school yet. We're not dealing with negative integers, so we want to stay with a positive number. So we're going to borrow and regroup. Now, some of you decided to rename as improper fraction, or fractions greater than one, and that's okay too. So, if you were able to do that, that would simply be multiplying sixteen by eight and adding the four to that. Angel, did you do that? Did you rename to improper fractions? Or you just borrowed and regrouped? I think you just borrowed and regrouped. Okay. All right, so, this is where we get the sixteen and twelve-eighths minus fifteen and seven-eighths. Okay. Now, the extra fraction that we end up is five-eighths and I'm going to show something real quick. When we're dealing with the sixteen and a half with models, you can do this two ways. You can actually start on a number line. Now, I know that you guys have been taught this since kindergarten, but you do not have to start a number line with zero. I was talking to group five and the phrase that came out that I really liked what they said was, we can start with the least number. So, the least number that was being used when we were subtracting was the fifteen and seven-eighths. So, from here, we can create a number line and have it start at the fifteen and seven-eighths, and what do we need to go up to?

STUDENTS [collectively]: Sixteen and—

MALLORY WILLIAMSON: Sixteen and a half. So, when we mark where our sixteen and a half is, or sixteen and four-eighths, we need to know all of the space in between. So, how can we figure that out? Roberto?

STUDENT: You can add?

MALLORY WILLIAMSON: Hm?

STUDENT: That you can add?

MALLORY WILLIAMSON: Add. So, I'm going to take your advice and I'm going to add certain fractional amounts to make it easier for me to figure out, so fifteen and seven-eighths, what comes next?

STUDENT: Fifteen and—Sixteen.

MALLORY WILLIAMSON: Fifteen and eight-eighths, which is equal to sixteen. Okay. What comes next?

STUDENTS [collectively]: Sixteen and one-eighth.

MALLORY WILLIAMSON: Sixteen and one-eighth.

STUDENTS [collectively]: Sixteen and two-eighths.

MALLORY WILLIAMSON: And...?

STUDENTS [collectively]: Sixteen and three-eighths.

MALLORY WILLIAMSON: Okay, so, from here, I can then decide to add or figure out the difference in between, so we've got one-eighth, two-eighths, three-eighths, four-eighths, five-eighths. Okay. Or you can either add or subtract to find your difference. Okay. Yes, Angel.

STUDENT: Can you divide?

MALLORY WILLIAMSON: What would you divide?

STUDENT: Sixteen and one-half.

MALLORY WILLIAMSON: By what? See, that, when you—

STUDENT: Five-eighths.

MALLORY WILLIAMSON: By five-eighths. So, if you divide sixteen and five-eighths, you're dividing them up into groups. We just want to know the space in between or the difference, okay? I'm not really placing each individual amount into a group, okay? Yes, sir?

STUDENT: I found my answer in a different way from that.

MALLORY WILLIAMSON: Okay.

STUDENT: How I found my answer is, when I was doing the math, um, I had two, um, five—five-eighths plus seven-eighths, and when I added that, it was six and four-eighths, so the five-eighths and seven-eighths gave—added one and a half, and then the fifteen had seven-eighths, just as before.

MALLORY WILLIAMSON: Mhmm [affirmative].

STUDENT: So, I thought, well, if I add five-eighths, that would add the one we needed to get to sixteen and the four-eighths, which is one-half.

MALLORY WILLIAMSON: So, you grouped the numbers that were easier to add first? And then worked your way up. That was a good strategy. The last question I want to use real quick is where you plotted that. So, Jaadiay, can you grab your line plot for me real quick? Or, Mister Greenlee. Thank you. [Inaudible]. Too many Mikes. Uh, okay, so the question was, if you have five-eighths, where would you plot that? So, group five, can you three talk to me about where you decided to plot your new marks?

STUDENT: Um, where it's two-eighths and three-eighths.

MALLORY WILLIAMSON: So, you guys see where the two-eighths is and the three-eighths is. So, what's two-eighths plus three-eighths?

STUDENTS [collectively]: Five-eighths.

MALLORY WILLIAMSON: Five-eighths. What's another way we could have plotted? Drake?

STUDENT: You could have put two marks on one-eighth and another mark on the three-eighths.

MALLORY WILLIAMSON: So, I could have put two marks on one-eighth. That's two-eighths total. And then another one on three-eighths. Angel?

STUDENT: Six-eighths and twenty.

MALLORY WILLIAMSON: So, you get five-eighths. So, I—Because, when I place a mark, I'm actually adding them up. Okay. I wouldn't know how—if that would—I wouldn't know if it was a subtraction because that would be taking something away. So because I'm plotting it, I'm adding to it. So, if you have another way we can create five-eighths, Angel.

STUDENT: One-eighth plus four—No.

MALLORY WILLIAMSON: Yes.

STUDENT: Two-eighths plus three-eighths.

MALLORY WILLIAMSON: Which is what that group decided to do.

STUDENT: One-eighth plus three-eighths.

MALLORY WILLIAMSON: So, I could put a mark—an x or a plus on one-eighth and one on the four-eighths. Okay. For the sake of time, thank you so much for hanging in there. If you guys would carefully make your way back to your seats for me. One person from each group, collect your assignment and turn it in to the basket. Make sure your name and date is on it.

MALLORY WILLIAMSON: A surprising fact was a lot of the students, when we're discussing how do we create a line plot, they had to refresh their memories about how to rename to equivalent fractions. Oftentimes they were getting the concept of a line plot and a number line confused. And you can create a number line to then create a line plot, but the purpose of a line plot is to plot the data that's been provided to you. So, having those discussions between listing the amounts from least to greatest, and you don't necessarily need all the eighths that were included when they listed them. However, if they did include them, it was acceptable. So, I was surprised to see that, and I was also surprised how a lot of students decided to draw pictures or models to support their answers, but at the same time a lot of them did not favor a number line, which is very similar to a line plot. So, when they went to go explain or model their—like how they found their solution, they were very hesitant to even try a line plot as a strategy.

I want to spend more time on how using a number line to find a solution. I think that's a valid strategy that can be used in fifth grade. A lot of the students easily were able to connect repeated addition to multiplication, so I don't need to spend a whole lot of time on that, but I do think that spending more time on the model factor or how to analyze a model or create a model, using multiple amounts of data. I think it's easy for them to model what one-third plus, you know, two-thirds is, or something that's like that with, you know, same denominators or like-denominators, but to have all of this data and then create a model, they don't necessarily know what's the most efficient way to do that.

It was interesting to see one of the groups start off with they know that halves is equivalent to fourths, and fourths is equivalent to eighths. So ultimately, they're going to have to create a number line in eighths, so a lot of the students started off with the halves or fourths before then going back and editing their line plot and recreating them into eighths. Another misconception or something that I saw that was a great discussion piece was, in one of the problems, in the first problem it asked you to repeatedly add three-fourths four times, or three-fourths and multiply it by four. So, a lot of the students, when they ask you, or you ask them, "Do you know how to model three-fourths?" they can easily show you that representation, but then to repeatedly add it, you have to keep drawing. One group shaded in what the answer would be, twelve-fourths, and could show you how and explain how they labeled each part, and one group actually drew a array model four times, and shaded in three parts out of the 4, four different times.

So, making that connection's really hard for them. If you are going to shade in repeatedly and you don't want to draw all of those arrays, we recommend using different colors to kind of understand that concept and how repeated addition is used to find that solution. But when you're using all the same color like a pencil, it's hard to see where you're repeatedly adding three-fourths over and over again. So, some groups decided to use that particular model, three-fourths, as that array over and over again, and other groups decided to draw the solution and label how they were able to connect the dots from three-fourths to the twelve-fourths, which is equal to three.

I think for lower-level learners, the advantage for separating your arrays where you can easily see the connection, "This three-fourths is modeled this way," and so they're able to see, "If I write out three-fourths four times, I can easily see four different groups of three-fourths." When

you shade all of them in at once, whether you're using different colors or labeling them, I think someone who is quick with their fluency is able to understand each group, "This is one group of three-fourths, and three-fourths, six-fourths, nine-fourths, twelve-fourths." So, sometimes I ask them to explain or label a little bit more in detail so they actually understand what is taking place, the concept that's happening, because in their mind they're counting by groups of three. So three, six, nine, twelve, but we're also dealing with fourths. So, I think for someone who wants a very detailed basic understanding it's easier to separate those arrays, but for someone who just wants to understand the concept of quickly multiplying or quickly, efficiently going by those groups, they're going to count by groups of three, and clump them together so you understand the relationship of what's being asked and how to find the solution.

When I went to a particular group, they had tried multiplying three-fourths times four, and one of the students had placed twelve down, and then another one of her peers decided to record twelve-fourths. When I asked what is it going to be, twelve-fourths or twelve, she then didn't necessarily understand—she thought she was wrong. She wasn't necessarily wrong, but there is a major difference between twelve as a whole number and twelve-fourths, which is equal to three. So, I needed her to reason through the difference between just placing twelve as our solution versus twelve-fourths, which is actually—when you show the model, you're only shading in three wholes. So, she easily was able to, like, recognize that mistake after the discussion, but if we were just to leave it, she would just write down twelve was her answer, and ultimately be incorrect.

In fifth grade, one of the important things that I really like to kind of stand by is, as I'm monitoring each group, I really like to visit each group and kind of tell myself that I'm going to ask at least, like, four to five questions before giving a statement. They're really eager to know whether they're right or wrong, and we're trying to refocus that and re-train their minds with the growth mindset of, "Just because I ask a question doesn't necessarily mean you're wrong." I want you to think about, "Okay, how could I use this to then spark an interest or the next step in my thinking, or better understand relationships?" So that's really important for me to ask those questions and have them feel comfortable with me asking questions so they understand that I'm not going to give them an answer. I want them to be able to find that solution to the problem and strengthen their problem-solving skills.