

BECCA SHERMAN: You know, it was impressive how eager they were, the students were to share out about multiplication and division. Um, what was going on in my head is sort of this stuck notion with the two's and not really, was hard to get them to expand even when I said, "Could that work with a different number?" and there were a lot of like, "No! Oh. Hmm!" Um, the idea of equal groups didn't come up naturally and, so I tried to put it out there a little bit and then gave a little heavier hand so that that would come out throughout the lesson, to bring in this idea of "Oh they are equal groups, hmm." You know, maybe that's important. Two things...something that I noticed that happened in both lessons was, they both brought up...the first thing they said about multiplication is, "it's like addition" and the first thing they brought up about division is, "it's like subtraction." And that to me was very much something that they were told, they couldn't back up at all. The more I thought about it, I almost said out loud "it's hard to explain how division is subtraction" because we don't tend to subtract iteratively like that, we don't use repeated subtraction. It's not a strategy I find very useful but for whatever reason, our kids, both groups of kids, and maybe we can condition them to know, "oh, this is sort of like subtraction." So another to investigate is why do we...is that useful information for them. They like to show that connection of multiplication and division. Because I was looking for the idea of groups when they couldn't explain the division and equal groups, I did chose to extend that warm-up longer, that sort of opening longer to try and push them into this idea of groups and let them explore, which I thought was pretty cool, how they came up with all the different types of groups, equal groups. Still they weren't using...I felt like I was putting on them, that, "Oh they're equal." When they were doing their drawings, that felt really good to me, that they were engaged and they were all thinking, and they...it was compared to that first lesson that I thought, "Oh my gosh, engagement" versus that start, like "I was just told what to do and I don't know what it is." Because they weren't given explicit instructions except draw it, draw something, yeah, they were thinking about it, they wanted to start with numbers and I just said, "Okay, put down your numbers first and then go ahead and make the drawing that fits with that." I liked it, it came up that there were no questions but they were already starting to think of answers to questions, "Oh, this was my answer, it's 8 times 3" or "this is my answer, Wayne would have \$8." Um, and then...so then I had to make a decision about where to go with how to share all those out because they were so different, and some of the kids did have the misconception of 24 times 3. So ultimately for time and other things, I decided let's get this model up here, they've looked at some other models, let's just see how they react to this particular model because that was our goal. And in doing that, the big AHA for me was, and I just looked at the papers, approximately half of them put, did use it incorrectly and wrote 24 three times to find out that Wayne had \$72, three times as much.

NORM FORBERT: I was working with, um, one of the students put 8 in there, came up with the answer of 8 but the other four at the table had 24. And so he switched it and I asked him why he switched it, and he couldn't explain it to me. So I asked him, "Was it because the others had 24?" And there was this dead silence, so I said, "Maybe you should trust your own thinking." But sometimes peer pressure has to do with it. And so the same question would be, it seemed like the other half of the classroom had it correct, and I wondered how many of the children understood it and how many had copied the correct answer.

LINDA FISHER: Most of the people I observed had the 24, 24, 24, and so, then Wayne also had 24. Um, and they did like, cookies, they did little squares, they did three Maria's with lots of hair, even though you told them not to do that, and the little Wayne picture. But somebody called Jarelle made three rolls of eight dots and then circled them, and he had it just like a text book solution, 24 divided by 3 is 8, Wayne is \$8, Maria is \$24. And then another little girl did eight 3's, so we know that it's...she did have the right answer, that Wayne had \$8 but it's really hard if Maria is 3 times as much as Wayne and he used the model this way instead of three groups of 8. How do you ever figure out what Wayne got? So, so it was just sort of a troubling picture because it doesn't show that mathematical relationship, it shows you just kind of a number sentence that equals 24.

BECCA SHERMAN: I asked her what...I don't remember the exact question I asked her but "What does this tell you?" And I said...oh, I think I may have said, "I noticed you said that Wayne has \$8, how does your picture tell you that?" And she said, "Well there are 8 boxes." So my...I agree with what you're saying. So she used the mathematics, understanding the mathematics to try to draw the diagram, then to represent it. And so, um, the 8 boxes doesn't make sense in the same way as having \$8.

LINDA FISHER: And then one little guy had 24 circles with 3 dots in each circle, which obviously isn't the relationship that you wanted, but he really got the idea of the pretense strategy. He goes, "Well, you know, I can make it cookies, I can make it pizza, it doesn't matter what group I choose for making my picture because the math will be the same," which, you know, that is a good mathematical concept that you can make it any friendly thing that you what to draw. So, I kind of like that but... And then the final thing that I just want to say is um, there was one table that pretty much all had the four 8's, and all four of them put 24 here too. And it wasn't until you then uncover the question, that one of the three changed their 24 to the 32. And all of them kind of wrote this sentence with the 32 in it but nobody, only one out of the four went back to change the number in this model. So they weren't really understanding kind of what this box out here was.

MEGAN MARTINS: One of the boys just was stuck in the 24 plus 24, plus 24. He actually, initially started with 24 times 3 and then got 62, and then got wrapped up in the whole multiplication of that but he insisted it was 62. And then he added different ways, tried to break numbers up and someone else in the group was like, "No, it's 72 and let me show you why." They were so wrapped up in that, in solving just that multiplication problem that the rest just...they couldn't follow the rest of the lesson. But one of the girls in the group, she drew out the bars and they were connected, but for Maria she had 24 in each box and for Wayne she had 8. When I asked her if she could explain how she knew Wayne had 8, she said, "Well, 8 times 3 is 24." And I said, "What does the 8 and 3 mean?" And she couldn't explain it. So there was a lot of misunderstanding at that table group.

LINDA FISHER: To, you know, not just teaching that process of multiplication but really trying to get kids to make that connection to equal groups, and then that's so critical for them in understanding what a unit is and to be able to do algebraic thinking. So you really want, you know, how do we get that connection when they first start making sense of multiplication, to really think about those equal groups so you don't have to force it out of them, or...you know, I think I had to even introduce the word "groups" to get them to say anything at all.

NORM FORBERT: It's really important that they build their algebraic and mathematical vocabulary. They just don't have it and it has to be repeated to them and they have to repeat it to feel comfortable with it. And I think that was the major part of what happened with reading the story problems, not cueing onto the wrong cues, and understanding the vocabulary and being able to use it.