TRACY SOLA: I just heard a really interesting conversation. Monica said that Ariana has the biggest feet because she has twelve.

STUDENT: My little feet.

TRACY SOLA: And then Salvador said, "No, she has the smallest." So now we have two different opinions --

STUDENT: Small.

TRACY SOLA: Monica thinks she has the biggest feet because she has the biggest number. And why do you think she has the smallest feet, Salvador?

STUDENT: Because of the ... the bigger that it is the --

STUDENT: The bigger it gets.

STUDENT: Hmm ...

STUDENT: Because the, because the bigger it is, then it's, the smaller that it is.

STUDENT: Oh yeah!

TRACY SOLA: The bigger the number, the smaller the feet?

STUDENT: Mm-hmm. [affirmative]

STUDENT: Yeah.

TRACY SOLA: Why?

STUDENT: Because they take little steps and, and then, um, they, they leave gaps.

TRACY SOLA: They take little steps and they leave gaps.

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: Does anybody else have something to say about that? Santiago, yes?

STUDENT: Some, some people are leaving gaps and some people are not.

TRACY SOLA: Some ... what do you mean some people are leaving gaps and some people are not? What do, what does that mean?

STUDENT: That means when -- when -- if somebody's leaving a gap, that means, that means that, that, that, that's, that somebody is, is, is, um, um ...

TRACY SOLA: Do you think you could sh--

STUDENT: Um, leaving a gap and, and that's why it's getting smaller.

TRACY SOLA: And tha--

STUDENT: The number.

TRACY SOLA: The, the number's getting smaller if you leave a gap?

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: Can you show me what you mean by "leaving a gap"? Can you walk on this line and showed me what you mean by that?

Oh, so if you take a step and there's some space between your feet, that's a gap?

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: And so if somebody leaves a gap, you think the number gets smaller?

STUDENT: Mm.

TRACY SOLA: What do you guys think about that? Santiago just said that some people are leaving gaps and so their number gets smaller.

STUDENT: I get it.

TRACY SOLA: Well, can you tell me? What that, it means?

STUDENT: It's like --

TRACY SOLA: I'm trying to ...

STUDENT: It's like --

TRACY SOLA: ... make sense of that.

STUDENT: They have, like, the ... like, they have smaller feet and since they have smaller feet, they make little, tiny ... like, they make small places so they --

STUDENT: And, and the number ---

STUDENT: So they make, so make it, to make it a lot.

TRACY SOLA: Oh, so if they have small feet, they take little tiny steps?

STUDENT: Yeah.

TRACY SOLA: And then their --

STUDENT: And then their amount is like, is 12.

TRACY SOLA: Oh, and then their amount is more.

STUDENT: Yeah.

TRACY SOLA: It takes more steps if you have small feet?

STUDENT: Yeah.

TRACY SOLA: Hm. What do you guys think of that? What do you think of what Kylie just said? If you have small feet ...

STUDENT: It makes up, it makes like, bigger, I mean small, like smaller places.

TRACY SOLA: It, it, it takes ... your, your feet ---

STUDENT: Watch, like, if my feet was like this big ...

TRACY SOLA: Yes.

STUDENT: I make more steps.

TRACY SOLA: It takes more steps if your feet are smaller?

STUDENT: Yeah.

TRACY SOLA: Do you agree with that?

STUDENT: Yeah, I agree.

TRACY SOLA: You agree? Me too? Thanks, Kylie.

Now, so there were a few things. There's that, and then Santiago's talking about gaps. Why would the number be smaller if there are gaps?

STUDENT: That makes no sense.

STUDENT: I already know.

STUDENT: Wait, what?

STUDENT: That makes no sense.

STUDENT: That makes sense.

TRACY SOLA: That makes no sense or that makes sense?

STUDENT: Sense.

STUDENT: That makes no sense, because if you leave gaps, that, that makes the number more s--

STUDENT: Smaller.

STUDENT: Tinier. And then you're saying that it makes the number more bigger.

STUDENT: No, I said smaller.

STUDENT: [Frustrated sigh] Let me show you. Let me show you again.

TRACY SOLA: So, what do other people think? Why don't you talk to your partner about this, decide what we think about this.

STUDENT: I'm confused.

STUDENT: Yeah, I'm confused too.

STUDENT: Okay, she agrees with me.

STUDENT: Well I'm not confused.

STUDENT: Um, I agree with Kylie. [crosstalk]

STUDENT: They, since they have smaller feet, if my feet was like this big, then I'll make smaller steps and I will make, and I'll make, like, more. No, more!

STUDENT: Wait. Wait. You say it.

TRACY SOLA: Does anybody else have anything they'd like to add to the conversation?

STUDENT: I do.

TRACY SOLA: What would you like to say, Santiago?

STUDENT: Um, if, if you le-- if ... the more gaps you make the, the smaller your number gets.

TRACY SOLA: Why?

STUDENT: Because if, if you leave more gaps and gaps, that means if, you won't have like, like enough, you, so, so you don't have to leave -- you, you do not need gaps because if you do, if you do the gaps, then, then, then, then your number will get smaller and smaller.

TRACY SOLA: If you leave gaps your number will get smaller and smaller? Well let's try that out. Remember how many I did the first time?

STUDENT: Yes, I do.

STUDENT: Yeah.

TRACY SOLA: How many did I do the first time?

STUDENT: Eight.

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STUDENT: Eight.

STUDENT: Eight.

TRACY SOLA: Eight.

STUDENT: Eight.

TRACY SOLA: I'm going to leave some gaps. Let's see how many it takes. Ready?

STUDENTS: One, two, three, four.

TRACY SOLA: Did my number get smaller when I left gaps?

STUDENTS: Yes.

STUDENT: Yeeeeeees.

TRACY SOLA: I think you're, I think you're right, then. I think that's true.

STUDENT: Ow.

TRACY SOLA: Okay. So there's something when we measure called accuracy. Can you say "accuracy"?

STUDENTS: Accuracy!

TRACY SOLA: Have you heard that word before?

STUDENTS: No.

TRACY SOLA: That words means that you're very careful, you're very careful not to leave gaps. That you're very careful to be accurate.

STUDENT: Accurate. I'm gonna --

TRACY SOLA: Very careful to be accurate. Can you say accurate?

STUDENTS: Accurate.

TRACY SOLA: So when we measure, we will be very careful to measure carefully and not leave gaps so we know what it really is.

TRACY SOLA: When the students were walking across the tape and seeing how many footsteps it would take to get to the end of the tape, most of the students were very careful to go, uh, heel to toe, heel to toe. Um, there was one student who was walking with some big gaps, and right away the other kids jumped in and said, "You have gaps! You have gaps." And, um, so they were thinking about that naturally without me saying anything about it or anybody mentioning it, and that just seemed very natural for them.

The other thing that was really interesting in that lesson was, in the debrief we were trying to think about, um, for the same length of tape, if one person had twelve footsteps, and the other person had eight footsteps, what did that mean? And several students wanted to say that if you have more footsteps, then your foot is bigger because there was that more and bigger correlation. But that's a counterintuitive idea and so that idea that if you have, um, bigger feet it takes less footsteps, what took a while for the group to come around to. But there were some people who understood that and so it took a little bit of peer discussion and group discussion to get to that idea. So that, that was really interesting. You could see the wheels turning and you could see them thinking about that apparent contradiction and why that would be, and then we had, I think Kylie came up and showed us very clearly, by stepping on the tape, why that was and, um ... I think it was Kylie, it might have been another student, but that was very interesting.