TRACY SOLA: This set of activities is part of a formative re-engagement lesson for 1st graders about non-standard measurement. And so, in all of the activities, students will use non-standard measurement items, cubes, feet -- they could use many things in life, you know, paper clips, hands, anything, to measure something. We will primarily be using feet and cubes in these lessons to measure things, to measure how long things are, and to think about what that means, and -- and just the -- the big ideas of measurement, and -- and what does it mean if we are all measuring with our feet, but our feet are different lengths. And, you know, so -- so surfacing some ideas about length, small, medium, and large. And that will be interesting to see what happens.

TRACY SOLA: This series of lessons is a part of a formative re-engagement lesson that is about non-standard measurement for first graders. So, one of the first things that I will do with them, is to establish the vocabulary of measurement. And we will be thinking about long and short as um, discrete things to describe something, but then we will also think about longer and shorter as words that can compare two or more objects together. Later in the lesson we will think about taller and shorter, and so we will think about how the word "short" is used in length, but also in height. And so that will be interesting.

This lesson starts with an activity where the students are walking off their footsteps on a length of tape and then measuring their uh ... how many steps it took to get to the end of a tape and then recording that on a piece of paper. And so what I will be looking for is how do they walk that tape? Do they walk that tape uh, heel to toe, heel to toe, with no gaps? Or do we see some gaps in their steps, um, do they notice that there are gaps? We will think about what that means. Um, we will go on to think about once everybody has recorded how many steps it took to walk across a piece of tape, we will think as a class about why did some people take more steps? And what does that mean? So it'll be really interesting to see what the kids come up with there.

TRACY SOLA: We are going to start the day by thinking about long and short. Long and short. What does long mean? Who thinks they might know what long means? Salvador.

STUDENT: Big.

TRACY SOLA: Big. And what does short mean, Jade?

STUDENT: Small.

TRACY SOLA: Small. Okay, so if we say long, that means it's big. And if it's short, it's small.

STUDENT: Yep.

TRACY SOLA: Now what if I say something is longer? What if I say it's longer, Jaira?

STUDENT: Me?

TRACY SOLA: Yes.

STUDENT: Um.

TRACY SOLA: Instead of, if I say something is long then you can say, oh that's long. But what if I say this is longer than something else, what does that mean?

STUDENT: Longest. It means longest one.

TRACY SOLA: The longest one.

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: So long, we say longer when we're comparing it to something else, right?

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: Yes, okay. So, we will be thinking today about longer.

STUDENT: And shorter.

TRACY SOLA: And, exactly. Shorter. So, the first thing we are going to do today--

STUDENT: Is longer.

TRACY SOLA: Well, we're going to do them both.

STUDENT: [laughs]

TRACY SOLA: We're going to be thinking about longer and shorter kind of all day. Um, you will need a partner, but we don't have an even group of people here so there will be one group of three. So can you two be partners? That's okay, and you two can be partners. And you two and you three boys.

STUDENTS: Yeah. Three.

TRACY SOLA: And Bryan, can you two be together? Is that okay? Great. Excellent, so you have your partner for this activity. This is what we're going to do. You need to work with your buddy, and do you see this piece of tape on the ground here?

STUDENTS: Yeah.

TRACY SOLA: Okay. There's one here and this is tape C, and here's poster C to go with it and there are some markers here, in the corner. Okay. Um, over in that corner there is a piece of tape on the ground. That's tape A and there's a poster A to go with it and the markers are just on the floor under tape -- or poster A. And then over in the back of the room is tape B, and I need to close the door and poster B will be on the door.

STUDENTS: Okay.

TRACY SOLA: And the markers are just down in the corner on the floor. Okay. So, I know you know this. When you get a marker and you use it, what do we need to do when we're done?

STUDENTS: Close the top really good.

TRACY SOLA: And you know how you can tell it's really good? Listen. Did you hear that click?

STUDENTS: Yeah.

TRACY SOLA: So you will hear a click and you'll know you've done a good job, yes?

STUDENTS: Mm-hmm. [affirmative]

TRACY SOLA: Great, okay. So, here's what you're going to do. Um, let's see. Ricardo, will you be my partner for one second so we can show the kids what we're going to do?

STUDENT: [nods head]

TRACY SOLA: Okay, come on up here. So, since you're my partner, I'm going to see how many of my footsteps long this piece of tape is. And so I'm going to walk on here and when I do that, Ricardo ,who's my buddy, my partner, is going to count my steps for me. And when I get to the very end, we are going to go over there together and I'm going to write my name and he will remind me how many steps and we'll put that number. And then we'll come back and we'll switch and Ricardo will have his turn. Can we show you?

STUDENTS: Yeah. Yeah.

TRACY SOLA: Okay. Are you ready to count for me? I'm gonna start here.

STUDENT: One, two, three, four, five, six, seven, eight.

TRACY SOLA: Okay. Now we come over here, and I will write ... Ms. Tracy and how many steps did I have?

STUDENT: Seven.

TRACY SOLA: Seven?

STUDENTS: No, eight.

STUDENT: Oh.

TRACY SOLA: Eight. Thank you though, it's a good thing you counted, they would never would have known. So, I had eight steps. Okay, so that's what I did, and now it's your turn and I'm going to count for you, and I heard my click. That was good.

STUDENT: Yay.

TRACY SOLA: Yay, okay. Now you come on over here, and you're going to step, ready? One, two, three, four, five, six, seven, eight. We got the same. Okay, now why don't you write yours. And then I remember you got eight, so you can write an eight over there. So, remind your partner if they forget to write their number, remind them to write their number because you, you're helping each other, right? Great. Thank you, Ricardo. That -- thanks for being my buddy for a second. Okay, so everybody understands what we're doing?

STUDENTS: Yes.

TRACY SOLA: We are going to do this for maybe ten minutes, so you'll have time. What we'll do is we'll start some of you here, some of you there, and some of you there. When you finish at this line, I want you to go over to that line.

STUDENT: To B?

TRACY SOLA: To B, yes, please. And when you're finished at line B I want you to go to line A. When you're finished at A I want you to come to ...

STUDENT: Rug.

TRACY SOLA: C, line C.

STUDENT: Again?

TRACY SOLA: Well, if you haven't done it yet. We won't all start in the same place, because then there would be a traffic jam, right?

STUDENTS: Oh.

TRACY SOLA: Yes. So, okay. Why don't we have Bryan and, can you move your hair so I can see your name? Adaia, yes, I remember. Bryan and Adaia, you can start at C. And why don't we have you three boys start at C, and Ricardo's already done his. Um, let's see. How about all of you girls go over to B and you ladies over to A, and get going. And let's shut that door ... because the chart is behind there.

STUDENT: Two, three, four, five, six, seven, eight, nine, ten.

STUDENTS: Two, three, four, five, six. Six.

STUDENT: Because your feet is more bigger than us. Yeah, you can get any color.

STUDENT: Wow, you're so close to me in seven. You're crazy, you're so funny.

STUDENT: Nine, ten.

STUDENT: Eight, nine.

STUDENT: Three, four, five, six. Six.

STUDENT: Four, five, six, seven. One, two, three, four.

STUDENT: How many did you got?

STUDENTS: Five.

STUDENT: Oh yeah.

STUDENT: Oh yeah, now me.

STUDENT: It looks better than mine.

STUDENT: I don't.

STUDENT: That looks like an A.

STUDENT: Do you have a hat? I wish I had.

STUDENTS: Four, five, six, seven, eight, nine, ten, eleven, twelve.

TRACY SOLA: I think it's really interesting that we all got different numbers, because if we're all taking steps on the tape, how come our numbers aren't the same? Why are our numbers not the same? Why don't you turn to a partner, right now, and talk to them about that. And think about, why are the numbers different?

STUDENT: Because our feet are shorter, this will help decide that we get bigger numbers.

STUDENT: Bigger?

STUDENT: Because it will take longer for us to get to the end, that's why.

STUDENT: It's like, this, that means, um, it will get smaller. When it's like this, then it will take a lot of steps.

STUDENT: Um, some of them are like um, different numbers. And some of them are--

STUDENT: Can't tell.

STUDENT: Shh.

STUDENT: And he did?

STUDENT: Yeah.

TRACY SOLA: So I heard some really interesting things out there. Who has an idea? What did you talk about? Why are they all different numbers? Why? Monica?

STUDENT: Because um, um, our friends have big feet, and some got medium feet, and some got little feet.

TRACY SOLA: Some of our friends ... Oh, okay. Turn to your partner and say what Monica just said.

STUDENTS: Monica said, "Some have big feet, and some people are medium, and some have little feet, [inaudible]."

TRACY SOLA: Great. Thank you. So, Monica said that, "Some friends have big feet, some friends have medium feet, and some friends have small feet." And so, by looking at these numbers, can we tell who have big ... has big feet, who has medium feet, and who has small feet?

STUDENT: Yeah. I have.

TRACY SOLA: How can we tell?

STUDENTS: Well, the numbers. By the numbers.

TRACY SOLA: Tell me more about that. Can you tell me more about the numbers? Who, who do you think on this chart, right here, has bigger feet? And, who do you think has smaller feet?

STUDENTS: Hmm.

TRACY SOLA: Hmm. Look, should we think of that, for a minute?

STUDENTS: Yeah.

TRACY SOLA: Let's ... Why don't ... Why don't I give you a moment to think about that, because there are different numbers up here.

STUDENT: Six?

TRACY SOLA: I see here a seven, a seven, a six, a seven, a seven, a six, a six, a six, a six, a seven. In fact, look, look at this one. There are lots of different numbers here, and here, too.

STUDENT: Jaycee has twelve.

TRACY SOLA: Miss Tracy has eight. Ricardo has eight here. Bryan has ten. Salvador has six. Santiago, I don't know how many ...

STUDENTS: Ten.

TRACY SOLA: Ten. Okay. Kylie has ten. Adaia has eleven. Jade has ten. Ariana has twelve. Jaira has nine. Jaycee has twelve. And Monica has ten. How can we tell who has larger feet, and who has smaller feet, by looking at those numbers? Talk to a partner about that, please.

STUDENT: If you have small feet, then you get bigger numbers because, pretend the line is your feet. And, this is my feet. And I'm gonna make big feet, one, two, three, that only took to three. And now, a small feet, a small feet, one, two, three, four, five, six, seven, only took seven steps. Because small feet was even smaller than the big feet.

STUDENT: I have to go.

STUDENT: Yeah, because it took a short ... The big feet took a shorter time to get there and the small feet took a longer time to get there. Yeah.

TRACY SOLA: I just heard a really interesting conversation. Monica said that Ariana has the biggest feet because she has twelve.

STUDENT: My little feet.

TRACY SOLA: And then Salvador said, "No, she has the smallest." So now we have two different opinions --

STUDENT: Small.

TRACY SOLA: Monica thinks she has the biggest feet because she has the biggest number. And why do you think she has the smallest feet, Salvador?

STUDENT: Because of the ... the bigger that it is the --

STUDENT: The bigger it gets.

STUDENT: Hmm ...

STUDENT: Because the, because the bigger it is, then it's, the smaller that it is.

STUDENT: Oh yeah!

TRACY SOLA: The bigger the number, the smaller the feet?

STUDENT: Mm-hmm. [affirmative]

STUDENT: Yeah.

TRACY SOLA: Why?

STUDENT: Because they take little steps and, and then, um, they, they leave gaps.

TRACY SOLA: They take little steps and they leave gaps.

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: Does anybody else have something to say about that? Santiago, yes?

STUDENT: Some, some people are leaving gaps and some people are not.

TRACY SOLA: Some ... what do you mean some people are leaving gaps and some people are not? What do, what does that mean?

STUDENT: That means when -- when -- if somebody's leaving a gap, that means, that means that, that, that, that's, that somebody is, is, is, um, um ...

TRACY SOLA: Do you think you could sh--

STUDENT: Um, leaving a gap and, and that's why it's getting smaller.

TRACY SOLA: And tha--

STUDENT: The number.

TRACY SOLA: The, the number's getting smaller if you leave a gap?

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: Can you show me what you mean by "leaving a gap"? Can you walk on this line and showed me what you mean by that?

Oh, so if you take a step and there's some space between your feet, that's a gap?

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: And so if somebody leaves a gap, you think the number gets smaller?

STUDENT: Mm.

TRACY SOLA: What do you guys think about that? Santiago just said that some people are leaving gaps and so their number gets smaller.

STUDENT: I get it.

TRACY SOLA: Well, can you tell me? What that, it means?

STUDENT: It's like --

TRACY SOLA: I'm trying to ...

STUDENT: It's like --

TRACY SOLA: ... make sense of that.

STUDENT: They have, like, the ... like, they have smaller feet and since they have smaller feet, they make little, tiny ... like, they make small places so they --

STUDENT: And, and the number --

STUDENT: So they make, so make it, to make it a lot.

TRACY SOLA: Oh, so if they have small feet, they take little tiny steps?

STUDENT: Yeah.

TRACY SOLA: And then their --

STUDENT: And then their amount is like, is 12.

TRACY SOLA: Oh, and then their amount is more.

STUDENT: Yeah.

TRACY SOLA: It takes more steps if you have small feet?

STUDENT: Yeah.

TRACY SOLA: Hm. What do you guys think of that? What do you think of what Kylie just said? If you have small feet ...

STUDENT: It makes up, it makes like, bigger, I mean small, like smaller places.

TRACY SOLA: It, it, it takes ... your, your feet ---

STUDENT: Watch, like, if my feet was like this big ...

TRACY SOLA: Yes.

STUDENT: I make more steps.

TRACY SOLA: It takes more steps if your feet are smaller?

STUDENT: Yeah.

TRACY SOLA: Do you agree with that?

STUDENT: Yeah, I agree.

TRACY SOLA: You agree? Me too? Thanks, Kylie.

Now, so there were a few things. There's that, and then Santiago's talking about gaps. Why would the number be smaller if there are gaps?

STUDENT: That makes no sense.

STUDENT: I already know.

STUDENT: Wait, what?

STUDENT: That makes no sense.

STUDENT: That makes sense.

TRACY SOLA: That makes no sense or that makes sense?

STUDENT: Sense.

STUDENT: That makes no sense, because if you leave gaps, that, that makes the number more s--

STUDENT: Smaller.

STUDENT: Tinier. And then you're saying that it makes the number more bigger.

STUDENT: No, I said smaller.

STUDENT: [Frustrated sigh] Let me show you. Let me show you again.

TRACY SOLA: So, what do other people think? Why don't you talk to your partner about this, decide what we think about this.

STUDENT: I'm confused.

STUDENT: Yeah, I'm confused too.

STUDENT: Okay, she agrees with me.

STUDENT: Well I'm not confused.

STUDENT: Um, I agree with Kylie. [crosstalk]

STUDENT: They, since they have smaller feet, if my feet was like this big, then I'll make smaller steps and I will make, and I'll make, like, more. No, more!

STUDENT: Wait. Wait. You say it.

TRACY SOLA: Does anybody else have anything they'd like to add to the conversation?

STUDENT: I do.

TRACY SOLA: What would you like to say, Santiago?

STUDENT: Um, if, if you le-- if ... the more gaps you make the, the smaller your number gets.

TRACY SOLA: Why?

STUDENT: Because if, if you leave more gaps and gaps, that means if, you won't have like, like enough, you, so, so you don't have to leave -- you, you do not need gaps because if you do, if you do the gaps, then, then, then, then your number will get smaller and smaller.

TRACY SOLA: If you leave gaps your number will get smaller and smaller? Well let's try that out. Remember how many I did the first time?

STUDENT: Yes, I do.

STUDENT: Yeah.

TRACY SOLA: How many did I do the first time?

STUDENT: Eight.

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STUDENT: Eight.

STUDENT: Eight.

TRACY SOLA: Eight.

STUDENT: Eight.

TRACY SOLA: I'm going to leave some gaps. Let's see how many it takes. Ready?

STUDENTS: One, two, three, four.

TRACY SOLA: Did my number get smaller when I left gaps?

STUDENTS: Yes.

STUDENT: Yeeeeeees.

TRACY SOLA: I think you're, I think you're right, then. I think that's true.

STUDENT: Ow.

TRACY SOLA: Okay. So there's something when we measure called accuracy. Can you say "accuracy"?

STUDENTS: Accuracy!

TRACY SOLA: Have you heard that word before?

STUDENTS: No.

TRACY SOLA: That words means that you're very careful, you're very careful not to leave gaps. That you're very careful to be accurate.

STUDENT: Accurate. I'm gonna --

TRACY SOLA: Very careful to be accurate. Can you say accurate?

STUDENTS: Accurate.

TRACY SOLA: So when we measure, we will be very careful to measure carefully and not leave gaps so we know what it really is.

TRACY SOLA: When the students were walking across the tape and seeing how many footsteps it would take to get to the end of the tape, most of the students were very careful to go, uh, heel to toe, heel to toe. Um, there was one student who was walking with some big gaps, and right away the other kids jumped in and said, "You have gaps! You have gaps." And, um, so they were thinking about that naturally without me saying anything about it or anybody mentioning it, and that just seemed very natural for them.

The other thing that was really interesting in that lesson was, in the debrief we were trying to think about, um, for the same length of tape, if one person had twelve footsteps, and the other person had eight footsteps, what did that mean? And several students wanted to say that if you have more footsteps, then your foot is bigger because there was that more and bigger correlation. But that's a counterintuitive idea and so that idea that if you have, um, bigger feet it takes less footsteps, what took a while for the group to come around to. But there were some people who understood that and so it took a little bit of peer discussion and group discussion to get to that idea. So that, that was really interesting. You could see the wheels turning and you could see them thinking about that apparent contradiction and why that would be, and then we had, I think Kylie came up and showed us very clearly, by stepping on the tape, why that was and, um ... I think it was Kylie, it might have been another student, but that was very interesting.

TRACY SOLA: I'm going to ask the students to all trace their footprint, and then we are going to compare the size of footprints and order our footprints. And I, so I'm also interested to see how they do that. Because um to get a true comparison, we would see them maybe lining up all the heels, uh, you know, at the bottom of something so that we can see how high the toes go. But, it's not clear to me whether the students will think to do that or not. It'll be interesting to see what happens there.

TRACY SOLA: So, are you ready for the next thing we're going to do?

STUDENTS: Yes.

TRACY SOLA: Great, okay. So, let me show you. You are each going to get a piece of yellow paper.

STUDENT: Yay!

TRACY SOLA: And once again, you are going to help your buddy. Okay, you're going to help your buddy ...

STUDENT: Buddy?

TRACY SOLA: And -- and this is what you're going to do. You're going to put down your piece of paper --

STUDENT: I know how to do it.

TRACY SOLA: You are going to take a marker, and I hope my ... you are going to put your foot on the paper, and you are going to trace all the way around your foot. Maybe your buddy can trace for you, you can help each other out. So, your buddy will trace for you ...

STUDENT: You painted your heel.

TRACY SOLA: Yep. And then inside of your footprint ...

STUDENT: We're gonna write our name.

TRACY SOLA: Yep. You're gonna write your name.

STUDENT: This is ... this is easy.

TRACY SOLA: And then the last thing you are going to do, is you are going to be as accurate and careful as possible, and you are going to cut out --

STUDENT: Your foot.

TRACY SOLA: Your foot. Well, I, why don't we cut out the paper cutout of our foot? How's that?

STUDENT: Yeah.

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TRACY SOLA: And when you're all done, whenever everybody on your team is done, because you're helping each other, right? Okay, you'll help each other trace? Then you, we will come back to the circle and we will think about our feet. Perfect. Okay. Why don't we have, let's see ... We've got a group of two ... why don't you ... um, Adaia, why don't you and Bryan go right there. Let's see Jaira and Ariana, to the back table. Um, oh, we need ... let's see ... How about you three boys here together. And I think all four of you girls can go to this table. I think you have the right amount of paper here. Well, your partner's going to help you. Okay, so you're just gonna come down here, and if you step on it, okay, just put your foot on there, and your partner will do it for you. There you are.

STUDENT: Now I need to cut it out. Kylie, you're the --

STUDENT: -- together. It's already done.

STUDENT: Again?

TRACY SOLA: Great. Santiago, are you ready? Let's go. I think we're ready. Okay, so this is what we're going to do. Um, I'm wondering if I can ask Jaycee, Bryan, and Monica to bring their feet up here. And I would like for us ... I would like for you to turn around and face your friends. There you go. Now, I would like for you to put your feet down on the rug in order of length. Okay, now I would like other friends to look, and can I ask you three now to sit back a little bit? Just sit right back here so everybody can see. Okay, I would like for everybody to look at what they did.

STUDENT: Oh, big, medium, small.

TRACY SOLA: Big, medium, small? Tell me more about that. Can you come and show me what you mean by big, medium, and small?

STUDENT: Because this one has a little bit bigger, this one is a little bit smaller, and this one's mo-- the most small.

TRACY SOLA: Okay, thank you, Kylie. So, I think they did something up here when they ordered them that was very useful, that was very useful. Why don't we put ours too down for a minute, okay? Let's put yours down. How can we tell which one is bigger and which one is smaller? Oh, why did you move it?

STUDENT: [shrugs]

TRACY SOLA: I thought that was a really good idea, but why do you think Santiago just moved his? Because I had it like that. And then when I asked who was, who's was bigger, he moved it down here. Why do you think he did that? Why was that a useful thing to do? Can you turn to your partner and talk to them about that? And you three can talk right here who are ... just talk to the person that's sitting next to you.

STUDENT: Because it was on the top and it's not even bigger than that one.

STUDENT: Because he's trying to compare.

TRACY SOLA: He's trying to compare them?

STUDENT: And he's trying to make it the biggest, he's trying to make it bigger.

STUDENT: Bryan's is bigger than mine. No, this one can go right here, can go right here, because this one is bigger than mine, and Jaycee's is smaller than mine. See? See?

TRACY SOLA: Okay.

TRACY SOLA: So what did you talk about? Why do you think, when I had it like this, why do you think Santiago moved it right down here to compare? Why, Jade?

STUDENT: Because your feet's taller and his is smaller.

TRACY SOLA: But how would we know that from here? Can't ...

STUDENT: His feet is lots smaller than yours.

TRACY SOLA: Well, he keeps moving it down here, which I think is a really good idea. Why do you keep doing that?

STUDENT: Because, because if I put it out there then, then it would not ... it would not mean that I would not be bigger because that's, that's, that's, because that's, that is cheating because I -- I do not do, do, do cheating.

TRACY SOLA: You do not do cheating? That's great news. Neither do I, I don't do cheating either.

Um, so, I'd like for us to look at these three. Wait, how were they before? Wait, we had them a certain way. And then we switched them. Are they ... oh. Now do we like it just like that?

STUDENTS: No!

TRACY SOLA: No, you had it a different way, Bryan. What did you do to them? Oh. Okay, what did he just do?

STUDENTS: Moved it!

TRACY SOLA: Why?

STUDENT: So they --

STUDENT: I know!

STUDENT: -- can be all the same height on the bottom.

TRACY SOLA: So they can all be the same height on the bottom. Why is that important?

STUDENT: I know.

STUDENT: Because ...

TRACY SOLA: Uh, because ...

STUDENT: So they can all almost be the same height. So we know which length. 'Cause if, 'cause if it wasn't the right length, 'cause it wasn't, if it wasn't the same length along the bottom then one of them would be cheating, on the bottom.

TRACY SOLA: Oh, they all need to start in the same place so it's fair?

STUDENT: Yeah.

STUDENT: Yeah!

TRACY SOLA: Oh, because there's no cheating allowed.

STUDENT: So it's fair and square.

TRACY SOLA: Fair and square. I like that. And that makes a lot of sense to me, and if we line them up on the bottom, then what? Okay, so can we now, one by one I will call on people and you will put your feet in the lineup. Where does, where would yours go Santiago? There. Can you decide where to put yours? There. Okay, what do you guys think? Salvador and Ricardo? Santiago just put his foot in the lineup. Do we think he put it in the right place? Okay.

STUDENTS: Yeah.

STUDENT: No.

TRACY SOLA: No, some people say yes, some p--. Why did you just move it, Monica?

STUDENT: Maybe it's, because it's bigger than mine.

TRACY SOLA: You think Santiago's is bigger than yours?

STUDENT: Wait.

TRACY SOLA: What do you think, Santiago?

STUDENT: Let me see.

TRACY SOLA: Do you think yours is bigger than Monica's? You don't.

STUDENT: No, they're not the same length on the bottom so --

TRACY SOLA: How can we tell?

STUDENT: Because they're not same at the, at the bottom.

TRACY SOLA: Oh, you put them the same at the bottom? Like that? And whose goes higher? It's, hmm, it's pretty close! Okay, so was it, Ariana, how about you add yours in?

You think yours is just a tiny bit bigger than Jaycee's? Okay, Kylie, go ahead. And then who's next? Ricardo, do you want to put yours in?

STUDENT: Ricardo's big.

STUDENT: Oh, that's bigger.

STUDENT: Ricardo's is bigger than yours.

STUDENT: No, I --

TRACY SOLA: Well, let's get, let's get them down. We'll do our best and then we can fix them if we want to.

STUDENT: I've got an idea. And I'm ... These ones, since they're, they're almost the same height, we go like this. And I go right here, and then Monica, I think, goes right here, and then that goes right here, and then Ariana goes here and then Jaycee goes here.

TRACY SOLA: Okay, thanks! Now, everybody back in the circle, please. Everybody back in the circle.

And Jaira, why don't you add yours in? Where should yours go? Right there?

STUDENTS: Yeah.

STUDENT: Maybe it should go right here?

TRACY SOLA: Do you --

STUDENT: In the middle? Of these two?

STUDENT: Maybe.

STUDENT: They're this big. Oh, this is good.

TRACY SOLA: Okay, who else? You want to go? Adaia, you want to go? Where does yours go?

STUDENT: Let me help you.

TRACY SOLA: Oh, I think she's got it.

STUDENT: Put it right here.

TRACY SOLA: Okay. Go ahead, Jade. Where do you think you should put yours, do you think? Uh, there?

STUDENT: I think! Yeah!

TRACY SOLA: Great. Salvador, I think you're the final foot. The final foot.

STUDENT: Melinna's going to be smaller.

STUDENT: Who's here? It's big.

TRACY SOLA: Where should I put mine?

STUDENT: You should put it here because it is the biggest.

TRACY SOLA: Can I put it here?

STUDENTS: No!

STUDENT: You can put it here.

TRACY SOLA: Why not?

STUDENTS: Because it's bigger. It's bigger.

TRACY SOLA: It's bigger? So, wow.

STUDENTS: Yeah.

TRACY SOLA: Great! This kind of reminds me of a graph. You, it almost reminds me of a, a graph. Which one is the longest?

STUDENTS: Yours.

TRACY SOLA: Which one is the shortest?

STUDENTS: Jaycee's!

STUDENT: And Ariana's.

TRACY SOLA: Well that was really interesting. So, I'm thinking back to when we did this.

STUDENT: Okay.

TRACY SOLA: When we did this, and we walked our feet on a ta-- we took footsteps on tape.

STUDENT: Yeah.

TRACY SOLA: To see how many footsteps it took to get to the end of a tape. And we said that if your feet are smaller, it takes more steps.

STUDENT: Yeah.

TRACY SOLA: Does that make sense when we look at this, these feet right here.

STUDENTS: Yeah.

TRACY SOLA: That makes sense? Hm. Okay, great.

TRACY SOLA: So what I found really interesting about this activity was when it came time to take their footstep cutouts and order them according to height, or length, um, three students put their footsteps down on the rug and one student immediately reached over and lined them up, um, on the bottom. So, as if there were an invisible line and the heels were all down right flush

against that line, and, um, the students were doing that naturally, but I, it was hard for them to articulate why they were doing it. They knew they needed to do it, but it was hard for them to say why it was important.

I then took, I didn't want to just say it, so I took my footstep and I took another student's footstep and I kind of put them, um, not even close to each other, and right away, the student, I think it was Santiago, pulled his down so once again the heels were aligned. But he could not articulate why, and I think, uh, after much discussion, we finally got to it, but I, it was interesting to see that that was a natural tendency but, um, but kids weren't really sure why they were doing it, they just did it naturally. That was very interesting. TRACY SOLA: The next lesson is a lesson where they will be measuring objects with cube trains. They will just walk around the room and explore. They have a cube train that is a certain length long and they will look for objects that are that long. So that's an exploratory activity that they could do all the time.

TRACY SOLA: I am going to give you and a partner a train of Unifix Cubes. Okay? And everybody ... you and your partner will have the same length of train. But you won't have the same length of train as another partner pair. And your job is to find things in the room--

STUDENT: Can I have blue? Are ours the same color?

TRACY SOLA: Hmm ...

STUDENT: Are they the same length?

STUDENT: No. Yeah.

TRACY SOLA: The same color or the same length. What have we been learning about today?

STUDENTS: Length.

TRACY SOLA: So, maybe the same length. Okay, so, if this is my cube train, I am going to walk around the room and try to find things that are that length. Okay? I have a question. What should we do ... Let's see, for example, hmm ... What should we do if something is in the middle of a cube? If it doesn't take up the whole cube? If it ... If it's all of ... Let's count... Well, let's count these. Okay, so this paper is not going all the way to the top of cube 12.

STUDENT: Go to 11.

TRACY SOLA: We should only go to 11? What do you all think?

STUDENT: 11.

STUDENT: 12.

TRACY SOLA: You think 12? Did you say? Some people think we should go to 11 and some people think we should go to 12. Why? How will we deal with parts?

STUDENT: Because -- because they're almost the same height, and if you took away this one then it's not gonna be ...

TRACY SOLA: Well, wait. Then it's not enough.

STUDENT: Then it's not enough. That's why.

TRACY SOLA: So you think that if it's not enough, we need to add another one?

STUDENT: Yeah.

TRACY SOLA: What do you think, Salvador? You had a different idea. You think we should count the 12th one? How come? I mean you -- you, oh wait, oh, you want to count the 12th one. You only wanted to say 11.

STUDENT: Yeah.

TRACY SOLA: Because it was 11 and some more? Tell me why you think that's a good idea.

STUDENT: Because 12 is bigger than 11.

TRACY SOLA: Because there's too much if we count number 12?

Okay, I'm not going to make a decision for us. I am going to let you and your partner decide how you want to deal with that. Okay? So, right now, let's see. How about I give you two these blues. Okay? Give me one second, here. Um, and before it's time to go, I want you and your partner to decide how many cubes you have in your train. Count them and make sure you agree. Count them and make sure you agree.

The only thing we are not going to measure are the things on that table. The only thing we're not going to measure are the things on that table. You can measure anything else. Let's see.

STUDENT: Are we gonna make some more of those?

TRACY SOLA: Yeah, later. We're going to do something. I give you three this.

What is ... what are the only things we are not going to measure in the room?

STUDENTS: That table.

TRACY SOLA: Anything on the table. That's going to sit there like it's invisible and it's not even there.

STUDENT: [Laughs]

TRACY SOLA: And we'll just pretend it doesn't exist right now. But you can measure anything else in the room.

STUDENT: Yay! What about the globe?

TRACY SOLA: If you can reach it, but without standing on a chair.

STUDENT: Oh.

TRACY SOLA: Or if you can ask me to get it down for you.

STUDENT: Oh.

TRACY SOLA: And then ... Do you want me to put the globe down somewhere where you can reach it?

Inside Mathematics

STUDENT: Yeah.

TRACY SOLA: Would that be fun to measure? Well that's going to be interesting.

STUDENT: Interesting.

TRACY SOLA: Why did I just say that measuring it will be interesting?

STUDENT: Because it might be so big or little, or *medio*.

TRACY SOLA: Because it might ... or -- or *medio*. Hmm ... it could be any of those. What else? It has a very interesting shape, doesn't it?

STUDENT: It might have a ... It's a sphere. It doesn't have ... it doesn't have um, the -- the ... it doesn't have any sides to count. It's only a sphere like a ball.

TRACY SOLA: Hmm ... it doesn't have any sides to count, it's a sphere like a ball. So that might be very interesting. I'll put it over here for you in case that's something you'd like ... Okay, why don't you get up and just try to find some things that are the length of your cubes?

STUDENT: This is humongous.

TRACY SOLA: Great! All right.

STUDENT: Yes.

STUDENT: Three, four, five, six.

STUDENT: It's one cube long.

STUDENT: It's only six.

STUDENT: One, two, three, four, five, six, seven.

STUDENT: It's hard to measure it because when you get here it has a curve and not a straight.

STUDENT: We need a shape that curves around this thing. If it's not, then it's not a shape that works.

STUDENT: Wow.

TRACY SOLA: Oh, what?

STUDENT: It has to go--

STUDENT: Mine does. [laughs]

STUDENT: Glasses.

TRACY SOLA: My--

STUDENT: Oh, it fits your glasses!

TRACY SOLA: Yep.

STUDENT: One, two, three, four, five, six, seven, eight, nine. Nine.

TRACY SOLA: -- that was as long as your cube train. What was something interesting that you found that was as long as your cube train?

TRACY SOLA: Monica?

STUDENT: Your head.

TRACY SOLA: My head? What else? Yes, Jade.

STUDENT: The globe.

TRACY SOLA: The globe? How did you measure the globe?

Inside Mathematics

STUDENT: You use friends.

STUDENT: You can connect them.

TRACY SOLA: With your friends?

STUDENT: You cannot measure with your cube because it's not, it doesn't have sides, or anything to measure and you need some curve or it doesn't even, like -- it's not like -- you have like, because you measure, like about all the same high because you use Ariana's too, and you took one away and that's how you measure it, to measure the globe.

STUDENT: Oh, you need to use this one? Here you go.

TRACY SOLA: What else did you measure that was interesting?

STUDENT: Um, feet.

TRACY SOLA: Feet? Okay. What else, Salvador?

STUDENT: My glasses.

STUDENT: We measured the box.

TRACY SOLA: The what? The box?

STUDENT: Box.

STUDENT: Oh, I know something.

STUDENT: Me -- me and Ricardo connect our cubes. And Santiago.

TRACY SOLA: You put all your cubes together?

STUDENT: Yeah.

TRACY SOLA: You needed all your cubes to measure the box?

STUDENT: But we needed to take one off.

TRACY SOLA: But then you took one off.

STUDENT: Yeah.

TRACY SOLA: And so what was the total measurement?

STUDENT: Uh ...

STUDENT: Uh, 20.

TRACY SOLA: 20?

Inside Mathematics

STUDENT: 20? No it wasn't. It was --

TRACY SOLA: How many?

STUDENT: It was 10.

STUDENT: What?

STUDENT: 30, I don't know. Was it like 30?

STUDENT: What?

STUDENT: That's what I said, 30.

STUDENT: 30.

STUDENT: You said 31.

STUDENT: 13.

TRACY SOLA: 13. How many do you each have?

STUDENT: Uh ...

STUDENT: 6.

TRACY SOLA: You each have 6. And you put them all together and took off one? That's something interesting to think about. Okay. What we are going to do now is, I am going to ask you, uh, if you have yellow, please put your cubes in, back in the bin.

TRACY SOLA: This activity, I gave them the choice to measure anything that was in the room and, um, one of the girls asked if they could measure a globe that was sitting up on a high shelf so I brought it down. And, um, another one of the girls said, "You can't measure that, because, um, it doesn't have a side." And I think what she was thinking was that it doesn't have an edge. But she didn't have the vocabulary for that and so she was saying, you know, that because it -and she used the term sphere, which was really interesting -- but because it's a sphere, it doesn't have a side, so you can't measure it. And I saw other students using their cube train to try to track it along the edge, you know, along the edge of that sphere and really thinking what it meant to, um, to measure something that was spherical in shape. That was very interesting. TRACY SOLA: They are going to get a bin of cubes that are not connected, Unifix Cubes that are not connected, and there is a large assortment of items on a table that are all labeled with, uh, a name. For, so example, a can of beans would just say can, and, um, a spoon would just say spoon. And, so, what I'm asking them to do, in partners, is to choose an item and to measure it using cubes. So, up to now, they've been measuring with a train of cubes, stacking them together to see how long something is. And, so, I will, uh, but the -- so I will ask them to do that. And, as they measure items, they will go and add them to the appropriate chart for, uh, if their item is five cubes high, they will go add it to the chart that says five cubes long. And, um, we'll see what happens. I think it might be interesting because, depending on how accurate they are, um, I think items might end up on more than one chart. And, um, I also hope to discuss with them what will happen if it measures into the middle of the cube. Do we count that cube? Or don't we count that cube? How do we deal with some leftover, or not quite enough. And I'm interested to see what kind of ideas they have for that.

TRACY SOLA: We are going to use the objects on the table now, and let me tell you what we're going to do. You must have noticed that I put up some posters.

STUDENT: Yes.

TRACY SOLA: What does that one say?

STUDENTS: Five cubes long.

TRACY SOLA: Right, and so you know what all the rest of them say.

STUDENT: Six cubes long.

TRACY SOLA: Right. So, here's what we're going to do. If you, you and your partner. There will, remember there will be one team of three. Wait, do we have everybody?

STUDENTS: Yeah.

TRACY SOLA: Four, five, six. Yes, okay, we'll get a tub of cubes. They're just, they're not connected right now, and you will get a marker for your team. Okay? And you will walk over to the table, and you will choose one item from it. Let's just say I take this one, and then you will walk to your table with your partner, and I want the two of you to work together using your cubes to measure how long this object is. Okay, so, let's see, Arianna, do you think you could help me for a second? You could be my partner? Okay, why don't you come over here. Why don't you and I measure, why don't you take some cubes and we will measure how long this is. Go ahead. Show us how it's done.

Think that's about right?

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: How many cubes are on it?

STUDENT: One, two, three, four, five.

TRACY SOLA: So this cup is five cubes long. Correct?

STUDENTS: Yes.

TRACY SOLA: Now, you will notice that on each object, I labeled what it is. Can you read that?

STUDENTS: Black cup.

TRACY SOLA: So, now that, since we're partners, we know that the black cup is five cubes long. So, you can, if you want to take your cubes with you so you remember, bring the cup and bring the cubes, and the other partner brings the marker, and let's go to the poster that matches the length of the cup. Okay?

There we go, excuse us, friend. Okay, so then, here we are. Why don't you stand over here by me. Here we are at the, the poster that says five cubes long. You may take the marker and you can write the name of the object right there. Okay?

Okay. There. And now that you've done that, you, you bring both things, you will -- oh, excuse me. You will bring your object back to table, replace it, and get ...

STUDENT: And then you get a new one.

TRACY SOLA: And then get something else, a new object, anything you want, and you go back to your station, and you do that again. And you keep doing that for as long a time as you have. We'll do about 10 minutes. So, you can measure as many things as you want in those 10 minutes.

Okay, are there ... Thank you for being my partner, my demonstration partner. Are there any questions about what we're doing right now?

STUDENTS: No.

STUDENT: Me.

TRACY SOLA: Yes, Santiago? You have a question? That's okay. You have a question, Monica, yes.

STUDENT: Can I -- can I ask you at the end?

TRACY SOLA: Mm-hmm. Oh, well would we like to switch partners?

STUDENTS: No.

TRACY SOLA: I'm okay with whoever you want for your partner. So long as nobody's left out. So long as nobody gets left out, I'm okay with, is ... How about Jade and Kylie, were you partners before? Can you be partners again?

STUDENT: Yeah.

TRACY SOLA: Great, thanks.

STUDENT: I want to be partners with him.

TRACY SOLA: Well, I think it's all working out just like it did before, isn't it? That's perfect. Okay, well, why don't you get started? Let's put ...

STUDENT: I'm throwing this out because it doesn't work. Three, four, five. Five.

STUDENT: One more.

STUDENT: Glue stick is -- oh! It's three inches.

STUDENT: It's four.

STUDENT: Oh. 'Cause there needs four.

TRACY SOLA: Do we need a poster for four?

STUDENT: We tried the glue stick and there's four.

TRACY SOLA: Okay. I'll make a poster for four.

STUDENT: Um, it looks like the same length. One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen!

STUDENT: Oh my.

STUDENT: You can write.

STUDENT: L. L-A-D-E-L.

STUDENT: Don't!

STUDENT: All right. One, two ...

STUDENT: Two. Connect up this.

STUDENT: Three, four ...

STUDENT: Four. Add another.

STUDENT: Five, six ...

STUDENT: Six. Seven. Oops. And eight.

STUDENT: Eight?

STUDENT: Eight pieces.

STUDENT: We're gonna do this.

STUDENT: Just the one side.

STUDENT: This way?

STUDENT: Yeah.

Inside Mathematics

STUDENT: And it takes up--

STUDENT: Now we can count it.

STUDENT: One, two, three, four, five, six, seven, eight, nine.

STUDENT: Okay. We got nine.

STUDENT: Got it. One, two, three.

STUDENT: This. One, two, three. Oops. Is it one, two, three, four, five, six. It's six.

STUDENT: I'm putting this on.

STUDENT: [laughs] I think I need two more. I think I need two more. Two more. One, two, three, four, five, six ...

STUDENT: No, wait, wait, wait, wait --

STUDENT: Seven..

STUDENT: Adaia, Adaia ...

STUDENT: Eight, nine, ten ... Um, it's eleven.

STUDENT: Look see. We need, just need a few more.

STUDENT: One more?

STUDENT: No, not one more. A few more.

STUDENT: Two more?

STUDENT: No, a few.

STUDENT: A few?

STUDENT: Now ...

STUDENT: Did you take ... One, two ...

STUDENT: One ...

STUDENT: Three more.

STUDENT: Aaah!

STUDENT: Now just the one we're looking for.

STUDENT: Uh, too long.

STUDENT: Take one away. Wait, you're supposed to take one away.

STUDENT: Mm-hmm. [affirmative]

STUDENT: Okay. One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen. It's sixteen.

STUDENT: No, let's do fourteen. One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen.

STUDENT: Me? Okay so you have to write the same thing.

STUDENT: No, no, I already wrote it.

STUDENT: We have to write the same thing.

STUDENT: Okay, fine then I'm gonna go and get ---

STUDENT: B-L-A-C-K-S-P-O-O-N.

STUDENT: Four five, six, seven, eight, nine, ten, eleven, twelve, thirteen.

STUDENT: Thirteen.

STUDENT: Count it again.

STUDENT: The glue stick is four. The glue stick is four.

TRACY SOLA: Measure one more thing. Measure your last item and then we are going to do clean-up and meet on the rug.

TRACY SOLA: What we're going to do now, is I am going to ask you to put, if they're not already in the bins, put the cubes in the foil bins. Take your objects, please, back to the table. Return the objects back to the table. Make sure your ... Here, hun, I can get rid of this for you. Make sure your pen is in your bin as well.

You boys and girls were very, very careful measuring scientists, weren't you?

STUDENTS: Mm-hmm. [affirmative]

STUDENTS: Is that, is that a microphone?

TRACY SOLA: That is the microphone. Yeah.

STUDENTS: It's tiny.

TRACY SOLA: Yeah. So, I'm looking at what we measured. I'm looking at all of our posters, and one thing I noticed was that it seems like on number six, everybody put can on number six.

Every -- everybody who measured a can, see ... I don't see "can" anywhere else, do you?

STUDENTS: Can, can, can, can ...

TRACY SOLA: I don't -- I only see can on number six.

STUDENT: Let me look at all --

TRACY SOLA: But, if I look at seven and four, I see the tea box on seven, and I see the tea box on four. And I'm wondering how they could be both. Because here's the tea box.

STUDENT: Oh, I know!

STUDENT: I know!

TRACY SOLA: Turn to a partner and tell your partner, why do you think tea box is on the poster for four and the poster for seven?

STUDENTS: Because the side ... the number seven ...

STUDENT: No, no, no, no, be--, be--, because, um, the number seven ...

STUDENT: Zip it.

STUDENT: You just punched me.

STUDENT: No, I didn't.

STUDENT: Yes, you went like this.

STUDENT: But that not punching.

STUDENT: Yeah, but it went like this.

STUDENT: I went like this.

STUDENT: I know why! Because they measured the bottom and they measured the sides.

STUDENT: That's what I said!

TRACY SOLA: So, so Bryan, can you tell me again what you just told me?

STUDENT: Um, some people measured on the--

STUDENT: Measured the top and the, and the sides.

STUDENT: No, it was --

TRACY SOLA: Go ahead.

STUDENT: -- some people measured on the long side, some people measured on the short side.

TRACY SOLA: Oh.

STUDENT: Yep!

TRACY SOLA: Okay.

STUDENT: It's because they measure it like this. They measure it like this for the seven and then like this for the four.

TRACY SOLA: So who would like to share with the group why tea box is on two different posters?

STUDENT: I know!

TRACY SOLA: Hmm, let's see. Adaia? What did Bryan say. You heard some -- Bryan just say something. What did he say? Do you remember?

STUDENT: I do not.

STUDENTS: I do! I do!

TRACY SOLA: Do you remember, Jade?

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: What did he say?

STUDENT: Bryan said some people put, put --

STUDENT: Measure.

STUDENT: -- measure the short side and the long side.

TRACY SOLA: Some people measured the short side and some people measured the long side?

Who measured the short side? Team Santiago, Salvador, Ricardo.

Who measured the long side? Ah, team Ariana/Jaira. Okay.

STUDENT: Jaira.

TRACY SOLA: Okay. How--

STUDENT: Jaira.

TRACY SOLA: Do you say your name Haira or Jaira?

STUDENT: Both.

TRACY SOLA: Both. Okay. Okay. Now, I noticed that the blue cup is on poster eight and poster nine. And I'm trying to think about how that happened.

STUDENT: I know.

TRACY SOLA: Does a, do people think they have any ideas?

STUDENT: No.

TRACY SOLA: No ideas?

STUDENT: Nope.

STUDENT: I do.

TRACY SOLA: Santiago, go ahead. What is your idea?

STUDENT: Some people measured the, the short side and some people measured the long side? [shrugs]

TRACY SOLA: Can you tell me where th--

STUDENT: Maybe they took off one and then, and then a little bit of the cube reached up?

TRACY SOLA: A little bit of the cube reached up, and then they took off one.

STUDENT: Mm-hmm.

TRACY SOLA: Hmm. Well, do you think we can measure this right now?

STUDENTS: Mm-hmm. Yeah.

TRACY SOLA: Is there, is there somebody ... Yeah, can you come up, Santiago? And show me exactly what it is you mean? Here, why don't you stand over here where I am, right over here, and, and, uh ... here. Turn around. There you go.

And here's -- and here's some cubes for you. So why don't you show us all what you mean by that? And who thinks they know what Santiago just said? While he's building, is there somebody who would ... can explain what they think Santiago was talking about? Go ahead, what do you think he was saying?

STUDENT: He said that, um, some people measured the, the long side and the short side because there's a long side and a short side. The top is a long -- the top is a short side, and the long one is a -- the long, the long side.

TRACY SOLA: And did -- and was there anything else he said? Does anybody remember anything else he said?

STUDENTS: Hmm.

STUDENT: It's actually eight.

TRACY SOLA: What?

STUDENT: It's actually eight.

TRACY SOLA: It's right at it?

STUDENT: It's actually eight, because look. If you put one more then it would not be nine.

TRACY SOLA: Okay. So, let ... well, let's see. So you're taking a look at this. And, so that's eight right there. And then, so why do you think some people wanted to put another cube on?

STUDENT: Because it look a little bit tiny.

TRACY SOLA: Because it looks a little bit tiny? Tell me more about that. I know what you're talking about, I think. Tell me a little bit more about that. You can come over here by me. So we can ... It looks a little bit tiny, what does that mean? You mean it's not quite to the top?

STUDENT: Mm-hmm.

TRACY SOLA: So maybe we need another one? And if we put another one on then it fills up that space.

STUDENTS: Mm-hmm.

STUDENT: Yeah, but it's big. And not --

STUDENT: Nine!

Inside Mathematics

TRACY SOLA: So, but now what? It's big?

STUDENT: Yeah.

TRACY SOLA: Do you think it's too big?

STUDENTS: Yeah. Yeah. It's too big.

TRACY SOLA: It's too big?

STUDENT: Yeah.

TRACY SOLA: Well, which one do you think is closer, eight or nine?

STUDENTS: Eight. Eight. Eight.

TRACY SOLA: You think eight's closer?

STUDENT: Eight is the right answer.

TRACY SOLA: Eight is the right answer? Thanks for helping us build that! You can have a seat, hun. But leave it here because I want to talk about it just a little bit more.

TRACY SOLA: Let's think about what just happened. We were trying to decide what to do when it wasn't perfect. Right? Because when he built this, we found out that that didn't go quite all the way to the top. And so, some people thought they better put that on, so it made it to the top.

STUDENT: I took that white one off.

TRACY SOLA: You took it off? You thought it was too much?

STUDENT: Yes.

TRACY SOLA: Okay. But some people thought we better at least make it to the top.

STUDENT: Who did that?

TRACY SOLA: Do you think ... Well, I -- I don't know. I mean do you think that both ways have good reasons for doing it that way?

STUDENTS: Mm-hmm. (affirmative)

TRACY SOLA: You do?

STUDENT: And I know, that's why I know, because I put mine right here.

TRACY SOLA: Okay, but I -- I'm wondering, I think that the person who put nine, I think had a good reason for doing it, too. Yes, Salvador?

STUDENT: A blue thing is over there, behind you.

TRACY SOLA: Oh, thank you. Okay, we can get that for later. Okay, so we -- so when we're measuring, we need to think about what to do when something isn't exact.

STUDENTS: Mm-hmm.

TRACY SOLA: And if we have a good reason for adding another one, or not. And that's really your call. I think we could argue either way, right? And you just need to make a decision and then defend it. Okay, finally, I think the pan -- I saw the pan on nine, but I also saw it way over here, two people put it on 14.

STUDENT: I put 14.

TRACY SOLA: Okay. But who put nine? Who put nine for the pan?

STUDENTS: Uh ...

TRACY SOLA: You did, Jade?

STUDENT: I put the first one.

TRACY SOLA: So, which way did you measure the pan?

Inside Mathematics

STUDENT: She did it on this side.

TRACY SOLA: You think you did it on that side?

STUDENTS: Wait, wait. We measured it -- wait.

STUDENT: She did it on this side.

TRACY SOLA: Ah. So, Jade did it on this side. And then the people who have 13 did it on this side?

STUDENTS: I did it inside.

TRACY SOLA: Inside? Inside of there. Oh, and you had 14? How did you get 14?

STUDENT: Because we did it at the bi-- the -- the large side.

TRACY SOLA: So that's really interesting. The people who measured ... so, first of all, some people measured this side.

STUDENT: Here.

TRACY SOLA: Some people measured this side, but inside.

STUDENT: I did.

TRACY SOLA: And some people measured this side, but outside, and they all got different measurements.

STUDENT: I got 13.

TRACY SOLA: So, where you measure makes a difference. Is everybody correct?

STUDENTS: Yes. Yeah.

TRACY SOLA: Yes they are. Because they all measured in slightly different ways. Well, I think that was really interesting to measure all those things.

TRACY SOLA: I was wondering what would happen when they got an object that you could measure, um, different dimensions of that object and that turned out to be an interesting thing that was a little bit dissonant for them. I -- it turns out that the tea box was, uh, four cubes in one dimension, and seven cubes in another dimension. And when, um, the one group went over and put it on the four poster, and the other group went over and started to write it on the seven poster, the boys who had measured it as four were trying to tell the girls who measured it as seven that it wasn't seven, and that needed to go on the four poster.

And -- and so that was a really interesting time to see them making sense of how -- how an object could have more than one dimension and -- and how it was okay for it really, to be both.

That -- that depending on which dimension you measured, it could be four or seven. And we saw that with the pan, we saw that with some other objects, too. I think, the pad.

Uh, the other really interesting thing that happened in that lesson was making sense of things that were not an exact amount of cubes high. I think, the blue cup was, uh, 11 and a little bit more high, and so some people had it on the 11 poster, and some people had it on the 12 poster. And uh, so it was -- it was interesting for students to think about, uh, what was the right thing to do with that.

And so, you know, some said, "Well, it didn't quite make it, so I needed to add another cube." And another ch-- student said something like, "Well, it's almost there, if we add another cube, it's going to be too much." And so, um, I purposely left that as unresolved, because it was making sense of a part, a remainder, and you know, that's something that we can start thinking about now, in 1st grade, but isn't really a standard until much later. So I just kind of left that as something to think about, saying, "You know, I don't care which way you go on this, but you just need to justify it, you need to give the reason why." TRACY SOLA: After we do those activities, we are going to bring in, uh, the problem of the month measuring mammals, and that activity has a picture of three giraffes that are small, medium, and large. And I am going to ask them to measure the giraffes, but this time I'm only going to give them one cube to do so. I'm not going to give them multiple cubes to make it easy and make a train. I am just going to give them one cube and see what happens, and if they can use iterative measurement to measure the height of that giraffe and, um, I'll be interested to see how they keep track and if ... We'll give them pencils and see if they use pencil markings.

How will they know, um, uh, you know, the idea of -- if the idea of gaps comes up earlier, how will they use that here in the iterative measurement piece. And then after they do that, I will give them more, uh, cubes and allow them to make trains to check their result to see if they got the same, and if they didn't to think about why that's happened. So they can think about really, um, what measurement means.

TRACY SOLA: Let's think about what we've done so far today. So far today we ... what was the first thing we did?

STUDENT: Ooh, I know.

TRACY SOLA: What was the first thing we did?

STUDENT: I measured.

TRACY SOLA: Santiago, what was the first thing we did?

STUDENT: I know. I know.

TRACY SOLA: Kylie.

STUDENT: We did this thingy.

TRACY SOLA: We walked on the tape to see how many footsteps? We walked on the tape to see how many footsteps it took to get to the end.

STUDENT: I know what is the next thing.

TRACY SOLA: And what was the next thing?

STUDENT: We, we measured.

TRACY SOLA: What did we measure?

STUDENT: We ... Stuff on the table over there.

TRACY SOLA: And there was two things we did in between that. What did we do, Monica?

STUDENT: First we -- not, no. First we traced our foot.

TRACY SOLA: First we traced our foot. And then there was one more thing in between. We got?

STUDENT: Um, um, we, we put the shoes biggest to smallest.

TRACY SOLA: We did that. And then do you remember the cube trains?

STUDENTS: Yes. Oh.

TRACY SOLA: And went and found things that were as big as that? Then we measured some objects. Now, last thing we're going to do today is we are going to do some measuring on this piece of paper. Now these words up here are quite small.

STUDENT: Who is taller?

TRACY SOLA: Who is taller?

So that's a new word that we haven't used yet today. It might not be a new word to you but it's a new word that we haven't used today for measuring. So can I please have Jaycee and Bryan come up to the front here. And let's see and how about, um, hm ... can we have you, Jaira, come up? Okay. So who's the tallest?

STUDENTS: Jaira.

TRACY SOLA: Who's the shortest?

STUDENTS: Jaycee.

TRACY SOLA: Um, who--

STUDENTS: Medium?

TRACY SOLA: Well, who's medium?

STUDENTS: Bryan.

TRACY SOLA: Bryan. Okay. Jaira, sit down for one minute. Now of the two people up there who's taller?

STUDENTS: Bryan.

TRACY SOLA: So he might not be the tallest person in the room, but when those two are together he's the tallest, right?

STUDENTS: Yes.

TRACY SOLA: Okay. So why doesn't everybody sit down, all our volunteers sit down. Thank you. You can go back to your seats. So our job today is to figure out who's taller. I'm going to introduce you ... First of all, what kind of animal is that?

STUDENTS: Giraffe.

TRACY SOLA: A giraffe. Okay. This giraffe's name is George.

STUDENTS: [laugh] George.

TRACY SOLA: George. This giraffe's name is Jeff.

STUDENTS: Jeff.

TRACY SOLA: And this giraffe's name is Jerry.

STUDENTS: Jerry.

TRACY SOLA: Yeah.

STUDENTS: That's my friend's name.

TRACY SOLA: What?

STUDENT: That's my friend's name. Jerry.

TRACY SOLA: Oh, great. Okay. So your job is to figure out who is taller. Now, it's not okay just to say who you think is taller, because you looked at it and that's what you think. If we are being very, very careful mathematicians we need to check to make sure that we know. How can we check to make sure? Yes?

STUDENT: So if you go like this and then you try to keep it together, and you keep it like that and then you know.

TRACY SOLA: Oh, huh. Okay, so we could use our hands. Keep it together. Okay. What else, Salvador, could we do?

STUDENT: You can use your hand?

TRACY SOLA: You can use your hand. What else, Monica?

STUDENT: You can use pencil to do, do a number line and can measure them how many size, I mean how -- how long it is.

TRACY SOLA: You could use your pencil and make a number line? Okay. So we have lots of tools. I've heard lots of good ideas. So you can use any of those tools, and I'm going to give you one more tool. The one more tool I'm giving you is one block. One cube. So far today, we have used many cubes and we've put them all together. And that was a pretty easy way to measure, wasn't it?

STUDENTS: Yes.

TRACY SOLA: But this time I'm only giving you one cube. You don't have to use it, but if you want to use it you can, or you can use anything else you have at your table to measure. You may not combine cubes with other people. You can only use one cube or something else you have. And so can somebody tell me, repeat to me what is your job when you get to your table? What is your job when you get to your table? Raise your hand. First of all, raise your hand if you know what your job is. If you know what your job is when you get to your table, raise your hand. Okay. Um, Jaycee, what is your job when you get to your table?

STUDENT: You, you um, get started.

TRACY SOLA: You get started, and what are we trying to find out?

STUDENT: What's the longest.

TRACY SOLA: The longest? Which -- which giraffe is the tallest -- the longest? The tallest.

STUDENT: I think maybe I know.

TRACY SOLA: Well, keep that idea right here and go check. Because I want proof.

STUDENT: I said maybe.

TRACY SOLA: You said maybe? Good. You have to check first. Okay, so we're going to go to our tables. We are going to check to see which giraffe is the -- the tallest. And we have to prove, prove it with numbers. Let's prove it with some numbers. Okay? Does everybody feel like they know what to do? Is there -- anybody have a question before we get started?

STUDENT: Do we write our names?

TRACY SOLA: Yes. Please do write your names.

STUDENT: Or -- or you don't know who it is.

TRACY SOLA: Exactly.

TRACY SOLA: ... you have measured, I want you to use either the blue or the green marker.

STUDENT: I don't have no green.

TRACY SOLA: Well you have blue. Oh, well here, let's put these over here, hang on. Take one of these. Is it the number over here? Here you go boys. That's okay. That's okay. Blue or green.

Well I want you to use either blue or green right now. Either blue or green right now. There we go.

STUDENT: Or orange?

TRACY SOLA: Not orange yet.

STUDENT: Okay.

TRACY SOLA: We're gonna save orange or pink or red for later. [crosstalk]

Oh. Here. You guys are sharing -- not, you don't need to share, here, here you go, here you go. All right and go ahead and start exploring. Try to figure out how tall each giraffe is and then prove who is the tallest. How tall is each giraffe? I would like to know. Yes, a pencil is a great idea. How tall is each giraffe?

Can you try to find out using one cube?

STUDENT: Yeah. I can.

TRACY SOLA: Can you try to measure each giraffe using just one cube?

STUDENT: One?

STUDENT: No. No.

TRACY SOLA: I bet you could figure out a way.

STUDENT: The tiny one? You cannot.

STUDENT: I do not know how to do this.

STUDENT: Me too.

STUDENT: Twelve? Twelve.

STUDENT: Eight, nine ...

STUDENT: Eight.

STUDENT: George is five tall, and Jerry is two.

STUDENT: So do you like mine?

STUDENT: I don't know how to do it. I said, 'cause I did -- I did like this and I didn't know ...

STUDENT: This is how I did it. I went like that and I went like that.

STUDENT: I know, but I put it longer. Oh, I'm not sure if I can fit that.

TRACY SOLA: They're the same?

STUDENT: I think.

TRACY SOLA: You think so?

STUDENT: Mm-hmm. [affirmative]

STUDENT: Two.

TRACY SOLA: You did trace it from the top of the block? Oh, so you put your block down on its side, and you traced it from the top of the block.

STUDENT: Four, five.

STUDENT: Miss Tracy, I'm already done.

STUDENT: I'm done too.

TRACY SOLA: Great!

STUDENT: What do I do when I'm done?

TRACY SOLA: Why don't you come and I'll look at your work and you can tell me what you did.

STUDENT: I counted, I counted the numbers up to the giraffes.

TRACY SOLA: You count -- so you wrote in your own numbers and you counted them up to the top of the giraffes.

STUDENT: Yep.

TRACY SOLA: Hmm. So I'm looking at this work and I see that the six, or the one, is very tiny and short. Yes?

STUDENT: Because the two's, and then I was writing up to ... [pointing]

TRACY SOLA: But I'm wondering about the eight. It seems very tall. Do you think uh, it -- what - do you think, some of these seem smaller than the other ones. Do you think that works still?

STUDENT: [nods]

TRACY SOLA: Yeah? And then you did the same thing here.

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: Okay. All right. And how did you end of here? You ended up ten here and eighteen here.

STUDENT: Can I draw up pictures on the back?

TRACY SOLA: Um, sure.

STUDENT: I want to draw a dog.

TRACY SOLA: Um, so if you just look with your eye, without measuring, do the, you told me there that George is how tall?

STUDENT: Ten.

TRACY SOLA: Ten. And how tall is Jeff?

STUDENT: Eighteen.

TRACY SOLA: Does it seem like they are really different in height?

STUDENT: No, they don't.

TRACY SOLA: You don't think they are?

STUDENT: But, but, but Jeff is bigger than George.

TRACY SOLA: Jeff is bigger than George?

STUDENT: Yeah because Jeff is fift--

TRACY SOLA: I know what you did but if somebody else came into the room how would they know what you did? Is there a way you can mark on your paper to show that work?

STUDENT: Oh, like this? Keep it like this?

TRACY SOLA: What if I took all the pencil and the cube away -- how would I know how you got the numbers? Is there a way you could show your work? Show your thinking.

STUDENT: I don't know how to do it.

TRACY SOLA: You counted it?

STUDENT: I went one, two, three ...

STUDENT: Oh wait, I could draw a picture of me doing it.

STUDENT: ... four, five ...

STUDENT: I could draw like, draw a pencil and then draw the cube next to it.

TRACY SOLA: I love that idea. I love that idea. Can you do that right up there next to Jeff or George or Jerry so we can --

STUDENT: I'm going to do it by George.

TRACY SOLA: Okay, do it by George!

STUDENT: I'm gonna do it right here.

TRACY SOLA: Okay, that's great.

STUDENT: I don't know how to draw a pencil. Does this look like a pencil?

TRACY SOLA: Okay, that's perfect, that looks like a pencil to me. [crosstalk] Okay.

STUDENT: I was going to put an eraser.

TRACY SOLA: Okay. So that's good enough.

STUDENT: And I put the cube right there.

TRACY SOLA: Okay.

STUDENT: And I put the little square

TRACY SOLA: Great, now, now keep going so I understand everything you did please. Okay, I understand, over and over. That makes sense to me. Great.

STUDENT: Go over and over.

TRACY SOLA: When I was watching you work, I saw three main strategies. I saw people doing it three different ways. The first thing I saw was Kylie and also Jade. There's a picture of a pencil here and also a cube, and it says, "Go over and over." So, what I saw them do, was I saw them put a cube here, and then I saw them put a pencil here. Then they put a cube there and a pencil there. And then they put a cube there and a pencil there and they kept going. I saw them do that. So when Kylie did that, she got five for Jeff and six for George.

Now, what I saw Bryan and Adaia doing is they took a cube and they put it this way. They put it there and drew a line. And then they put it there and drew a little line. And they put a cube and drew a little line. So they could keep track of how high the cube went up each time, and then they tried to put it right above that. So I want us to think about before we look at the last way, how are these strategies different and what does that mean for accuracy in measurement? Turn to your partner and tell your partner how they are different.

STUDENT: Kylie did it a different way and so did Bryan because they didn't count the same way.

STUDENT: Kylie did it a different way because she did one number and Bryan did more numbers.

STUDENT: Oh, and I almost forgot that Kylie put the cube first and then the pencil.

TRACY SOLA: So, Adaia, what did you say to Bryan?

STUDENT: They -- we draw lines, and they do the pencil and it's different.

TRACY SOLA: And it is different. Do you think that the measurement will turn out exactly the same if you draw a line between your cubes or if you put a pencil between your cubes? I see some people saying yes and some people saying no. Santiago, you are shaking your head no, why not? You don't know.

Well what do we think? Do you think it'll be exactly the same or do you think it will be different?

STUDENTS: Different.

TRACY SOLA: Why?

STUDENT: Because they are all different. That one only uses a pencil and a block, and that one only uses the cube, and that one only uses the pencil, um --

STUDENT: And the last one does nothing.

TRACY SOLA: Well, but these two I'm trying to think about. These two. If I take a cube, maybe I'll just do it right over here. If I take a cube right here, and if I put it at the bottom of Jeff, I saw them draw a line at the bottom of Jeff. If I put my cube right there and draw a line on top of it, and then do it again, and then do it again. I keep going. Do you think I'll get a different measurement? Maybe I'll use a different color. Then if I put a cube, there's the cube right there,

and then I put a pencil. Let's see if I can do it. That's not very easy up here. Then I put a cube and I think we started here and then I put a pencil, and then I put a cube. So here's my first cube, my second cube, and my third cube. And for these, here's my first cube, my second cube, and my third cube. Are they the same or are they different?

STUDENTS: Different.

TRACY SOLA: They're different. Why are they different?

STUDENT: Because you draw two lines.

TRACY SOLA: Why did I draw two lines? What was in that space? Do you remember how I did that one? There was that -- whoops, that's okay. There was that and then there was that. That, that, that, What was in that space?

STUDENT: A pencil.

TRACY SOLA: The pencil was in there. How high is the pencil? A little bit. Do you think it's the same height as the line?

STUDENTS: No. Mmm ... no ...

TRACY SOLA: Maybe? No? Which one do you think is taller? The line or the pencil?

STUDENTS: The pencil.

TRACY SOLA: The pencil's taller? So if we put a pencil in between each one, it gets a little bit taller, right? Now, how did these people measure? Turn to your partner and talk about this one. What did people do here?

STUDENT: They wrote numbers and they got their answer.

TRACY SOLA: So what did they do? Someone who hasn't talked for a while. Ariana, what did these people do?

STUDENT: Three of them?

TRACY SOLA: This one right here. Some people used this strategy. In fact, I think you used this strategy.

STUDENT: So, she used a line between the giraffe and she started to do the numbers.

TRACY SOLA: Oh, okay. So what did Ariana just say? Santiago, what did she just say?

What? Why did she just say, Jaira?

STUDENT: She said, "I put a line in between the giraffe" and she put numbers on it.

TRACY SOLA: And then some numbers? Okay.

TRACY SOLA: ... heard some people say at the beginning they were thinking about using a number line model. Is this kind of like a number line model?

STUDENTS: Yep. Yeah. Not really.

TRACY SOLA: Not really? Not really?

STUDENT: It looks like it.

TRACY SOLA: A little bit?

STUDENT: Yep.

TRACY SOLA: Except it's, uh, straight up and down instead of across, maybe? And then these people both used cubes stacked on top of each other. This team used cubes stacked on top of each other and this team used cube, pencil, cube, pencil, cube, pencil. And this team built a number line. Which one do you think would be the most accurate, and why? You remember what accurate means?

STUDENTS: Yep. No. No.

TRACY SOLA: Accurate means --

STUDENT: Something that you have to be careful.

TRACY SOLA: Something that you have to be careful with, very precise. Just --

STUDENT: Like fragile.

TRACY SOLA: Uh, well, it's a little different than fragile. Fragile means it might break. Right. I think if we're going to be accurate ... what?

STUDENT: Like glass.

TRACY SOLA: Yeah, that's fragile. If we're accurate, then we wouldn't ... Remember when we were walking on the number line and we said that we didn't want gaps between our footsteps?

STUDENT: Yeah.

TRACY SOLA: Because we wanted it to be very accurate, we wanted to know exactly.

STUDENT: Mm-hmm. [affirmative]

TRACY SOLA: So which one of these do you think would be the most accurate, and why? Santiago?

STUDENT: Um, counting numbers, drawing the numbers.

TRACY SOLA: The numbers? Well, let me ask you a question, then. Do you think that one right there is the same size as that eight? Do you think --

STUDENT: No.

TRACY SOLA: No. Do you think it's important, uh, for the numbers to be all the same size, or do you think it really matters --

STUDENT: It matters.

TRACY SOLA: If you're using that strategy. It does matter. Why does it matter?

STUDENT: Because if you do this -- if you do the same strategy but if you don't get the same answers, then, then, if, if, if, if, if ...

TRACY SOLA: That's okay. I appreciate your thinking. I appreciate your thinking. So do we think it's okay for the numbers to be different sizes?

STUDENTS: Yep. Yes.

TRACY SOLA: We do. Okay. Are the cubes different sizes here?

STUDENT: Yes.

TRACY SOLA: They are?

STUDENT: No.

TRACY SOLA: Well I don't know.

STUDENT: Yes.

TRACY SOLA: How come? The cubes are different -- this is a different size every time?

STUDENT: Yes.

TRACY SOLA: Is it?

STUDENT: No.

TRACY SOLA: No?

STUDENT: Yeah, it is.

TRACY SOLA: Why?

STUDENT: I cannot see it straight and it doesn't look like a square, it looks like a cube.

TRACY SOLA: Oh, depending on how you hold it? Hm.

STUDENT: It depends on how you hold it.

TRACY SOLA: Okay. Well, who do you think is the, uh, tallest giraffe?

STUDENTS: The top one. The bottom one.

TRACY SOLA: The bottom one?

STUDENTS: Jerry. George.

STUDENT: I think it's this one.

TRACY SOLA: You think it's that one? Well, Bryan seems to think they're the same size. Um, we've got a twelve here and a ten here. We've got a six here and a five here. I think we are still thinking about George and Jeff, aren't we? We're not really sure? Um, but are we certain about Jerry? Is Jerry the tallest?

STUDENTS: No. No.

TRACY SOLA: Is he the shortest?

STUDENTS: Yes. Yes.

TRACY SOLA: We know he's the shortest, we're still not sure about those two. That, that's something for us to keep thinking about. Yes?

STUDENTS: Yes.

TRACY SOLA: I would like to thank you so much for working with me today, it's been just a ...

STUDENT: You're welcome.

TRACY SOLA: ... a pleasure working with such dedicated mathematicians. You guys are, are great thinkers. Will you continue to work really hard about thinking in math?

STUDENT: Yes!

TRACY SOLA: Wonderful. Thank you so much.

TRACY SOLA: The culminating activity of the measuring mammals with the three giraffes, uh, Jeffrey, George, and Jerry, was interesting because I gave them one cube, and I told them they could use any strategy they wanted. And what I was really looking there, for there, is to see if they could use iterative measurement. Having only one cube, they couldn't make the cube train, which would have been easy. They had to either use the cube multiple times, or create some other method. And, and we saw several things happen. We saw that, uh, two students did that. They took their cube, and they tried to be very precise. They drew a line at the bottom where the giraffe started, they would place a cube, draw a line at the top of that cube, and they kept doing that and doing that, and they were very precise.

TRACY SOLA: Another group wanted to use that idea but didn't know how to keep track of where the cube had been and where to put it next. And they finally devised the system of putting a pencil on top of the cube, and then holding the pencil in place and moving the cube up one, and then moving the pencil up one, and they kept doing that. And, uh, so, of course, not realizing that the pencil added some height. And ... Or really not thinking about that, or maybe that was okay. Maybe that was our unit of measurement, you know, cube-pencil. Um, but it came out different than just cube-cube-cube. So, that was something to think about because if, uh, you know, they're using cube and pencil, it's two, things of two different length combining to make nonstandard measurement, which is fine. But, um, to think about doing that intentionally.

TRACY SOLA: And then, the third group, um, decided there had been a lot of talk about a number line, and they decided to make a number line. And so, they just started stacking numbers, writing numbers from the bottom, one, two, three, four, five, six, just up in a vertical line. And many of them drew a line, and then, uh, wrote their numbers next to the line. And, um, I think the thing that happened that made it ... Well, one of the hallmarks of nonstandard measurement is whatever your unit of measure is, it stays the same. And so, they were writing numbers all different sizes. And so, I don't think they were really in a place to realize yet, or in that short period of time, that all the numbers had to be exactly the same height for that to work. So, that's just that idea of scale on a number line that is also, I think, um, a, a standard in, in subsequent grades. It's not as standard now.

TRACY SOLA: But, uh, but it was interesting to see them trying to make use of a number line to figure out that problem. And I don't think, even though the two large giraffes were essentially about the same size, some of the students got, you know, 19 for one and 12 for the other one, or something like that, and that didn't seem to bother them. So, so, just stacking up numbers and feeling like they had done their job, that seemed kind of, like, where they were in their development. And so, there're a lot of interesting places that we could go from here with those pieces of work.

TRACY SOLA: So, the other interesting thing that happened in this lesson was, my plan had been to give them one cube and have them use it, um, iteratively. And then, to give them more cubes, and have them stack cubes and see if their iterative measurement, uh, matched their, their cube train measure. But wh-- I changed my plan mid-lesson, because when I saw that some kids were using a number line were, weren't using cubes at all. I didn't wanna force the

lesson in a certain way because they were using a strategy that they had invented for themselves. And most of the class did that. And so, I didn't want to negate that strategy as valid, so I just decided not to go in that direction and to talk about all the different strategies we saw, and because that's where the kids were.