AMY BURKE: My name is Amy Burke and I am the Teacher on Special Assignment supporting the high school math program here in San Lorenzo Unified School District.

DEIDRE GREVIOUS: I'm Deidre Grevious. I'm the math department co-chair at Arroyo High School.

AMY BURKE: And we're going to be working today in Deidre's fifth-period mathematical modeling class. So these students are 11th and 12th graders.

DEIDRE GREVIOUS: Amy, how do you plan on intentionally integrating in the standard mathematical practice?

AMY BURKE: So, today's lesson is going to use the students compiling a dataset and then fitting a regression curve to model the behavior of that dataset. So we can definitely see the modeling with mathematics is one of the practices that we'll use. They will also be making sense of and persevering in problem solving by just beginning with the conjecture and then coming back to their conjecture to correct it as they move forward. During partner talking table talk, they will be putting forth their own ideas and then critiquing the reasoning of others. And then there's also absolutely the use of appropriate tools within this lesson because students will be doing a bit of work using Desmos.

DEIDRE GREVIOUS: And do you see any or have any foresight into anticipated glitches or anything that you might need to support?

AMY BURKE: Some of the things that I think will be hold ups for students are maybe when they get into Desmos, scaling their axes, determining how to make sure that they can see the correct window so that they can see the dataset. I'm also curious to see if students will know what to choose as their independent and dependent variable. I'm imagining that some students might compare the length versus volume instead of what I'm imagining -- or the independent variables of actually the cut size versus the volume. So, I'm thinking those are two things that students might struggle with.

DEIDRE GREVIOUS: And are there any particular strategies for teaching that the standards are calling out in this lesson? So, do you think there are any specific strategies around your teaching that are getting called out by these standards?

AMY BURKE: So, I think that we've worked to make sure that the students are going to be talking to one another; so, we've built in some pair share. And also we're going to be using the group roles so that all students have a, you know, job to do, and an expectation and a responsibility for the group process and their conversation. There are some questions that I think will elicit conversation around, once we have the dataset plugged in: Will the quadratic function fit it best or will the cubic function fit best? So, I think, and also, of course, the pausing that we'll do to go back and revise the conjecture throughout.

DEIDRE GREVIOUS: And what particular skill sets are the kids supposed to have, previous to this, that you think that they're going to make use of in this lesson?

AMY BURKE: So, I think really the work with Desmos, their facility with that tool is going to be important. So, just being able to put values into the table to set the window for the graph and to use the regression function to get a model for the data — those are some of the skills that they really need.

DEIDRE GREVIOUS: And then, have you planned some probing questions for them to help them articulate their thinking?

AMY BURKE: Yeah. So I'm thinking that while they are working with their table group to determine which function will best fit the data, and also while they're looking at understanding what the *x* intercepts mean, I have some questions set that just ask them to go further with, Why are they thinking that?, or maybe asking about a different point on the model to see if that can help them to better understand the intercepts.

DEIDRE GREVIOUS: And I know that you've been super-thoughtful about how the lesson is going to pan out; so, do you have some points at which you're going to stop the students for a group discussion or independent discussion with their partner?

AMY BURKE: Yes, yes.

DEIDRE GREVIOUS: And these are marked out for you already?

AMY BURKE: Yes, yeah.

DEIDRE GREVIOUS: What ways have you thought about differentiating the lesson to accommodate any student needs that might be in the room?

AMY BURKE: Right, well, first I want to thank you for sharing your students with me. This is not my typical class, so I don't know the students all that well. And so, I think that some of the strategies that I've put in place around the group work, so that peers can be supporting one another through the, the task, is one way for differentiation.

AMY BURKE: Also, I made decisions about just really basic things, like, just each pair will get the paper, so that that will ensure that someone is reading the questions aloud. And again, with the Chromebooks, that there will just be one per pair, so that, again, students are working together instead of just working individually or not working individually.

AMY BURKE: And again, I think the pauses that we've built in to ensure that students have time on their own, to have their own idea before coming together and contributing.

DEIDRE GREVIOUS: Yeah, so it sounds like you've been really thoughtful about the different ways that students are putting themselves out there and having a responsibility to one another, making sure that there are conversations that you get to help facilitate, but not control.

AMY BURKE: Mm-hmm (affirmative).

DEIDRE GREVIOUS: Yeah, that's good.

AMY BURKE: Yeah.

DEIDRE GREVIOUS: So, when we originally looked at this lesson, it kind of felt flat in that, if you just put it in front of the students, it's just there. And it seemed like it would be relatively challenging independent work. So, what did you do to look at the lesson and bring it more active, for the students to become better participators?

AMY BURKE: Right, so when we looked at the lesson, one of the things that jumped out was that there were going to be multiple representations of this phenomenon, mathematical phenomenon.

AMY BURKE: And so, definitely building in the time for students to have their hands on, and sort of, building the boxes is one way to bring that out, so that they have that tactile understanding of what we're actually talking about when we're talking about the cut size versus the volume.

AMY BURKE: Also, just looking at filling a class table and asking students to then take a moment to digest that table and talk to each other about their original conjecture, I think, will be a way for them to really interact with the mathematics in front of them, rather than just writing things on their own paper, without thinking necessarily.

AMY BURKE: Going into the Desmos portion, where students will see table and graph next to one another, I think that we've built in some prompts, or I'm hopeful that we've built in some prompts that will have them kind of making those connections between. And I'm planning to come back at the very end of the lesson, back to the physical model, to see if we actually really understand what the graph was telling us.

DEIDRE GREVIOUS: So, there's also some interpretation as well as looking and touching and plotting. But all of these things then bring it more alive for them.

AMY BURKE: Right.

DEIDRE GREVIOUS: Yeah, great.

AMY BURKE: Yeah.

AMY BURKE: We're going to do a great, a great — we're going to do a great task today, guys. Am I selling it? Do you believe me? It's going to be a task that asks you guys to be multimodal, meaning you're going to build something, then you're going to gather your data and organize it in a table, then you're going to use that technology that we've been practicing, the Desmos, to model your data set. And then you're going to interpret your data.

All the while, we are hoping that you guys are having conversations that are sharing your own thinking and pushing the understanding of the other people who you're sitting with. So, to help those conversations we have on your tables, and I know you have these in Miss Grevious's classroom too, some group roles. So, I want you to please pause and to take a look at your own group role and to read the words to yourself about, What are the questions and contributions that I'm expected to make at my table. So I know these are familiar to you. They're not new. So, but I do want to give an opportunity, if there's any questions or anything you're unsure of for your role to help support your group today.

Okay, great. Then I'm going to pose the problem. The problem is this: You're going to be given a rectangular grid that is on centimeter paper. There are, as I've written here, 19 centimeters across the top, 25 centimeters along the side. What you're going to be asked to do is to figure out, How could I use this amount of grid paper to build a box with the largest volume? So, what I'm going to ask actually is for one student to help me pass out some paper. Don't all jump up at once. Thank you so much Stacy. Thank you. Okay.

And right now, you're not going to do any cutting, but you are going to just get one of these pieces of paper and you can draw onto it. You would cut squares. Sorry, I need to be more clear. You can only cut squares out of the side. You cannot cut like a rectangle. You cannot cut something that's a three by two. You need to cut an actual square out, being a three by three or four by four, and you'll cut those out of each side. No one's going to cut right now. I just want you to have the paper so you can look at it to help you visualize what you'll do. Yes, ma'am.

STUDENT: Are we making a top to cover the container?

AMY BURKE: That's such a great question. Thank you for asking. We are not making a lid. No lid. So it's an open-top box.

Okay, so I'm going to ask you individually right now, so this is not yet time to talk to a partner, but I want you to take a look at number one, and it asks you to make a conjecture about how to cut squares from the side of your rectangle so that you would build a box with the largest volume. Build an open-top box with the largest volume. Will you please take a moment on your own to think about it and to write into question one, what you think.

AMY BURKE: Okay. Will you please turn to your partner and share your original thinking? We may come back and revise this after we do some data collection. Please turn to your partner and share your original thinking.

STUDENT: I said, "I think the one-cut will maximize volume of the box because it will still have a big size, and it'll only be cut by one square rather than one or more."

STUDENT: Mine is actually kind of similar, because I think one cut will minimize the volume of the box because it reduces the box size and shape.

STUDENT: All right.

STUDENT: And you only take off, like, this much.

STUDENT: I know what you mean.

STUDENT: You know what I mean?

STUDENT: So, you can cut the two out right here on each side, and then how tall it would be, and how wide it would be? You get more.

STUDENT: Yeah. Since you're taking out less, it would, like, have more.

STUDENT: Yeah. I know what you mean.

STUDENT: Yeah. You know what I'm saying.

STUDENT: I mean, I said the five would, because you take out – you pretty much take out five units and it would be kind of taller. Know what I mean?

STUDENT: Mm-hmm (affirmative).

STUDENT: It would be so it could hold more. That's what I was thinking. Or like the same width.

STUDENT: Okay.

AMY BURKE: Okay, please wrap up those initial ideas. And I want to remind you, these are just our initial ideas. We have not gathered any data yet. I want to clarify two things. One thing is a student asked me, What did I mean by a size cut? So, I should have said this before: If I cut out a square like this, I'm going to call this a four-centimeter cut. Why is it centimeters? Because we're using centimeter paper. Okay, so the cut for the square is a four-centimeter cut. The second question that I had was, What did I mean by, "What type of model?" And so over the past couple of days, we've been looking at data sets and using different regression curves to fit those data sets. We've looked at a linear model, we've looked at a quadratic model, and we've looked at a cubic model. So, that was what I was intending there. Again, these are our initial thoughts. Thank you for doing that, sharing with your partners.

AMY BURKE: I should have also said that you may have a camera, like, just very close on your face, so I don't want you to be too startled by that. We're really trying to capture what you're saying and what you're doing. Like if you're pointing at something or if you're, like, turning your paper, even just, "How are students interacting around this?" Okay. So, you guys are great. Thanks for being good sports. Okay, cool. So, you know what, we are going to go ahead and, um — actually, I'd like to hear, maybe, two students who would share their original conjecture with all of us.

STUDENT: I thought of a four-by-four, so I can get the middle to be wide, but the size to also be tall.

AMY BURKE: Good. Thank you. Thank you so much. Alexander.

STUDENT: And I said a nine-by-nine because it's, like, directly half — or almost directly half of nineteen, so then you could use that to spread out the whole thing, and you have, like, a big middle base and then you also have, like, a wide spread.

AMY BURKE: A wide spread. Can you say another sentence about the wide spread please?

STUDENT: Like the wide spread on the grid. So it's like it'd be nine over instead of just, like, small and [inaudible]. So, then there's a longer volume, a longer base.

AMY BURKE: Thank you. One more. Thanks. Vincent

STUDENT: I think the one-centimeter cut will maximize the volume of the box because we're just getting rid of four boxes total.

AMY BURKE: Wow. So I just heard a four-centimeter cut is going to maximize, a nine-centimeter cut will maximize the volume, and a one-centimeter cut.

AMY BURKE: I'm super curious. Aren't you guys? Let's check it out. So ... because of our time in the class I've pre-cut ...

AMY BURKE: ... Resource manager to head over near the Chromebooks and grab one set of tape, please? You're just going to build the pre-cut ones that I have.

AMY BURKE: Will you guys please go ahead and build the box and then send your recorder reporter up here to fill in, what is the cut size, what is the length, the width, the height, and the volume of what you are building right now, okay? And what are each of these measurements in terms of? I want to talk about units quickly. The cut size? What are the units that we're using for that?

STUDENTS: Centimeters.

AMY BURKE: Centimeters. Okay. How about the length? What are we counting there?

STUDENTS: Centimeters.

AMY BURKE: Again, centimeters. How about the width? What are we counting there?

STUDENTS: Centimeters.

AMY BURKE: Centimeters ... The height?

STUDENTS: Centimeters.

AMY BURKE: And the volume?

STUDENTS: Centimeters cubed.

AMY BURKE: Centimeters cubed. Interesting. Okay, great! Cool. Will you build that very quickly and then send your recorder reporter up here ... with the actual box? Thanks.

STUDENT: I think the cut size is a four-by-four because if you could count it, there's four boxes here. There's four here, they are both coming out. It'd be a four-by-four square. So that's the cut size. Then, I'm not sure about the length though. I'm not sure if it would be this, so that's the full length and everything, right? This right here. Is that the full length then?

STUDENT: Shouldn't we build it first? Tape it and then let's figure it out.

STUDENT: I was going to do it [inaudible].

STUDENT: So, what are you doing, measuring all of them?

STUDENT: Yeah.

STUDENT: He's counting the length and width so he can find the volume of the square.

STUDENT: We're just doing a bunch of comparisons for this —

AMY BURKE: So get it taped, counted up, and then we'll get it up here quickly. Thanks guys.

STUDENT: Could you have seven cut, and try nineteen from seven and twenty-five from seven?

STUDENT: Oh yeah.

STUDENT: Did you get it?

STUDENT: Yeah.

STUDENT: Let me see this. So, the height is one, two, three, four, five, six, seven. The width is one, two, three, four, five. And then five, ten, eleven. All right. Yeah.

STUDENT: So what do we notice? All right, so we notice that it was cut by an eight-by-eight on the corners. What do you notice about the side heights?

STUDENT: Not sure.

STUDENT: Wow, look at that.

STUDENT: Aye, lit.

STUDENT: Why are you guys using so much tape?

STUDENT: I don't know, I thought it would be easier if you just get the whole thing done in one go. But now that I think about it, it would be easier if we just did like —

STUDENT: I'll tape the next one.

STUDENT: Yeah we need —

STUDENT: I drew 225.

STUDENT: Yeah guys, collaborate.

STUDENT: Thirteen by twenty-five, right? 325 not 295 —

STUDENT: You said thirteen? It's fifteen.

STUDENT: Yeah, it's fifteen.

STUDENT: Oh yeah, I'm tripping, I'm tripping.

STUDENT: I already calculated it.

STUDENT: You calculated it? What did you get?

STUDENT: Huh? How? It's ...

STUDENT: Seven times seven times seven.

STUDENT: What? At first, I did fifteen times five. This is what I did. I did fifteen times five, which gave me seventy-five. Then I multiplied that by nine. What?! Did I do it wrong? Hold on.

STUDENT: Because, you know, when you just see everyone else's, it looks so much bigger.

STUDENT: Where's the second base?

AMY BURKE: Will you send your reporter recorder up to put the info up, please?

AMY BURKE: Which is yours?

AMY BURKE: I'm going to ask you now. The next prompt on the worksheet has you taking a look at this table, which includes the actual open-top boxes and asking you to write down what do you notice from this table and what do you wonder about this table? So, you are invited right now to talk aloud or to take a moment maybe first to write your own observations and then to share.

AMY BURKE: Will you take maybe three minutes together to write down what you notice and what you wonder now that we have some data?

STUDENT 3: I have a question. What is this word?

AMY BURKE: I don't even know what that is. It's from Ms. Grevious's class. I'll take these guys back up.

STUDENT 4: -- has the most volume with 798 cubic squared and then nine has the least with sixty-three cubic squared that's kinda, and then it shows that these ones are a lot higher than the other ones are. They have a lower base or ... the width is a lot smaller, or length is smaller than the height. You get what I'm saying like that?

STUDENT 5: Just wondering is, like, they're all from the same paper, but they all have different volumes.

STUDENT 4: Yeah. I think it's because of the cut size. That's a good question though.

STUDENT 6: Okay, I wonder.

STUDENT 7: What are you wondering?

STUDENT 6: Why does ... I wonder why does the bigger numbers have the smallest volume. What did you guys put? What do you wonder?

STUDENT 8: What do I wonder? Oh yeah. What I put was basically what we just solved or concluded which was the bigger ... the bigger, like you say, the length and the height, the greater the volume.

STUDENT 9: So I wonder if the cubic volume that is shown on the table will be grafted onto graphing, plot form into like, a parabola, almost.

STUDENT 10: Can you repeat that first part?

STUDENT 9: So, I wonder if the cubic volume that is shown on the table will be grafted into, like, a parabola or something?

STUDENT 10: Parabola?

STUDENT 8: I think so, yeah.

AMY BURKE: I know that you guys have noticed and wondered certain things about this data set. Will you please make sure that you've answered the questions about -- from the table, from the table. What's the maximum volume, and what is the cut size that gives us that maximum volume?

STUDENT 11: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21.

AMY BURKE: You want to fix something? Yeah that'd be great. Great. Ah, I see people taking away one of the questions. Yeah. There was a small error. So, it was a five before. Now it's a six.

AMY BURKE: Okay, I'm going to ask you to revisit now, and revise your conjecture. So, I noticed some students weren't sure what to put down. They weren't sure what idea they even had at the start. So, please don't go back to problem one. Instead, flip your paper, and take another moment. Now that you see this data set, what is the cut size that you believe will maximize the volume?

STUDENT 1: It's twenty-two.

STUDENT 2: What?

STUDENT 1: Twenty-two.

STUDENT 2: -- from both sides.

STUDENT 1: There’s twenty-eight -- there’s twenty-five.

STUDENT 2: Oh, there’s twenty-five. If you get rid of one, it will be twenty-three.

STUDENT 1: Oh.

STUDENT 2: Times seventeen.

STUDENT 1: But it's still wrong.

STUDENT 2: Yeah, I know.

AMY BURKE: Yes sir?

STUDENT 1: I have a question about the cut size for the two-centimeter one, because they put their dimension by twenty-one, nineteen and two. But if we get rid of two for the nineteen side, you would only get fifteen, so wouldn't it be, the volume would be 630 inside?

AMY BURKE: Interesting. So that is for the cut size of two?

STUDENT 1: Two, yeah.

AMY BURKE: I think you have a valid argument there, but I want to allow for our data set to stand as it is, unless folks are recognizing their own. We'll see how that argument might come out later.

STUDENT 1: Okay.

AMY BURKE: Okay, thank you for that.

STUDENT 1: All right.

STUDENT 3: The centimeter one is a lot bigger than ... it has the most volume.

STUDENT 3: And is eleven the maximum cut? Yeah, so the biggest cut here is nine. I don't know.

AMY BURKE: Okay.

STUDENT 3: Yeah, that's the maximum.

AMY BURKE: Okay, so, will you please ...

AMY BURKE: Send your resource manager over here to grab just two Chromebooks -- you're going to share with your partner, okay? So please grab two Chromebooks and follow along. Facilitators, will you make sure that you're moving your groups into step four?

STUDENT: Okay, oh the *x* and *y* are probably the cut size and the volume.

STUDENT: Yeah.

STUDENT: All right, so it starts off with 6.

STUDENT: 5.

STUDENT: Tell me the numbers as it goes down.

STUDENT: For cut size?

STUDENT: Yeah. So, it starts with?

STUDENT: Then it's 5. Then it's 6, 7, 5, then it's 2, and then it's 7, 9, 8, 7, 3, 8.

STUDENT: Wait, what's before that?

STUDENT: 7, 9, 8.

STUDENT: 2, 7, 4, and 3.

STUDENT: 8.

STUDENT: All right, and then next you got to read out -- let's do -- what do you do for one?

STUDENT: 3, 8, 5, 7 and 3, 8, 5 and then it's 741, 48 -- yeah 4, 748, 9, 63.

STUDENT: 63?

STUDENT: Yeah.

STUDENT: What are the numbers?

STUDENT: What, like that? Those numbers?

STUDENT: Wait, but how would you do that?

STUDENT: Pick a different table.

STUDENT: [inaudible]

STUDENT: Huh?

STUDENT: [inaudible]

STUDENT: Wait, so how would we put the data in? Wait, what?

STUDENT: The cut size and then whatever the volume is.

STUDENT: The length of the sizes, the weight, and then the ...

STUDENT: Oh, oh wait no, you just put the cut size and then volume at the end, right?

STUDENT: Yeah.

STUDENT: Yeah, okay, I got you. 6 ...

STUDENT: Go in order, go to like 9.

STUDENT: That number up there, that's -- Anthony ...

STUDENT: 6. The cut size, right?

STUDENT: For which one?

STUDENT: The first one.

STUDENT: 6.

STUDENT: The first one is for length?

STUDENT: No, for cut size.

STUDENT: Just what I said, I said 6.

STUDENT: 6? All right, and then the second one.

STUDENT: Yeah, the first one is 6, 5.

STUDENT: 5.

STUDENT: 2.

STUDENT: 2.

STUDENT: 7, 3, 4.

STUDENT: 7, 3, 4.

STUDENT: Wait, wait, 2, 7 ...

STUDENT: 3, 4, 9, 8.

STUDENT: 3, 4 ...

STUDENT: 8.

STUDENT: 8, 9?

STUDENT: 9, 8.

STUDENT: 9, 8, all right, I got it all. All right, now it's 5, 4, 6.

STUDENT: Is the cut size and then y is going to be the volume, so -- but like up there, the first one is 6, then the second one is 5. Type it.

STUDENT: 5, 4, 6.

STUDENT: 2, 7, 8.

STUDENT: So, this is all in *x*'s.

STUDENT: Yeah, *x*'s are the cut size.

STUDENT: 3, 4, 9, 8. They're not in order.

STUDENT: Yeah, they're not in order, it doesn't matter, it's just going to make a scatter plot.

STUDENT: Oh yeah, you right, you right.

STUDENT: So, it doesn't have to be like a going up thing.

STUDENT: Hey, can I see the table?

STUDENT: Hold on ...

STUDENT: He's almost done, yeah, we only have like two more. So, you got the *x* already, right? Oh, what? That's not right, hold up.

STUDENT: Oh yeah, wait, it's all on the line.

STUDENT: Oh no, it's just ... Wait, what? I'm confused. Like because the increase is so ... Wait, why is it like that? Where is it? What? Why is it just going straight up?

STUDENT: It's because one of them is 63? Or I did. Maybe it's the wrong order.

STUDENT: Huh?

STUDENT: Maybe we had to set it in order. That wouldn't affect it, right?

STUDENT: No. Okay, that's ...

STUDENT: 6 like ...

STUDENT: No, because why, why is it like that? See, if it was right it would be, like … because it's on the *x* axis, right?

STUDENT: Maybe because you can barely see it, yeah, because it's zoomed out so it all looks the same, in the same area maybe.

STUDENT: Wait, are you doing -- are y'all experiencing the same problem?

STUDENT: Yeah, it's all straight, because it's going straight up, right?

STUDENT: Yeah.

STUDENT: Yeah, that's not right. Is it because of the ...

STUDENT: Maybe if it's like closer, you know?

STUDENT: That doesn't work ... What? That's so weird. Hey, Ms. Burke?

STUDENT: Like 6 to 8.

AMY BURKE: So Anthony, can you say what you're doing right now so that Ivan can do it too, because he can't see what you're pressing.

STUDENT: So what I just pressed, I went to the wrench up here in the corner.

STUDENT: The graphs setting.

STUDENT: The graph settings, go to graph settings.

STUDENT: Yeah, what about it?

STUDENT: Go to that, and put it on projector mode.

STUDENT: Projector mode? Uh huh.

STUDENT: And then take off the minor grid lines.

STUDENT: Minor grid lines off.

STUDENT: Are the *x*-values from here to here?

STUDENT: All right.

AMY BURKE: So Karla's asking about the *x*-values. So you're thinking from where to where?

STUDENT: 6 to 8.

AMY BURKE: 6 to 8? Do you guys think the *x*-values going from 6 to 8 will capture all the data?

STUDENT: No.

STUDENT: Do you know what I'm talking about?

STUDENT: No, because you're still missing other parts of that.

STUDENT: So 1 to 10.

STUDENT: Yeah, 1 to 10.

AMY BURKE: What do you think about that?

STUDENT: 1 to 10.

AMY BURKE: Okay.

STUDENT: 1 to 6.

AMY BURKE: All right.

STUDENT: And then what could we do for the *y*?

STUDENT: 263.

STUDENT: 10. Can you start at 10 and go up to 800 [inaudible] and then count by tens? Could that be it?

STUDENT: Connected?

STUDENT: Maybe it should be the--

STUDENT: To what number?

STUDENT: I think they're all positive.

STUDENT: Orange? What number you want to go up to?

STUDENT: Up to 800.

STUDENT: We're going to ... Because maybe we did it the wrong way, so we're going to try the [inaudible].

STUDENT: There's two dotted lines.

STUDENT: I don't think it's [inaudible].

STUDENT: Wait a minute.

STUDENT: I think it's put a negative [inaudible].

STUDENT: No, that's--

STUDENT: Wait, did I put a negative? Yeah, I did.

STUDENT: Dude, can you manage here to put them in order?

STUDENT: Oh, we forgot this step.

STUDENT: Oh. There's 1 to 10, right?

STUDENT: Yeah.

AMY BURKE: Is any group ready to move forward? Oh, great! What type of function do you think it's going to be? You're thinking cubic?

STUDENT: Isn't cubic the one that goes up, like that?

AMY BURKE: Is that what a cubic is?

STUDENT: Wait.

STUDENT: Yes. It goes like that way.

STUDENT: No, like it starts from the bottom. Then goes up, then it goes back up, right?

STUDENT: Yes.

STUDENT: Yeah. Okay. Cubic.

AMY BURKE: Do you think this data fits a cubic?

STUDENT: Yeah.

AMY BURKE: Well let's check it out.

STUDENT: Or a linear.

AMY BURKE: You guys are using one of the computers, right?

STUDENT: Yeah.

AMY BURKE: And you guys use the other? Okay. So, facilitator, I need you moving your group forward.

STUDENT: Come on, facilitators.

AMY BURKE: How are we doing here?

STUDENT: So one, use a regression feature to find a function to model the data. So, you can find the [inaudible] from yesterday from the cubic function and then the other function.

STUDENT: With the equation?

STUDENT: Huh?

STUDENT: With the equation.

STUDENT: Yeah. Can you find the [inaudible]

STUDENT: I have one.

STUDENT: You have one? Which one do you have?

STUDENT: Quadratic.

STUDENT: Quadratic, all right.

STUDENT: What is it? I'll put in for this one.

STUDENT: You have a quadratic for that one?

STUDENT: Yeah.

STUDENT: I have a quadratic for this one.

STUDENT: What is it?

STUDENT: It's Y1.

STUDENT: Y1.

STUDENT: Approximate.

AMY BURKE: Say words to Valentin. What did you do so they can do it too?

STUDENT: I opened up the, is it called the wrench?

AMY BURKE: They've been calling it the wrench, because it's for the tools.

STUDENT: So for my *x* axis I made the, what is it called?

STUDENT: The numbers?

STUDENT: Yeah, the numbers. I changed the numbers from 0 to 10 so it could capture the *x* axis because my, like, let the … the smallest number is 2, and the greatest number is 9, and then my *y* axis, I made it go from 50 to 800. And then my step for my *y* axis, I made it to 100, and the step for the *x* axis, I made it 1.

AMY BURKE: Are you getting all that, Devin?

STUDENT: Yep.

AMY BURKE: Did it make sense where she got all those numbers?

STUDENT: Yeah, I think.

AMY BURKE: Can you tell me why?

STUDENT: She was using the lowest point in the graph -- she's using the lowest and the highest point of the graph to then fix the way it looked.

AMY BURKE: Awesome, thank you. Great.

STUDENT: [inaudible] Wait, it changed, it should be …

AMY BURKE: So have you guys had a chance to talk about this question yet? What type of function could we use to model this data?

STUDENT: No, we haven't had a chance to.

AMY BURKE: Okay.

STUDENT: The equation for the …

STUDENT: It's *y* and then it's the tilde.

STUDENT: The tilde, *a*, *x*, cubed.

STUDENT: Make sure there's a *y*, 1, tilde, the *a*, *x*, 1, cubed.

STUDENT: [inaudible] You said not to, don't connect the dots.

STUDENT: Yeah plus *v*, *x*, 1, squared, plus *c*, *s*, 1, plus *d*.

STUDENT: [inaudible] 1 plus *d.*

AMY BURKE: What do you guys think the function might be?

STUDENT: I think that from when these, the cuts, I think it decreases every time just, like, a little bit and it gets a more dramatic cut every time you go down one number. Each cut size, the volume goes down.

AMY BURKE: So what type of function do you think might model this data then?

STUDENT: Decrease.

AMY BURKE: A decreasing function?

STUDENT: Yeah, something like subtraction or something.

AMY BURKE: So we've talked about a linear function, a quadratic, and a cubic. So which one of those do you think it might be?

STUDENT: Probably a quadratic then.

AMY BURKE: You think a quadratic, okay. Can I ask you guys to hit your wrench? Because I'm concerned about that you’re not seeing the data as well as if you set your window differently. So I see you're hovering over the *x* values, right? What are the *x* values and how might we get a better picture of it?

STUDENT: So you put 0, right?

AMY BURKE: Why 0?

STUDENT: Because then it shows like the grid [inaudible].

AMY BURKE: Okay.

STUDENT: And this one is 100 isn't it? Or is it 10?

AMY BURKE: What are you looking at to determine what to put in there? 100 or 10?

STUDENT: [inaudible]

AMY BURKE: Sorry?

STUDENT: I think you could put in 11 because it starts at 2, so we're going back 2, so we could go up 2.

AMY BURKE: So you're going up 2 from what?

STUDENT: From 9.

AMY BURKE: From the 9, ah! What do you think of the graph that you had now? Do you think that's a better picture?

STUDENT: Yeah.

AMY BURKE: Right? Larisa, do you need to ask Carlos to say it again?

STUDENT: Yeah, you put 11, right?

AMY BURKE: Why did he put 11?

STUDENT: Because it's more -- 2 higher than the *x* values.

AMY BURKE: Um hmm, great, I might disconnect those just so you can see the data set and then I'm going to ask Alexander, that you move your group forward, on to this please.

STUDENT: Their computer won't give them a color for this. Yeah, I pressed this. So it shouldn't, but then it didn't, and it says, "I don't understand this" and then ...

STUDENT: Our parabola is missing.

AMY BURKE: Hm, isn't yours working?

STUDENT: Yeah, ours is working.

AMY BURKE: Can I ask you guys to say we are having a technical difficulty and the whole group work off of that one? So we can move forward, facilitator? I need you facilitating your group forward on this work.

STUDENT: So just log off.

AMY BURKE: You're already doing the mathematics that's on here [inaudible]. Thank you.

STUDENT: So you see how I did it?

STUDENT: Wait, hold on.

STUDENT: You still putting them in order?

STUDENT: Yep.

STUDENT: Because once you're done, you're gonna click the wrench.

STUDENT: Just write it down.

STUDENT: Mm-hmm. [affirmative]

STUDENT: I was about to say like, "Oh yeah, I'm just going to write down--"

STUDENT: You're subtracting.

STUDENT: Yeah, I got it.

STUDENT: Wait, do you want to switch these two? Or you just want to leave it?

STUDENT: What do you mean?

STUDENT: Do you want to switch it? Or you're talking about adding this and the other one up there?

STUDENT: Yeah, but I think ... I think we got it.

STUDENT: Okay.

STUDENT: With the way it looks.

STUDENT: Okay, use your model to find the maximum volume of the box.

STUDENT: Mm.

STUDENT: I don't know how we're going to start with that.

STUDENT: I think it's just the -- the intersections with the purple--

STUDENT: Oh, we plug them in.

STUDENT: Yeah, with the purple line and then with the black line. Like, to see what intersects.

STUDENT: Yeah. So, take a look at some of these numbers, see what happens if we ...

STUDENT: One centimeter.

STUDENT: One centimeter?

STUDENT: Yeah.

STUDENT: One centimeter is going to maximize, um, the volume for--

STUDENT: What do you guys think ... I know why it maximizes it but I don't know how to put it into, like, mathematic terms.

STUDENT: I mean, I just put, "The smaller the cut, the more grid space left over to make the box."

STUDENT: No, the first one. Try the first one. Oh, wait, the first one, it can't be. It can't be the first one because it's negative so ...

STUDENT: 9.286.

STUDENT: So that -- it's -- Wait a minute. Okay, so that's the cut size, volume--

STUDENT: The *x* intercept is 9.286, meaning -- what is the cut size? It’s 0 centimeters?

STUDENT: ... so this is the *x*. [inaudible]

STUDENT: Yeah, but the equation would always be like that.

STUDENT: So, uh, do you have 5 yet?

STUDENT: Wait, wouldn't the equation always be like that? So why would we count it?

STUDENT: Hm?

STUDENT: This part? Why would we ... why would we get the *x* intercept from here if it's always like this when it's not what we're--

STUDENT: Just so you don't have to worry.

STUDENT: Yeah, you don't have to worry about this line because uh ...

STUDENT: It's negative.

STUDENT: Yeah, it's negative. On the *x* intercept. But if it’s like right here, then that would be like the *y* intercept right there. Because it's on the *y* axis.

STUDENT: No, I'm asking because our data doesn't go to … on an *x* axis, but the equation here, it will always hit an *x* axis, so why are we using it?

STUDENT: That is true. Let me see.

STUDENT: What is this question even asking?

STUDENT: Like, this is 963 but it can -- it can -- it can go more though, right?

STUDENT: If it keeps going ...

STUDENT: Yeah. [laughs]

STUDENT: So, it could be this, I think.

STUDENT: Can it go more? Can it go more though? Let me add another one.

STUDENT: Our data can't. Unless you added 10.

STUDENT: Yeah. If you add more data, it's kinda gonna follow the same line.

STUDENT: Ten. Can someone find 10 centimeters on here?

STUDENT: We don't have 10, it only goes up to 9.

STUDENT: All right, so if we had a box of 10, so ... if this is 19, that's 25 so ... 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. So, 1, 2, 3, 4, 5, 6, 7, 8 ...

STUDENT: That's what we did, right?

STUDENT: So this will be the cut. That -- but that's too large.

STUDENT: Oh, I -- wait, what's the value for the height? I think that would be for 6, like, because 10 is too big.

STUDENT: Mm-hmm. [affirmative]

STUDENT: I don't know because then if you look at the graph, like, 9 is up here, 8 is here, but then 10 is like below it so that's negative. You would get negative volume.

STUDENT: So we don't have it.

STUDENT: So it's impossible.

STUDENT: Yeah. It only goes to 9, that's why.

STUDENT: So, we don't have an *x* intercept.

STUDENT: We can fight that we don't have one. [laughs]

STUDENT: Unless we can use the negative? I know we can't but, like, is v-- well, it's volume, it's not telling you how many people ... it's not by people, it's by just, you know, a graph. So what if we can use negative, too?

AMY BURKE: Okay, wrap up your conversations.

STUDENT: Our data doesn't hit the *x* intercept, and I don't think we should use like--

STUDENT: The negative.

AMY BURKE: Hm. Why not?

STUDENT: Because, yes, this is, like, the cubic equation, but it's the equation from -- it's the cubic equation so no matter, like, whenever you input it, it's always going to hit the *x*--

STUDENT: Over at zero.

AMY BURKE: Hm.

STUDENT: Because it's not our data, I don't--

AMY BURKE: Mm-hmm. [affirmative] Okay.

STUDENT: It's just better to, like, see if our data is like ...

AMY BURKE: Is fitting the model.

STUDENT: Yeah.

AMY BURKE: Okay, okay. I hear you.

AMY BURKE: I want to really appreciate how you are all engaging in conversation around the graphical representation that we have of our data set. And then of the model that I see you guys have put into Desmos. I wish we had more time because I have more things planned for us to discuss, but we have only a few minutes left in class. I'm going to actually use all of the minutes for us to talk together. You don't need to stress about putting the computers away or even stacking the papers. I will handle all that. What I want is for the next four minutes, full participation please.

So, we gathered our own data set. There was one mistake that a group had caught with the 6, right? And we didn't go through and double check everyone else's data. So really we're using some data that, had we had maybe a bit more time, we might've double checked everything together. We then represented our data set with a table, and found the maximum from our table. And we then represented the data about the volume on two Desmos with a graph. And you guys have been using your model to determine what is the maximum volume and what does that mean? I was just in an interesting conversation over here regarding the *x* intercepts for a model. And I'm going to ask Linette, for you to speak really loud about your group's question because I think this is something for us all to think about.

STUDENT: -- cubic equation. But I thought because our data doesn't necessarily hit the *x* intercept, that, why would we record the equation when it's not our actual data? So I was asking what is our *x* intercept?

AMY BURKE: And I thought there was something in particular about the 2 that showed up on your model, right? Wasn't there one that was more concerning from your model?

STUDENT: What do you mean?

AMY BURKE: An *x* intercept?

STUDENT: Well, what I thought in the *x* intercept was the parabola, where it ends off in the *x* intercept, which was like 9.286. So for that the volume will be 0 centimeters cubic, but it didn't really hit off in our table that we brought it in. So I don't know if we should write the parabola that hit the *x* intercept or the table in our data.

AMY BURKE: So I'm hearing the question I think from this table is, can we use the model to answer this question or do we need to stick with just what we gathered as a class? Is that your question? What do other tables think about that question? Can we use the model to talk about what might happen, or do we need to just stick with our actual data that we found?

Okay. That's one thing I want us to think about. The second thing that I wanted to share, I'm not able to for some reason get it up there, but would the front group share what you guys found, how you modeled your data and what happened?

STUDENT: The 2-centimeter cut size, the volume was wrong. It's not 798, it's actually 630, and we entered it into the Desmos. The cubic model, the regression line fit perfectly with the points that we entered in.

AMY BURKE: Awesome. Okay. So there was a mistake that you found in our data and when you corrected it, you noticed that the cubic function fit the data much better. Okay. Interesting. You guys, thank you so much. Have a wonderful long break.

DEIDRE GREVIOUS: So how are you?

AMY BURKE: I'm good. Yeah.

DEIDRE GREVIOUS: Good. How do you think the lesson went?

AMY BURKE: I think that the students were really engaged with the mathematics. Not just with the pieces that are kind of easy to engage with, like building a box and coming up to share that in the class table, but I also saw them at the start of the period really thinking and building their own argument when they made their first conjecture. And then I, after conversations that I heard between pairs, I know that many of them shifted their thinking when they were asked to make a second conjecture. So I felt like that was successful from an, like a, engagement level around really turning their brains on.

When we transitioned into using Desmos, there was a lot of conversation around -- around, you know, what is the independent variable and the dependent variable, which is something that I think it's important for students to think about. I didn't see when I walked around the classroom that anyone chose anything other than cut size versus volume. I could be wrong, because I didn't get to see everyone, but I thought that might happen, and I didn't see anyone choosing length versus volume.

And then, there was a lot of conversation about scale and figuring out how to see the data. I went to many groups and looked at what was on their screen, and saw just, you know, the points in--

DEIDRE GREVIOUS: In the cluster.

AMY BURKE: ... a line. And so, sometimes with prompting from me, students were able to figure out, "Oh, we need to change the domain and the range." And sometimes it was simply showing the other side of the table what yours looks like versus theirs, and that conversation would happen.

So I think that their use of the technology and kind of the right questions were being asked of themselves around that. Vincent, up front, recognized from the table that there had been a mistake in calculating. And he ... I loved his reasoning, and I wished that I had found time to have it really shared with the class. But he was noting that ...

Vincent had a great discovery right at the start, when the table went up, that there was an error in the class data and his reasoning was wonderful. He didn't just recognize that something seemed off. It was in the 2-centimeter row up there, and he noted that the 19 centimeters for the width did not make sense. And he showed me on his centimeter paper. He said, "This is 19. If this is 2 and this is 2, then the remainder is 15."

DEIDRE GREVIOUS: Oh, right.

AMY BURKE: And I was like, "That is what ..." Like, "Wow, what a great argument." And he really wanted to tell the class. I made a decision right then not to stop the class to correct that. I was thinking, "Oh, it'll come up. When they look at the graph, they'll recognize an outlier." But tables did not recognize that as an outlier that I witnessed. Did you see some tables?

DEIDRE GREVIOUS: Well, so that was ... I noticed that the same table, on one side of the table, they used the incorrect data and then the other side of the table used the correct data. And then they were looking at and comparing it. “Is it possible that the incorrect data is true?” Right? Like they were saying, "But it's got to be fitting the curve."

And so when they started to fit the curve, then they realized that it's not a possibility. So I think having the incorrect data right next to it was a great argument about what made it wrong, and which one was wrong.

AMY BURKE: Right. Right.

DEIDRE GREVIOUS: So the students were able to determine that for themselves. Yeah.

AMY BURKE: Right. That's great.

DEIDRE GREVIOUS: Yeah. That was great.

AMY BURKE: I knew that they had ... They told me, "We have it differently on ours," but I didn't get to hear all of that. So that's really cool to hear.

I also really liked, at this table and at that table, the groups were really ask ... They were calling me over to ask me more questions, because they were kind of working through what do the *x* intercepts really mean in this case. And so I'm going to call that a positive for the lesson, because I think that it's bringing up real mathematical questions.

DEIDRE GREVIOUS: So how did you push their thinking when they were struggling in that way?

AMY BURKE: Well, I was trying to ask students to really reframe what, what they understood, and then asking them to state again why something didn't make sense. And I thought that I actually did that with this table, but when I walked away, I could tell they were, felt like they didn't understand it any better than when I had walked over. Well, I thought ... Their question was ... They had graphed the data with a quadratic, and they had a negative intercept, and they had said to me, "This doesn't make sense. We can't have a negative 3." And I said, "Okay." So that was not a pushing their ... I mean, I thought I pushed their thinking by asking them to re-explain it to me, but I don't know. They just kind of still seemed like, "Wait, what?" after that. So I'm not sure--

DEIDRE GREVIOUS: They weren't sure about the reality versus the model. Yeah.

AMY BURKE: That was what then came out in the share-out at the end of the class, was they didn't know if they could trust a model, versus the actual data that they had compiled. I knew that that was part of their concern, so that's why I had sequenced their brief share-out right before Vincent's brief share-out, because I thought, "Hey, if we think about this question of, can we use a model versus real data, can we trust it?" And then I thought, "Oh, Vincent's then going to come in and share, actually, our data was a little bit off, and by using the model, we are able to correct ..." So I feel like there were some missed opportunities due to the timing.

DEIDRE GREVIOUS: But also, what about the role of technology? Do you think that the technology was helping them or slowing them down?

AMY BURKE: Um hmm, right. Because something that ... I mean, we could have taken the piece out where they are actually inputting the technology. I could have provided them ... I could have quickly put that into Desmos and made the window and put it up on the screen, and then ask them the same questions.

DEIDRE GREVIOUS: But would that take away or add to their learning, do you think?

AMY BURKE: I think that would take away, though, because I think that those discussions of how do we see our data and ... You know, cause had I done that, I think they would've been quicker to find the model, or I would have just been typing it in. Right. And I don't ... I mean, I think that that would have taken away from their learning opportunities. Yeah.

DEIDRE GREVIOUS: So do you want to look at some of the student work?

AMY BURKE: Yes, let's take a look.

DEIDRE GREVIOUS: And how do you want to sort them?

AMY BURKE: Yeah, so we have two piles right now. One is their individual papers, where they were asked to make a conjecture, to consider the data and have their own noticings and wonderings about it, and then to come back. So I'd like to see if they, if any of them revised their original thinking.

DEIDRE GREVIOUS: Okay.

AMY BURKE: So maybe we can--

DEIDRE GREVIOUS: We'll split it--

AMY BURKE: You and I can just do like a quick--

DEIDRE GREVIOUS: Yeah.

AMY BURKE: Flip to see if ...

DEIDRE GREVIOUS: If there's anything different?

AMY BURKE: If there's anything different. And then--

DEIDRE GREVIOUS: And by different, do you mean the number or by the model?

AMY BURKE: I'm thinking the number.

DEIDRE GREVIOUS: Okay.

AMY BURKE: What I'm seeing so far is that they're all putting 2.

DEIDRE GREVIOUS: Mm-hmm. [affirmative]

AMY BURKE: Because they have that data. Because they have that information from the table, and so they're just saying, "Oh, okay, it's from the table. So that must be correct."

DEIDRE GREVIOUS: Not all of them.

AMY BURKE: No?

DEIDRE GREVIOUS: Nope. Some of them are consistent with their solution from their original conjecture, despite the fact that it was wrong.

And then do you want to sort them for the ones that made the adjustment to the true number? So I have some that ... They changed their opinion.

AMY BURKE: Mm-hmm. [affirmative]

DEIDRE GREVIOUS: But it went from 5 to 1.

AMY BURKE: Mm-hmm. So, but even those that changed it to the 2, I think that they made that change because that's what the table says. It's the maximum.

DEIDRE GREVIOUS: Right, but there are some that still changed it to a 1.

AMY BURKE: I have some that changed it to a 1 as well.

DEIDRE GREVIOUS: Okay, so let's look at those differently then, I guess.

AMY BURKE: Okay.

DEIDRE GREVIOUS: Ooh, I got a 4.

AMY BURKE: And that was left off of the boxes, the one with the one cut. So maybe they're thinking, the smaller the cut.

DEIDRE GREVIOUS: Well, there are also some here with 4s.

AMY BURKE: That changed to a 4?

DEIDRE GREVIOUS: Yes.

AMY BURKE: Oh, interesting.

DEIDRE GREVIOUS: Yes. I'm trying to see if they're at the same table. No.

AMY BURKE: No. These two were up at this table.

DEIDRE GREVIOUS: Yeah.

AMY BURKE: Where was Antonio? Is he at the back corner? Does he wear glasses?

DEIDRE GREVIOUS: No, Antonio has curly hair. He was sitting up here, I think.

AMY BURKE: Oh, okay.

DEIDRE GREVIOUS: Emilio has glasses, so he was ...

AMY BURKE: Oh, yeah. Mm-hmm. Okay.

DEIDRE GREVIOUS: So you did have a number of them then. If you want to measure that, your stack of the ones who changed their--

AMY BURKE: Who changed their--

DEIDRE GREVIOUS: Opinion.

AMY BURKE: Opinion. So that's the majority. So 16.

DEIDRE GREVIOUS: Mm-hmm.

AMY BURKE: Out of ... Who was in here? Maybe--

DEIDRE GREVIOUS: 31.

AMY BURKE: 36. 31?

DEIDRE GREVIOUS: Or 35. You're right.

AMY BURKE: 30 ...

DEIDRE GREVIOUS: 8 times 4 ... 30 ... 31 students.

AMY BURKE: 31. So 16 out of 31 changed their conjecture after looking at the table.

DEIDRE GREVIOUS: Mm-hmm.

AMY BURKE: And these students changed it to 2 centimeters based on what they see in the table.

DEIDRE GREVIOUS: Mm-hmm. And do you think that if you hadn't taken that pause, the thought process of making a conjecture and revising it would have come naturally?

AMY BURKE: I don't think all students would do that naturally. No.

I'm actually realizing I have a ... That's one in there.

DEIDRE GREVIOUS: And so, what do you think was the move that you made as a teacher to help them get to this space? Because we are talking about now the majority of the students.

AMY BURKE: Mm-hmm. Well, I think just that the design of the lesson to have a pause and to ask them to revisit and invite them to revise. I think just that practice of slowing down what we're doing so that students have the time to sort of be metacognitive about their own learning process, you know? That's what, that's what strong thinkers do -- is we think about, and we revise, thinking as we go.

DEIDRE GREVIOUS: Mm-hmm.

AMY BURKE: Right? I'm not calling myself a strong thinker. I'm just saying the royal we. I know.

I put this one on top because I'm interested to read. He says, "I think it will be a 2-centimeter cut, because the smaller the cut size, the greater the length and width even though the height is small."

DEIDRE GREVIOUS: Mm-hmm.

AMY BURKE: So with this student, I don't recognize this handwriting or the name. Who is that?

DEIDRE GREVIOUS: Jeannette.

AMY BURKE: Oh.

DEIDRE GREVIOUS: Linette, sorry.

AMY BURKE: Linette. Oh, so she's really trying to reason through why that might provide the greatest volume.

DEIDRE GREVIOUS: Mm-hmm.

AMY BURKE: Which I don't see as necessarily true of all of the students who did revise their thinking. I see more frequently, “I think the 2-centimeter cut will maximize the volume because ...”

DEIDRE GREVIOUS: I'm looking at ... Yeah.

AMY BURKE: It's the one that has the greatest volume, you know? And there I see maybe three who are really thinking about why is it 2 and relating it to the length and the width that are also considered.

DEIDRE GREVIOUS: But I think you gave them the opportunity in the beginning to justify their original thinking.

AMY BURKE: Mm-hmm.

DEIDRE GREVIOUS: And so that there were some ideas put out there. Because some of the students were talking about the area of the base--

AMY BURKE: Of the base. That's right.

DEIDRE GREVIOUS: And were being considerate of that.

AMY BURKE: Mm-hmm.

DEIDRE GREVIOUS: And then I also heard another student that was talking about the same idea about just using a 1, because that leaves me with the most on the bottom--

AMY BURKE: Base area.

DEIDRE GREVIOUS: Yeah.

AMY BURKE: Right. Right, yeah. Right, and I think also often in math class, teachers and students are about getting answers and moving through content at a pace. And so, I think anything that slows that process down helps learners step up and have the opportunity to really learn instead of just have things thrown at them.

DEIDRE GREVIOUS: Mm-hmm.

DEIDRE GREVIOUS: And so, what were the different things that you think that you introduced into this lesson to help with their reasoning?

AMY BURKE: Honestly, I think that even just providing the little sentence frame that we did allows, kind of walks them down the path of you're going to need to share your reasoning about this. This isn't just going to be an answer. And I think also the turn and talk. So hearing from someone else allows them to build their own argument, that mathematical practice of critiquing the reasoning of others. You're doing that internally whether or not you're asked to like rebut at that moment. And I think also the writing piece, the individual writing piece allows for that.

DEIDRE GREVIOUS: And so, there were some struggles like you said about scaling the axes. Were there other struggles that you noticed as you were trying to get them -- I know we were short on time, but as you're trying to get them to figure out which model to use? So ...

AMY BURKE: Yeah. I didn't, honestly I don't have a great sense of what everyone used as their model so I'd love to look at this, this, because in the moment I wasn't able to recognize who was using quadratic and who was using cubic.

DEIDRE GREVIOUS: Oh right, sort them by ...

AMY BURKE: So maybe we could just go through and look at groups who did quadratic versus cubic. So interesting.

DEIDRE GREVIOUS: Yeah.

AMY BURKE: So, I was expecting coming into the lesson that 100% of groups would use the cubic function, but I think that I had not even anticipated, which is ridiculous that I didn't anticipate, but I had not anticipated there would be an error in calculation.

DEIDRE GREVIOUS: Yeah. Right.

AMY BURKE: And that's -- I mean we're humans, of course there might be an error. So, because of that outlier maybe that's me putting a reason in where I don't know if that's true or not but we have three of the eight groups who chose a quadratic to model the data.

DEIDRE GREVIOUS: Well only because it fits a quadratic relatively well. So, how do you think that you would be able to help these students see that the quadratic model was actually a better fit than the -- I'm sorry, the cubic model was a better fit than the quadratic?

AMY BURKE: Right. Well, I would have loved to have the time to have. I had the front table's Chromebook queued up to try to share theirs, so.

DEIDRE GREVIOUS: And it was a cubic?

AMY BURKE: It was a cubic and it had corrected the data. And Vincent said to me, "When we corrected this one point, the cubic just fell right onto it." I mean those weren't his, those might've been his exact words, but it's just so clear …

DEIDRE GREVIOUS: Right.

AMY BURKE: … when you have the correct data in there. So, on this side of the lesson I'm not surprised that some students didn't get to the cubic and I don't ... I'm thinking about it, how might I have done the lesson differently. And I think, I wonder about whether stopping earlier and asking Vincent to share that correction might have supported everyone in getting a cubic.

But then again, to me that's not really equal to success. I'm not looking at this and being like, "Oh, these kids got quadratic instead of cubic. They didn't understand the lesson today." The lesson was focused on multiple representations of mathematics, and I think that students had that opportunity. And it was -- another takeaway we wanted was for students to do some sort of deep thinking and building an argument and revising that argument as we moved through the lesson. And I feel like we see evidence of that.

DEIDRE GREVIOUS: And do you think then, that because they've experienced this in a completely different way from just doing it on a piece of paper that you'd be able to return to it and revise the work today? If a student was convinced it was a quadratic would you easily be able to readdress it?

AMY BURKE: I think so. Absolutely, right. Not through my voice though, through one of the student’s voices, you know?