

Understanding and Interpreting Fractions

Silicon Valley Mathematics Initiative's

Formative Re-Engaging Lesson

Grade 4

Understanding and Interpreting Fractions

Mathematical Goals

This lesson unit is intended to help you assess how well students are able to translate between different representations - the fractional parts of sets, area models, the language of equal groups, measurement, word problems and symbolic notation - of the part/whole relationship, and to identify and help students who have difficulty:

- constructing fractional parts of a whole;
- using the language of "*equal parts out of*" and "*equal parts of*" to make sense of fractions and to translate between representations;
- understanding the meaning of fractions by knowing that the denominator represents the total in a set and the numerator represents the quantity being counted;
- comparing and ordering fractions by their placement on a number line diagram.

Standards Addressed

This lesson relates to the following Common Core State Standards:

Third Grade Geometry

- 3.G.2. Partition shapes into parts with equal areas.

Third Grade Number and Operations – Fractions

- 3.NF.2b Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts.

Fourth Grade Number and Operations

- 4.NF.1. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $(n \times a)(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions are the same size.

This lesson relates to the Mathematical Practice Standard #4, **Model with Mathematics**: Mathematically proficient students can apply the mathematics they know to solve problems. At early grades, this might mean being able to write an equation or problem to demonstrate a situation.

Introduction

This lesson unit is structured in the following way:

- Students work on their own, completing an assessment task that is designed to reveal their current understanding and difficulties.
- Students work in pairs on collaborative discussion tasks. As they do this, they translate between fractions represented as a part of a set and different representations for these quantities.
- Students take the individual assessment again, compare their second attempt to their first attempt and reflect upon what they have learned from this lesson and what they are still struggling to understand.

Materials required

Each student will need one copy of the assessment task.

Each pair will need the following items - cut, paper clipped, and placed into a baggie OR just copied [to be cut later when pairs of students select and match cards]:

Copies of Card Set A, *Concrete/Pictorial Models*

Copies of Card Set B, *Verbal Descriptions*

Copies of Card Set C, *Contextual/Fractional Situations*

Copies of Card Set D, *Number Line/Measurement Models*

Copies of Card Set E, *Numeric/Fractions Represented by a Number*

Please Note:

It is helpful for sorting and maintaining control over the many different cards to have each Card Set copied in a different color.

If possible, counters, pattern blocks, fraction pie pieces, grid paper, fraction strips, fraction bars, or Cuisenaire rods, and blank paper or mini whiteboards should be accessible to students.

Painter's blue tape and a large sheet of construction paper or butcher paper for making a poster are needed. The poster material should be available for the first pairing of Card Set A and Card Set B.

Please Note:

If blue painter's tape is used instead of glue to attach the cards to the poster, the cards may easily be re-arranged as students determine and change their minds regarding equivalent representations.

Time needed

The lesson will need at least two one-hour sessions. Timings given are only approximate. Exact timings will depend on the needs of the class.

Resources

Heibert, James 1997. Making Sense. Portsmouth, New Hampshire: Heinemann.

"How to Teach Math as a Social Activity." Edutopia.org video on building community norms around math discussions.

Inside Mathematics (www.insidemathematics.org/index.php/classroom-video-visits).

This website will provide you with video clips of different teachers doing Math Talks with their students. Watching these may be helpful for those teachers who are unfamiliar with Math Talks/Number Talks. The introduction to this lesson begins with a Math Talk/Number Talk.

Before the lesson

Individual Assessment Task: *Brittany's Cat's Kittens*

The assessment task, *Brittany's Cat's Kittens*, should be completed before the lesson. Ask students to attempt the task on their own. Explain that they should not worry too much if they cannot understand or do everything, because you plan to teach a lesson using a similar task that should help them.

It is important that students are allowed to answer the questions without assistance, as far as possible. If students are struggling to get started, then ask questions that help them understand what is required, but don't do it for them!

Possible Questions to Pose to Students Who May be Struggling

What is the question asking you to do?

What do you know about the part and the whole that may help you?

Assessing student responses

Look at and sort all student responses to the task; make some notes on what their work reveals about their current levels of understanding. Look for similarities among the papers regarding student understanding and difficulties. The purpose of doing this is to forewarn you of the difficulties students may experience during the lesson itself so that you may prepare carefully. Do not grade students' work at this stage. Research shows that this will be counterproductive, as it will encourage students to compare their grades and will distract their attention from the mathematics. Instead, try to understand their reasoning and think of ways in which you can help them.

As students work on the lesson, consider using the suggested questions and prompts listed below that arose from student work on the pre-assessment task, *Brittany's Cat's Kittens*, in the trialing process.

| COMMON ISSUES | SUGGESTED QUESTIONS AND PROMPTS |
|----------------------------|--|
| Student cannot get started | <ul style="list-style-type: none">• What information have you been given?• What might you be able to do in order to solve this problem?• What decisions do you need to make to help you get started?<ul style="list-style-type: none">• Are there any words that you need to understand in order to move forward?• What does the word "fraction" mean to you?• What facts do you have to help solve the problem?• How have you approached similar problems in the past? |

| | |
|--|---|
| <p>Student interprets the quantity value of the fraction to be the number of specified kittens over that same quantity.</p> <p>For example, the student says that the fraction for the gray kittens is $\frac{4}{4}$; the fraction for the white kittens is $\frac{2}{2}$.</p> | <ul style="list-style-type: none"> • Have you carefully read all the information given? Tell me. • What do you know? Tell me. <ul style="list-style-type: none"> ○ Explain your fraction to me. ○ What does your numerator represent? ○ What does your denominator represent? ○ Please identify "the whole". ○ Please identify "the parts". ○ What do you know about "the whole" and "the parts" when understanding fractions? |
| <p>Student has difficulty justifying their solution or makes an error in their solution process, be it a drawing, picture, or numbers.</p> | <ul style="list-style-type: none"> • Tell me what you know and understand about the question. • Show me what you know and understand by drawing a picture or using numbers. • What does the $\frac{3}{4}$ mean? • Can you use this understanding to help you in this problem? • Can you draw a picture to help you understand this problem better? • Are there ways you can make your solution easier to understand? |
| <p>Student answers incorrectly.</p> <p>For example, the student says that the solution is $\frac{1}{8}$ rather than $\frac{7}{8}$.</p> | <ul style="list-style-type: none"> • What information are you given in this problem? • What do you need to find out? • What fractions represent the whole story of this problem? <ul style="list-style-type: none"> ○ Tell me how you know. ○ Explain your fractions to me. ○ What does the denominator represent? ○ What does the numerator represent? |
| <p>Student answers incorrectly on one or both parts.</p> <p>For example, the student says that there is more than $\frac{1}{3}$ of cod or there are 4 cans of tuna.</p> | <ul style="list-style-type: none"> • What information are you given in this problem? • What does this information mean to you? • How can you use the drawing of the cans to help you understand this problem? <ul style="list-style-type: none"> ○ Please identify "the whole". ○ Please identify "the parts". • How can you compare the parts? <ul style="list-style-type: none"> ○ What do the parts mean? |

Suggested lesson outline

Class Introduction: Two Math Talks [25 minutes]

Please Note:

Manipulatives are needed for this first Math Talk. Manipulatives may be counters, cm cubes, color tiles, linking cubes, unifix cubes, or beans.

First Math Talk: Write the following word problem on the white board or document camera.

“Mrs. Clark has 20 students in her class. Eight of her students are boys and 12 are girls. What part of the students in her class are boys?”

Say to students, **“Please use the manipulatives to help you solve this problem. You are to think and work silently. Give me a thumbs up when you are ready to share your thinking.”**

Have students share with their shoulder partners and have a conversation about the similarities or differences between their representations.

After the individual think time and pair/share, ask, **“Who has an idea what the solution may be?”**

Collect the responses and write them down for all to see. Then ask, **“Who is willing to share and explain how they got their answer? Which answer are you justifying?”**

The teacher is to record the students’ response on the white board/document camera and ask probing questions such as: **“Can you tell me more about this? What were you thinking?”**

And then pose this question to the entire class, **“Are there any questions you would like to ask of Student A?”**

Then ask students, **“Who has used a different strategy and is willing to share?”**

Your job is to help facilitate the discussion on the mathematical reasoning of each student. For each student strategy, the teacher is to help students identify the key mathematical idea[s] around that particular strategy, be it part, part whole, parts of a set, or a fractional equivalency, etc.

Please Note:

For this second Math Talk the manipulatives should be put away. A mini-whiteboard or blank sheet of paper is needed.

Second Math Talk: Write the following word problem on the white board or document camera. Students may use blank copy paper or mini-whiteboards for their work.

“Mary baked a cake. She shared it equally with her five friends. Mary cut the cake into six equal pieces. How much cake did she have for herself?”

Say to students, **“There are some interesting vocabulary words in this word problem. Take a moment to think about the word ‘equally’. What does this mean to you?”** Have different students share their ideas with the whole class to build a robust definition of “equally”.

Say to students, **“What does the “five” mean in this problem? You are to think and work silently. Give me a thumbs up when you are ready to share your thinking.”**

Have students share out as a whole group. Then follow with, **“What does the ‘six’ mean in this problem?”** After the individual think time, ask, **“Who would be willing to share?”** Students may show their solution with words, numbers, a fraction, and drawings.

This is an opportunity to look for different representations such as numbers, words, circles, rectangles, number lines, etc., divided into equal parts or pieces.

The teacher’s job is to gather and record the different representations for the whole class to see and to ask clarifying and probing questions such as: **“Can you tell me more about this? What were you thinking? Do you think X’s drawing gets the same answer as Y’s drawing? How do you know? Are there other drawings or representations that may describe this situation? Talk with your partner and see if you can come up with another drawing or representation.”** Continue to ask students, **“Who has used a different representation and is willing to share?”**

This Math Talk is an opportunity to set the stage as an example of the discussion, discourse, and justification expected in the collaboration tasks of this lesson.

If the class has not generated different representations, you might say, **“I saw other students who used X representation. Would that work? Talk and work with your partner to see how such a representation might or might not work.”**

Please Note:

If one doesn’t have mini-whiteboards, one can put blank white copy paper inside of plastic sheet protectors and students may use washable markers and baby wipes to write and clean their faux mini-whiteboards.

The Lesson

LESSON EXTENSION POSSIBILITIES

There can be a one-to-one correspondence between sets. The blank cards are an integral part of the sets because some students may not see this correspondence and will choose to use the blank cards to complete their matches. This provides invaluable information to you, the teacher, about student understanding and misconceptions.

Furthermore, to increase the cognitive demand of these collaborative tasks, selected cards of your choice may be removed from one or more sets of representations forcing students to fill in the missing representation[s] on their own and providing deeper insight into student thinking and understanding.

Whether you use the complete sets or remove some of the cards, letting students know that there are blank cards available is important. A student's use of the cards provides additional information about his or her understandings and difficulties.

Collaborative Task #1: Matching *Concrete Models* with *Verbal Descriptions* [15 minutes]

Materials needed: Card Set A, Card Set B

The first task is designed to help students describe parts of a set in fractional terms emphasizing "equal groups of" and "equal parts." Organize students into groups of two. If you have an uneven number of students in class, please think carefully about which three students you will place together. Since this task is designed to allow access for all students within the range of a heterogeneous class, you need not pair students with like abilities.

Give out the Card Set A (*Concrete Models*) and Card Set B (*Verbal Descriptions*) only after the class understands the expectation of discourse and justification. They will also need blue painter's tape loops and a large piece of butcher paper to be used to make the poster.

Please note:

Students are determining the language to describe a part of a set or whole.

Tell the students, "**Your task is to match each picture set with the corresponding description. First, you and your partner will take a few minutes to look at and read Card Set A and Card Set B. As you read, make a mental note of a match or two that you see.**"

The student with the longest hair will be Partner A. Partner A will select a picture card and the verbal description card they think matches. This student will place the two cards on the Master Discussion Template if the cards have been pre-cut; if not, the two cards will be cut at this time by the partner pair.

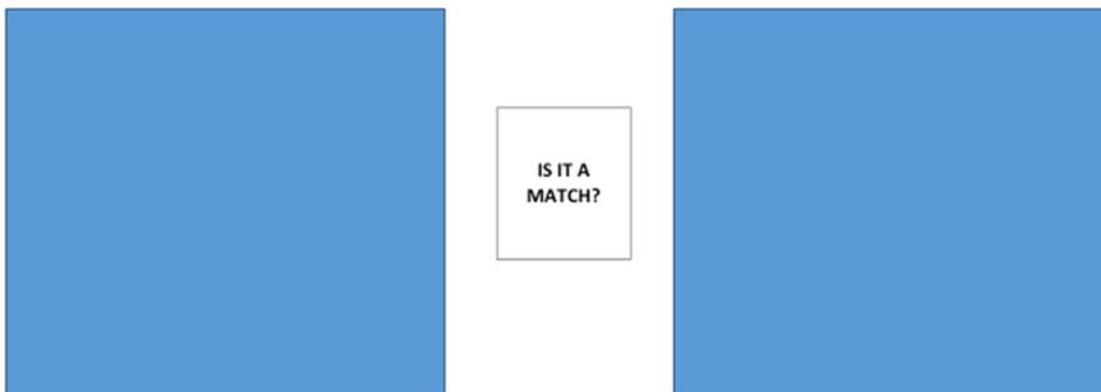
Please Note:

This decision to use pre-cut cards or not will be up to you and your class. Pre-cut cards have a tendency to “get lost”; conversely, some students struggle with motor coordination skills using scissors. Thus, the decision is yours to make.

Third, the partners may use the sentence frames from the “Matching Discussion” sheet to help them discuss their reasons for this match; this provides support for productive talk. Together, they must agree upon a written JUSTIFICATION card written by Partner B, which is to be placed with each match. (Please see the resource “The Matching Process” in the Support Materials at the end of this guide.)

The Matching Process

1. Partners A and B **spread out the card sets and think about cards that might be matches.**
2. Partner A chooses the first pair of cards that he/she thinks is a match and **puts one card on each of the blank squares and says why the two cards are a match.**
3. Partner B **continues the discussion by either agreeing or disagreeing** with Partner A about the cards matching and **says why or why not.**
4. When both partners agree, **stick the matching cards in a column using a tape loop on the large chart paper.**
5. Partner B **completes a Justification Card** about the reasons why the cards match and sticks it below the two matched cards on the chart paper.
6. Repeat the process to **match another pair of cards**, but this time **switch roles**, so Partner B chooses the cards and begins the discussion and Partner A completes the Justification Card.
7. **Repeat until all possible matches have been made.**



“Take turns with your partner to choose a picture card and the verbal description to match it. When you and your partner are in agreement with each other, then and only then may you tape your match to your poster. The partner who did not select the match will write the explanation on the Justification Card. If you cannot find a matching card, write your own using the blank cards.”

Remember
Please use your “Matching Discussion” sheet with the sentence frame starters to help you in your discussion.

When a pair of students is satisfied with all their matches for Card Sets A and B, have the pair share their matches with another pair of students from a different part of the room. If the two pairs are in disagreement, a discussion around the mathematics should ensue. When all the matches have been discussed, each pair has the opportunity to make changes to their original matches.

Please note:

This interim additional student sharing and processing may be done at the teacher’s discretion after each addition of another card set.

Since the expectations for this card sort and match may not be familiar to your students, you may wish to take the opportunity to provide time for the whole group to process and share together. Please note that the purpose is **not** to have all the correct matches and to create boilerplate replicas, but to provide an opportunity for students to articulate, reflect, and improve upon their own understanding and thinking.

Different Scenarios

Thus, you may ask of your students, **“Who would be willing to share a match and its justification for a pair that you strongly feel is a match?”** Let several different student pairs share with the class and provide the opportunity to let other classmates ask clarifying questions of them.

And/Or

Thus, you may ask of your students, **“Who would be willing to share a match and its justification for a pair that you THINK is a match but aren’t as convinced?”** Let several different student pairs share with the class and provide the opportunity to let other classmates ask clarifying questions of them.

Or

Thus, you may say to your students that you would like to focus on what makes a complete justification. You may ask of your students, **“Who would be willing to share the match which has your best or most complete justification?”** Have students self-select and move their match and justification card to a mini-whiteboard for display using a document camera. Then you may ask the class if there is anything that could be added to make the justification more complete. This complete justification may be recorded by the teacher on chart paper for all to see. Depending upon your class, this may be done with 2 or 3 matches.

After one or more of these scenarios, allow students to reflect upon their matches of Card Set A with Card Set B; give them the time and opportunity to make changes and/or additions to their justifications and matches to make them more complete.

Some students may notice that a few of the verbal descriptions represent equivalent fractions.

You do not need to comment at this stage.

This is a noticing that warrants discussion with the whole group.

Throughout this collaborative activity, students will be asked to occasionally create representations for area, measurement or sets. Drawings provide opportunities for you to assess students understanding. It is not until a student is asked to draw a fraction representation that it becomes apparent that the notion of equal parts may not have been incorporated into his or her emergent understanding of fractions. It is also important to distinguish between incorrect drawings of fractions due to drawing skills and those that are the result of mistaken ideas.

Collaborative Task # 2: Matching *Concrete Models, Verbal Descriptions*, with *Contextual/Fractional Situational Cards*. [15 to 20 minutes]

Additional materials needed: Card Set C

Give out the Card Set C (*Contextual/Fractional Situational Cards*) and ask students to:

- First, **“On your mini-white board, sketch a ‘concrete model’ of the fraction this story represents AND write a ‘verbal description’ for this fraction.”**
- Second, **“Compare your work with your partner and see if you agree or disagree. Use the sentence frames to help you in your discussion. When you reach consensus or agreement with your partner, look on your poster for the ‘match.’”**
- Third, **“Using a tape loop, add this card to your match. Write your justification card and add this card to your poster. Be sure to explain how and why this particular C card represents the fractional part of the corresponding *Concrete/Pictorial Model* and the *Verbal Description*.”**

Please Note:

Some contextual/situational problems may be missing. Students may need to create their own versions. The situational models help to connect concrete models and verbal descriptions to problems with real-world fractions.

Collaborative Task # 3: Matching *Concrete Models, Verbal Descriptions*, *Contextual/Fractional Situational Cards* with *Number Line/Measurement Models*. [15 to 20 minutes]

Additional materials needed: Card Set D.

Give out the *Number Line/Measurement Models* (Card Set D) and ask students to:

- First, **“On your mini-white board, sketch a ‘concrete model’ of the fraction this story represents AND write a ‘verbal description’ for this fraction.”**
- Second, **“Compare your work with your partner’s and see if you agree or disagree. Use the sentence frames to help you in your discussion. When you reach consensus or agreement with your partner, look on your poster for the ‘match.’”**
- Third, **“Using a tape loop, add this card to your match. Be sure to explain how and why this particular C card represents the fractional part as the corresponding *Concrete/Pictorial Model* and the *Verbal Description*.”**
- Some models may be missing. Students may need to draw their own versions.
- HINT: Many students will find it helpful to also draw a representation of the number line card that is chosen. It will help them to make sense of the number of equal parts in the whole as well as the number of equal parts delineated on the card.

Please Note:

Justification cards are not required for this match.

With measurement models, length is compared instead of area. The use of manipulatives provides more opportunity for trial and error, exploration and discussion. Encourage students to justify how and why a particular measurement model represents the same fractional part of a corresponding set and the contextual/situational problem.

Collaborative Task # 4: Matching *Concrete Models, Verbal Descriptions, Contextual/Fractional Situational Cards, Number Line/Measurement Models with Numeric/Fractions Represented by a Number.* [15 to 20 minutes]

Additional materials needed: Card Set E.

Give out the *Numeric/Fractions Represented by a Number* (Card Set E) and ask students to match these to the card matches already on their poster. Some models may be missing. Students may need to draw their own versions.

Please Note:

Justification cards are not required for this match.

This task is designed to help students interpret the quantity of fractions represented by a number and realize that these fractions define: the area of each equal-sized piece of the whole; the number of equal-sized pieces; a portion of a set; a distance on a number line, and a contextual problem.

Collaborative Task #5: Creating a Number Line

Using twine or string, make a number line to hang across the classroom. With a clothespin, put in the "zero", the "one", and the "two". Give students a replica of this number line on a worksheet.

- Say, **"We have a number line here and our goal is to add fractions to this number line. Please mark on your number line where you think the fraction "1/2" should go. Share with your partner. Have a discussion about whether you agree or disagree and why?"** Hold a whole class discussion and come to consensus. Then, using a clothespin, place a card with "1/2" on the number line. Do this for "1 1/2" as well.
- Ask the class if there are any groupings that represent any of these quantities on the number line. Hold a discussion with the class and when the class reaches consensus, clip these representations (from a teacher set) to the appropriate quantity on the number line.
- Next, say, **"We have many more groupings. Where might some of these go on our number line? Talk with your partner and we will share out as a whole class and reach consensus on placing other strips on our number line."** Continue in this manner as far as you feel your students are ready to go.

This discussion provides the students with an opportunity to find and express equivalencies of fractional quantities. Students will be able to justify their reasoning by using the various representations from the card sort with a focus on how the number and size of the parts differ, even though the two fractions are the same size.

EXTENSION: ORDERING FRACTIONS

Have students cut their posters into strips with each matching group on one strip. These strips may then be organized from least to greatest or greatest to least and glued to another poster in the preferred order as directed by the teacher. Students should write a reflective statement as to how they justified their comparison of fractional quantities and decided upon their order.

Plenary Discussion [30 minutes]

Hold a whole class interactive discussion to review what has been learned over this lesson.

First, ask each student to reflect upon the following two questions. Collect their responses to look at and assess later. (Please see the resource “Fractional Parts of a whole can be expressed in multiple ways using various representations” in the Support Materials at the end of this guide.)

*“Which representation[s] made the most sense to you? Why?
Which representation[s] was the most difficult for you to understand? Why?”*

Second, have students use mini whiteboards and the following questions to assess students’ learning. For each fraction have the students represent the quantity on a number line, as part of a set or area model, in a contextual problem, or with a verbal description. It is your choice to select a representation; you may want to consider letting students select their own representation for one of the fractions.

After each fraction, ask students to turn to their partner and see if they agree or disagree with their partner’s representation. *Then ask if anyone has a clarifying question to ask of the class, OR have the class hold their white boards up so that all may see the different representations used.* Use your discretion and observations as the teacher to guide you in this endeavor.

$\frac{2}{3}$

$\frac{4}{6}$

$\frac{3}{4}$

$1\frac{1}{5}$

$\frac{24}{10}$

Third, ask several representative pairs of students to justify one of their groupings and explain to the whole class why the different types of models convey the same fractional quantity. *Ask the class if anyone has a question for the presenters or anything to add on to their thinking.*

Fourth, pass out a large piece of construction paper to each student. Say, **“Based upon the presentations you have heard, everyone has the opportunity to share, change or modify one grouping of their choice. You and your partner will decide who does which grouping. You will tape all the cards of this grouping EXCEPT the justification cards onto your construction paper. Below this grouping, you will write your answer to the following questions:**

- **Why did you select this grouping?**
- **How do you know that all the cards represent the same quantity or numerical value?**

- **When sharing your work with other pairs of students, what did you learn from that discussion?"**

[Please see the resource "Answer these questions on your individual poster" in the Support Materials at the end of this guide.]

Individual work [30 minutes]

Finally, give students a new copy of the task *Brittany's Cat's Kittens* (2011 MAC Grade 4). Ask students to have another go at it. When students have completed the second attempt, pass out their original work to them. Ask them to write a reflective sentence or two about what they have learned from this lesson which helped them on their second attempt, and what they are still struggling with regarding their understanding of fractions with different representations.

Solutions

This table is for your convenience only. It is helpful not to refer to cards by these letters in class, but rather to the content of the cards.

| Concrete or Pictorial Model | Verbal Description | Contextual or Situational Problem | Number Line or Measurement Model | Numeric or Fraction Symbol |
|-----------------------------|--------------------------|-----------------------------------|----------------------------------|--|
| A1 | B4 | C3 | D11 | E6 E3 E1 |
| A2 | B5 | C2 | D7 | E1 E3 E6 |
| A3 | B2 | C4 | D9 | E3 E1 E6 |
| A4 | B12 B13 | C10 | D4 | E1 |
| A5 | B8 B11 | C12 C1 | D1 | E2 E11 E9 E12 |
| A6 | B10 | C11 | D3 | E2 E11 E9 E12 |
| A7 | B15 B17 | C14 | D13 | E13 E16 |
| A8 | B12 B13 | C10 C15 | D14 | E18 E10 |

| | | | | |
|------------|--|------------|------------|--|
| | | | | E7 |
| A9 | B14 B16 B18 | C13 | D15 | E14 E15 |
| A10 | B3 | C7 | D8 | E18 E10 E7 |
| A11 | B6 | C6 | D5 | E2 E11 E9 E12 |

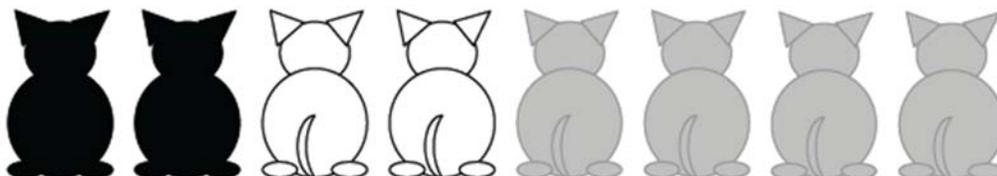
Brittany's Cat's Kittens

This problem gives you the chance to:

- work with common fractions.

Brittany's cat had 8 lovely kittens.

2 are black, 2 are white, and 4 are gray.



1. What fraction of the kittens is gray? _____

What fraction of the kittens is white? _____

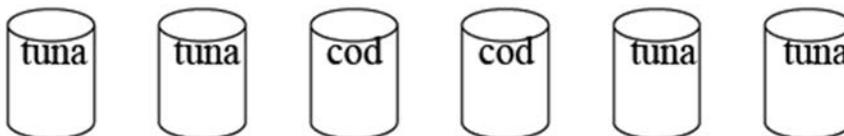
2. Brittany has found homes for three quarters of the kittens, the rest she will keep herself. How many will she keep?

Show how you figured this out. _____

3. Only one of Brittany's kittens is female.

What fraction of the kittens is male? _____

4. Each day, the kittens share these 6 cans of kitten food.

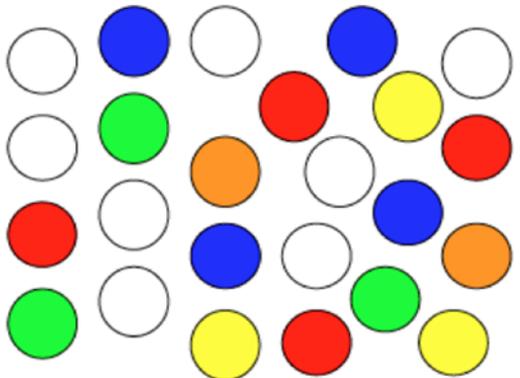


Brittany said that less than $\frac{1}{3}$ of their food was cod.

Explain how you know she is wrong?

How much of their food is tuna? _____

8



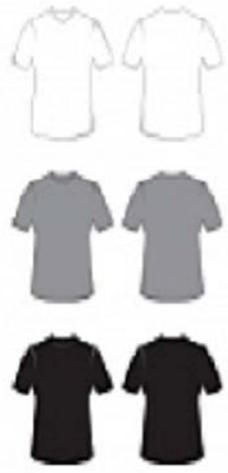
What part of this group of dots is white?

A 1



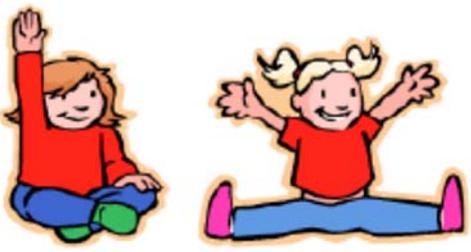
What part of this group of girls is sitting?

A 4



What part of this group of T-shirts is black?

A 2



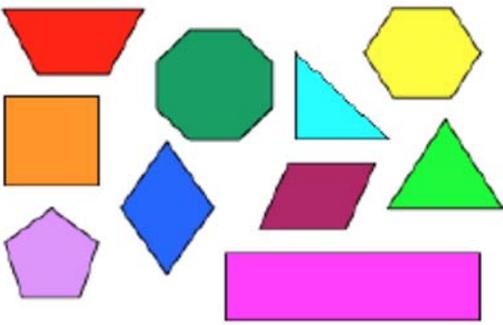
What part of this pair of girls has her legs folded?

A 5



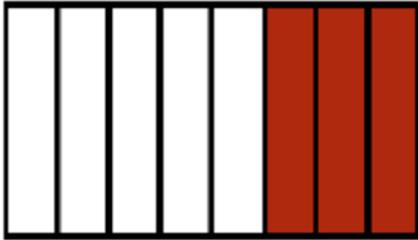
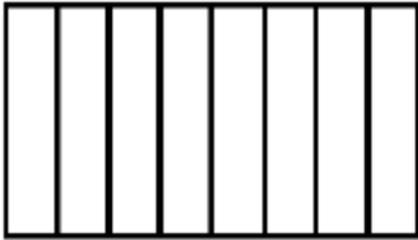
What part of this group of workers is the policeman?

A 3

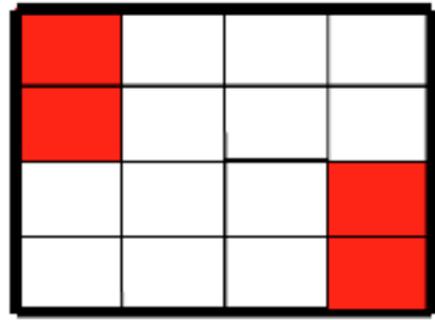


What part of this group of shapes is the four sided shapes?

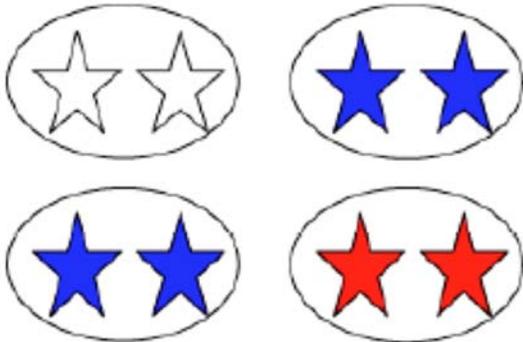
A 6



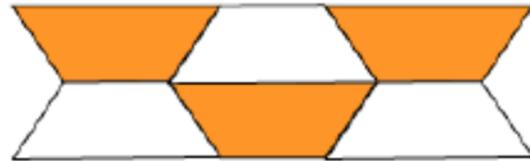
What part of these two pans of brownies was eaten? A7



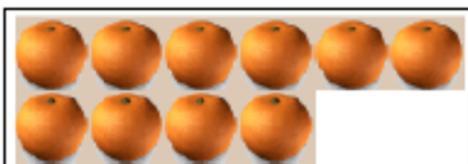
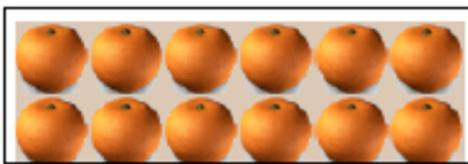
What part of this shape is shaded? A10



There are 4 groups of 2 stars. What part of the stars is white? A8



What part of this shape is shaded? A11



Each box holds 12 oranges. Some oranges from one box were eaten. What part of the two boxes of oranges is left? A9

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| <p style="text-align: right;">B1</p> <p style="text-align: center;">four out of twelve equal parts</p> | <p style="text-align: right;">B4</p> <p style="text-align: center;">eight out of twenty-four equal parts</p> |
| <p style="text-align: right;">B2</p> <p style="text-align: center;">one out of three equal parts</p> | <p style="text-align: right;">B5</p> <p style="text-align: center;">two out of six equal parts</p> |
| <p style="text-align: right;">B3</p> <p style="text-align: center;">four out of sixteen equal parts</p> | <p style="text-align: right;">B6</p> <p style="text-align: center;">three out of six equal parts</p> |

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| <p style="text-align: right;">B7</p> <p style="text-align: center;">four out of eight equal parts</p> | <p style="text-align: right;">B10</p> <p style="text-align: center;">five out of ten equal parts</p> |
| <p style="text-align: right;">B8</p> <p style="text-align: center;">one out of two equal parts</p> | <p style="text-align: right;">B11</p> <p style="text-align: center;">two out of four equal parts</p> |
| <p style="text-align: right;">B9</p> <p style="text-align: center;">one out of six equal parts</p> | <p style="text-align: right;">B12</p> <p style="text-align: center;">one out of four equal parts</p> |

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| <p style="text-align: right;">B13</p> <p style="text-align: center;"><u>two</u> out of eight equal parts</p> | <p style="text-align: right;">B16</p> <p style="text-align: center;"><u>one</u> and <u>five</u> out of six equal parts</p> |
| <p style="text-align: right;">B14</p> <p style="text-align: center;"><u>one</u> and <u>ten</u> out of twelve equal parts</p> | <p style="text-align: right;">B17</p> <p style="text-align: center;"><u>one</u> whole and <u>five</u> out of eight equal parts</p> |
| <p style="text-align: right;">B15</p> <p style="text-align: center;"><u>thirteen</u> <u>eighths</u></p> | <p style="text-align: right;">B18</p> <p style="text-align: center;"><u>twenty-two</u> <u>twelfths</u></p> |

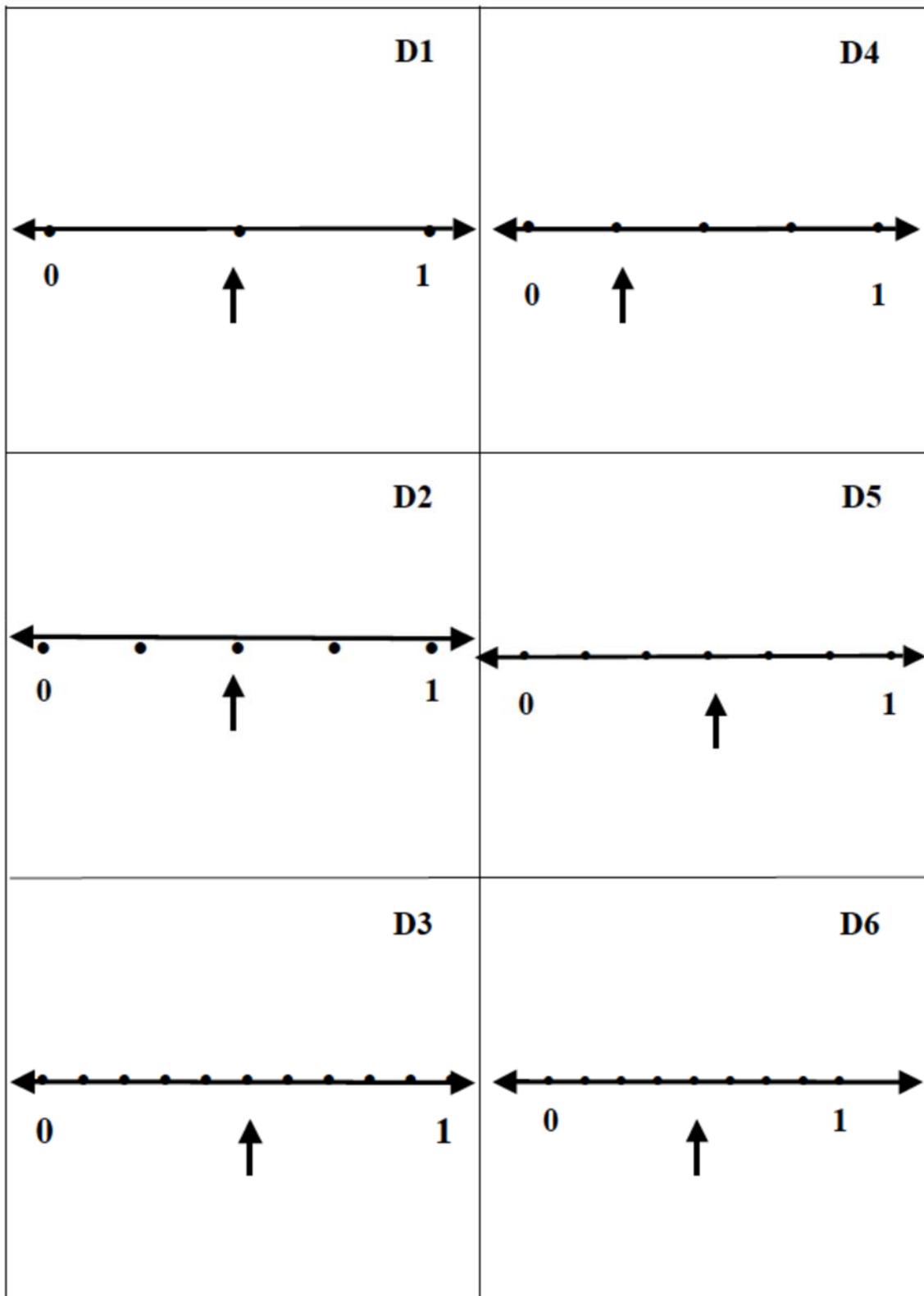
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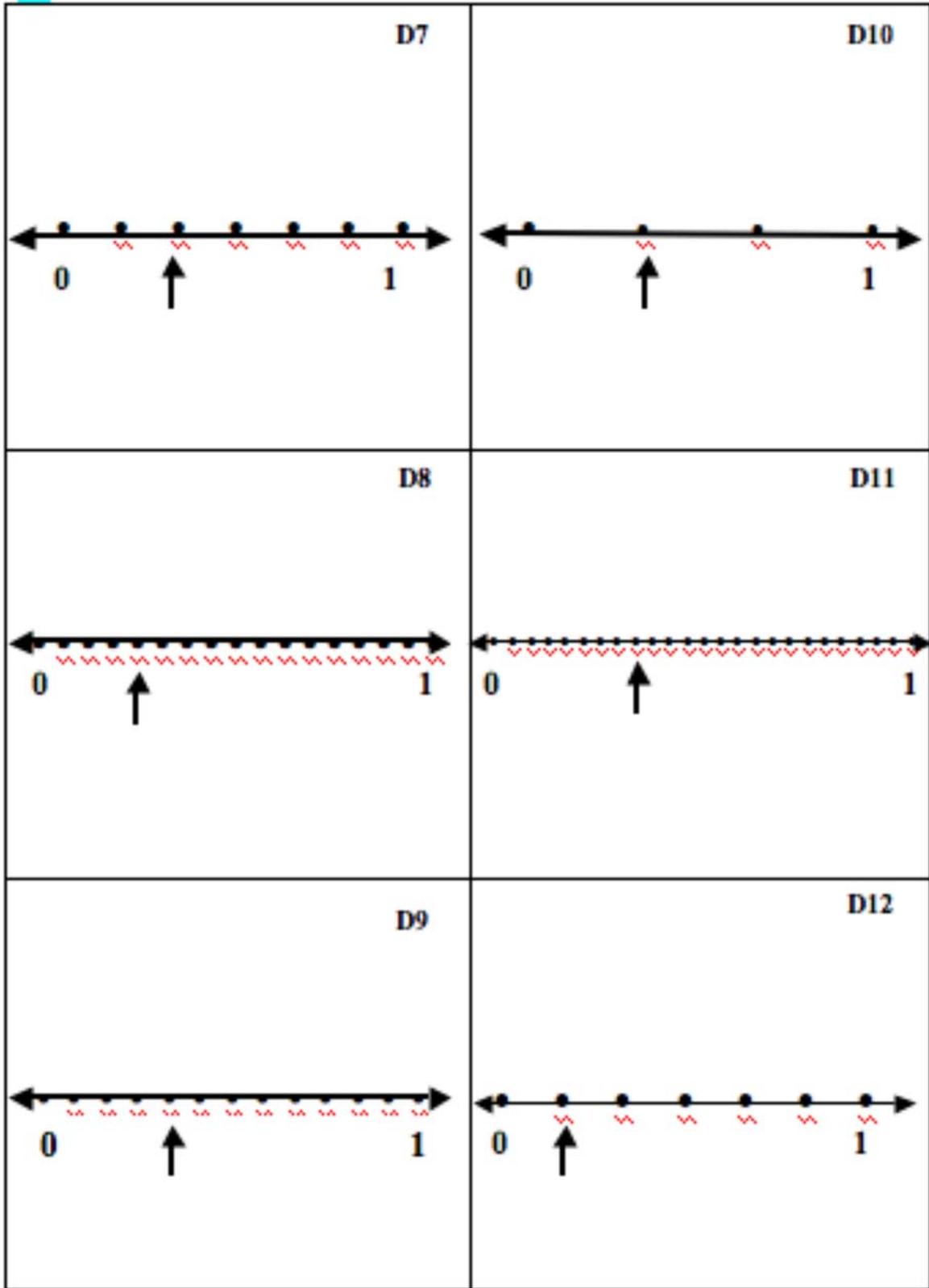
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| <p>Caitlin wanted to share her brownie with her friend. She divided the brownie into two equal pieces. How much of the brownie did each of them get?</p> <p style="text-align: right;">C1</p> | <p>Daniel's mother made enough of his favorite dessert for three days. What part of the dessert did Daniel get the first day?</p> <p style="text-align: right;">C4</p> |
| <p>Six of Ashley's friends came to her birthday party. Two of them stayed for a slumber party. What part of the party guests stayed overnight?</p> <p style="text-align: right;">C2</p> | <p>There were twelve players on Scott's soccer team. Four of the players scored goals in the first game. What part of the team scored goals during that game?</p> <p style="text-align: right;">C5</p> |
| <p>Eight of Mia's students had dogs as pets. Mia had twenty-four students in her class. What part of her class had pet dogs at home?</p> <p style="text-align: right;">C3</p> | <p>Four of the sixteen boys in Sarah's class had red hair. What part of the students in Sarah's class had red hair?</p> <p style="text-align: right;">C6</p> |

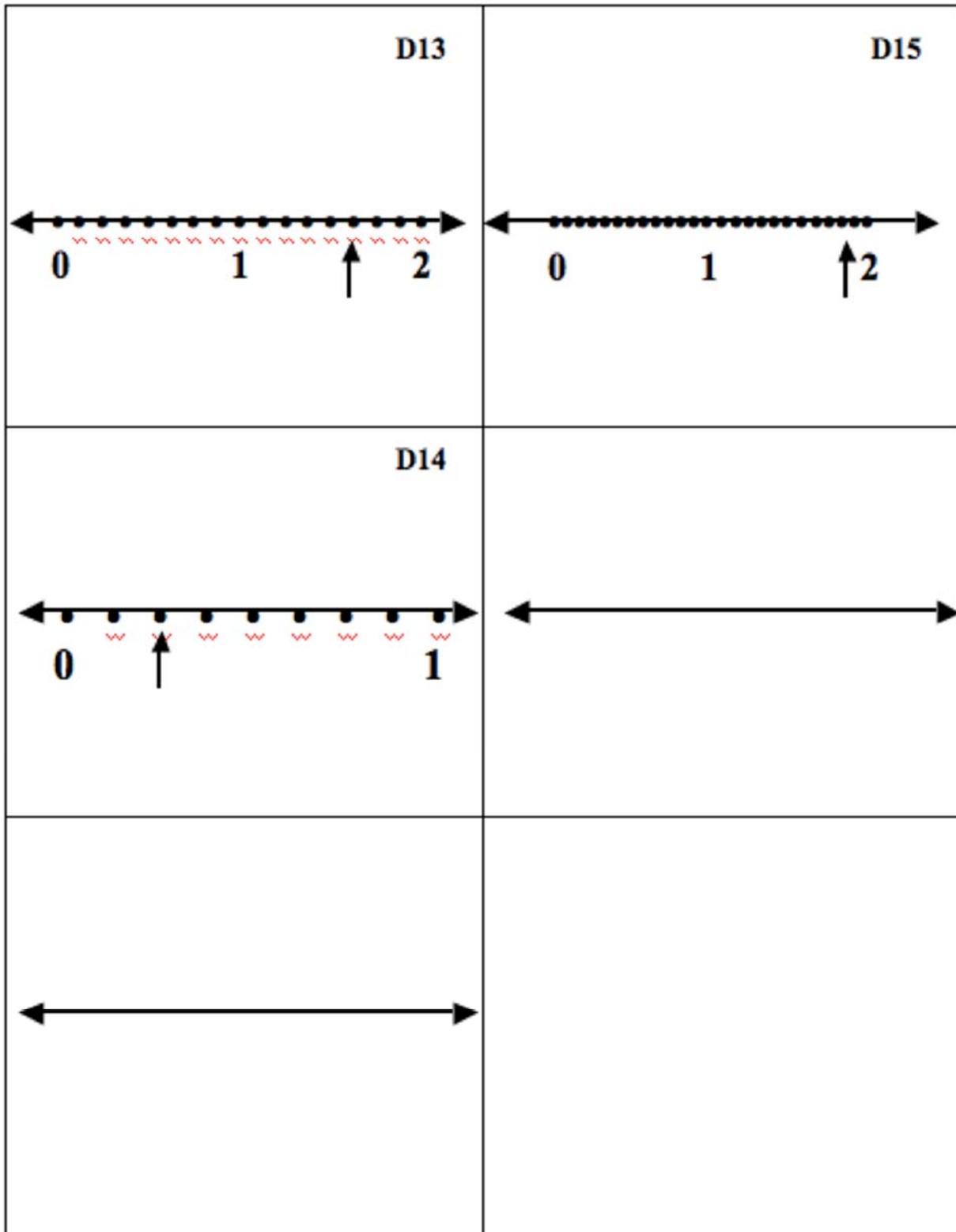
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|---|---|
| <p style="text-align: right;">C7</p> <p>Two of Debbie’s friends came to her house, and they all went for a bike ride. Their ride was 6 miles altogether, and 3 miles of it was uphill. What part of their ride was uphill?</p> | <p style="text-align: right;">C10</p> <p>One out of the four third grade classes had the same number of boys as girls. What part of the third grade classes had an equal number of boys and girls?</p> |
| <p style="text-align: right;">C8</p> <p>One of the flowers in the vase was a red rose. There were six flowers in all in the vase. What part of the flowers in the vase was red roses?</p> | <p style="text-align: right;">C11</p> <p>Sally read ten books in one month. Five of them were books about gardening. What part of the books she read was about gardening?</p> |
| <p style="text-align: right;">C9</p> <p>Four of the eight fish in the fish tank are goldfish. What part of the total number of fish in the tank is gold fish?</p> | <p style="text-align: right;">C12</p> <p>Two of Becca’s four closest friends live California. What part of these friends live in California?</p> |

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| <p>Joe bought oranges ^{C13} that came in boxes of twelve oranges each. He bought twenty-two oranges. How many boxes of oranges did he buy?</p> | <p>Jemma baked two ^{C15} pans of brownies. Each pan had eight brownies. She gave thirteen brownies to her friends. How much of the brownies did she give away?</p> |
| <p>^{C14} Debbie had eight pets at home. Two of her pets were dogs. What part of her pets is dogs?</p> | |
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Set D Number Line







Set E 3 - Numeric

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| <p style="text-align: right;">E1</p> <p style="text-align: center;">$\frac{2}{6}$</p> | <p style="text-align: right;">E4</p> <p style="text-align: center;">$\frac{4}{8}$</p> |
| <p style="text-align: right;">E2</p> <p style="text-align: center;">$\frac{5}{10}$</p> | <p style="text-align: right;">E5</p> <p style="text-align: center;">$\frac{1}{6}$</p> |
| <p style="text-align: right;">E3</p> <p style="text-align: center;">$\frac{4}{12}$</p> | <p style="text-align: right;">E6</p> <p style="text-align: center;">$\frac{8}{24}$</p> |

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|----------------------|----------------------|
| $\frac{4}{16}$ E7 | $\frac{1}{4}$ E10 |
| $\frac{1}{3}$ E8 | $\frac{1}{2}$ E11 |
| $\frac{2}{4}$ E9 | $\frac{3}{6}$ E12 |

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|---------------------------------|-------------------------------|
| E13 $1 \frac{5}{8}$ | E16 $\frac{13}{8}$ |
| E14 $1 \frac{10}{12}$ | E17 $1 \frac{5}{6}$ |
| E15 $\frac{22}{12}$ | E18 $\frac{2}{8}$ |

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