STUDENT: Well, we're drawing Gemma with her sixteen brownies and her giving away thirteen of them to her friends. And we're talking about how we think it's...the blue card should go here. So this is her brownie and she's giving away thirteen of them out of sixteen to her friends, and it's thirteen eighths because there's eight in each pan, and she's giving thirteen away to her friends.

MICHELLE MAKINSON: And you've got...like, I notice this one has a justification card.

STUDENT: Yup.

MICHELLE MAKINSON: How did you decide that this blue card matched these two other cards? What did you...

STUDENT: Because the answer of this is one fourths and the answer of this green card is the same thing.

MICHELLE MAKINSON: So you've got to back up. How do you know it's one fourth?

STUDENT: Because the question for this one is: how many girls are sitting down?

MICHELLE MAKINSON: And how many are sitting?

STUDENT: One.

MICHELLE MAKINSON: One out of four? And how does that connect to the white card?

STUDENT: Because it is one out of four third grade class is...

MICHELLE MAKINSON: No, no. It's this card.

STUDENT: It's one out of four equal parts.

MICHELLE MAKINSON: Okay, so each person is equal to each other and one of them is sitting?

STUDENT: Yes. And this is one out of four class has same equal amount of girls and boys.

MICHELLE MAKINSON: Okay. So only one class is like that, the way there's only one girl that is sitting, the way that there's only one out of the four equal parts.

STUDENT: Yes.

MICHELLE MAKINSON: Okay. Interesting. Thank you!

MICHELLE MAKINSON: Can you tell me why this blue card matches the green and the white? And Noelle, I'm going to need your involvement, too.

STUDENT: Because there are...it says what part of the stars are white. There are two stars out of eight stars. And this one has eight pets at home. Two of their pets were dogs. What part of the pets are dogs? So that'll be two eighths, just like that it's two eighths.

MICHELLE MAKINSON: So is there a way that they were grouped that within the whole group, that helps you connect it to this?

STUDENT: Yes. Because for...

MICHELLE MAKINSON: So I know we can say that there are eight stars, but is there another way that we can say that picture? That they are...

STUDENT: One twelfths.

MICHELLE MAKINSON: Well, not just a fraction, but how many groups are there?

STUDENT: There are four groups.

MICHELLE MAKINSON: Oh, there are four groups, but there's also, like, eight total. So there's four groups total, eight total. How do you connect...so how does this fit in there because that doesn't even have the number eight?

STUDENT: Because there are four groups, which is the denominator and one of the groups has white.

MICHELLE MAKINSON: Okay. Is that similar to what's going on with the pets? How so?

STUDENT: Because, um...

MICHELLE MAKINSON: Is there something similar between the white stars and the dogs?

STUDENT: Well, um, there are two white stars and two dogs.

MICHELLE MAKINSON: What do you think, Noelle? Do you agree with that?

STUDENT: Yeah.

MICHELLE MAKINSON: Tell me more. Why do you agree?

STUDENT: Because there are two white stars and there are two dogs.

MICHELLE MAKINSON: And how many stars are there all together?

STUDENT: Eight.

MICHELLE MAKINSON: Eight. How many pets are there all together?

STUDENT: Eight.

MICHELLE MAKINSON: And so you have two out of eight, two out of eight, but then you get to one out of four. How does that work? How are they...are they saying the same thing? Can you explain how that's the same thing?

STUDENT: Um, because these two are the same.

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MICHELLE MAKINSON: But how is this the same as these two? Pull in that white card. How does that work? Tommy, feel free to jump in. Why is one out of four equal parts a match to these two cards?

STUDENT: Because they are equivalent.

MICHELLE MAKINSON: In what way are they equivalent? How do you know they are equivalent?

STUDENT: Um, because... By two eighths...two divided by two is one and eight divided by two is four, so it's one fourth.

MICHELLE MAKINSON: Okay. So they are talking about the same amount of stuff?

STUDENT: Mm hm.

MICHELLE MAKINSON: If the wholes were the same?

MICHELLE MAKINSON: He bought twenty-two oranges. How many boxes of oranges did he buy?

STUDENT: Twenty-one because he can only buy...because twelve plus twelve...

MICHELLE MAKINSON: Well, what if he...imagine that maybe he got ripped off. Like, the other day I ordered eggs and a couple of my eggs were broken. So it's all about how you interpret it, right?

STUDENT: Maybe they caught on fire.

MICHELLE MAKINSON: Well, think about what that would mean.

STUDENT: Unless he tore apart the package and...

MICHELLE MAKINSON: It's a great question. And you're going to have to grapple with it; you're going to have to work with it to see how you interpret it, how you see it, what you think the real meaning is. So don't be afraid to do that. Remember they have tape on the back. We can move them, right? You can change your mind.

STUDENT: Do it by sixes!

MICHELLE MAKINSON: [Speaking to coach Sally Keyes] They're asking realistic questions about the oranges. It's like, well if they come in packs of twelve and he only bought twenty-two, then he must have only bought one box.

SALLY KEYES: Oh!

MICHELLE MAKINSON: So there's an issue. Yeah. So I said just grapple with it. If you find an error then, you know, justify that you believe it doesn't match, you know, or for whatever reason make it match. What could you do to change the card itself, right? The other day two were broken, so did I really buy twelve eggs or did I buy ten. Maybe two were rotten. I don't know.

SALLY KEYES: (Inaudible) you know what I mean?

MICHELLE MAKINSON: Yeah! And that's what he's arguing. He's like, "This problem doesn't make sense."

SALLY KEYES: Yeah. He's looking.

MICHELLE MAKINSON: He's listening. He's always listening.

MICHELLE MAKINSON: I think you might be over thinking it. Do, you know, do you know how we make cards when there wasn't something? What if you made that made more sense? I mean, what could you do if you think that it doesn't make sense?

STUDENT: Scratch it out.

MICHELLE MAKINSON: Well, but I mean...and Alondra was asking, "Well, could we look to see what exists on these ones to guide you?" Right? So maybe there is an error. Find it, fix it, change it. Make it make sense. Your job is to make sense of the world.

STUDENT: Joe bought oranges that came in boxes of twelve oranges each. He bought twenty-two oranges. How many boxes of oranges did he buy?

MICHELLE MAKINSON: And what did you think about that, originally?

STUDENT: Well, twelve plus twelve equals twenty-four so he couldn't have bought twenty-two oranges if twelve plus twelve equals twenty-four.

MICHELLE MAKINSON: Because your background knowledge was telling you what?

STUDENT: That that's not the same...

MICHELLE MAKINSON: Or, like, in real life you were saying that this doesn't make sense in real life. How does it not make sense?

STUDENT: Because he couldn't have...because it said he got twenty-two but if you do twelve plus twelve, it equals twenty-four and those answers aren't the same.

MICHELLE MAKINSON: So the first box would have how many oranges?

STUDENT: Twelve.

MICHELLE MAKINSON: And so to get to twenty-two, how many would be in the second box?

STUDENT: Ten.

MICHELLE MAKINSON: Ten. Does that ... why didn't that make sense to you?

STUDENT: Because we thought you only would be able to buy boxes...

MICHELLE MAKINSON: That are...

STUDENT: Full.

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MICHELLE MAKINSON: That was full? Okay. So how did you solve that problem?

STUDENT: We (Inaudible) to help us solve our problem and we saw twenty-two twelfths. So we decided that that was the correct answer because the question fraction was twenty twelfths...twenty-two twelfths. Because there are twenty-two oranges that he bought and each had twelve in it.

MICHELLE MAKINSON: Where does the twelfths come from?

STUDENT: Um...

STUDENT: The boxes.

MICHELLE MAKINSON: Tell me more. How does that make it a twelfths? Because the boxes...

STUDENT: Have twelve in them.

MICHELLE MAKINSON: Twelve equal things in them, in this case, the oranges. And so when you didn't have a complete set you had...

STUDENT: A part...

MICHELLE MAKINSON: A fraction where the numerator was...

STUDENT: Bigger.

MICHELLE MAKINSON: Bigger than the denominator because you did... Did you have two complete wholes?

STUDENT: No.

MICHELLE MAKINSON: No.

MICHELLE MAKINSON: Yeah, I love that they were thinking about real life and going, "This doesn't make perfect sense to me. I know that when I go to a store, I can't just rip open a package and buy the parts that I want to buy." So JT was really thinking hard and he was like, "If the oranges come in sets of twelve and two sets is twenty-four, but this guy only bought twenty-two, what did he do? How does that even work in reality?" And when they started scanning down the already matched cards, they realized that it was about it being in twelfths.

And that you could have more than a whole but less than two wholes, and still measure it in twelfths. And the minute they saw that "improper fraction" where the numerator is bigger than the denominator, they went "Oh, I get it!" And they were able to let go of the realism part of "Well, you can't buy ten out of twelve oranges. The package is closed," to go, "Oh, they just mean to say that he bought twenty-two twelfths." And he'll still argue to the day he dies that you can't do that, but he found the math of it.