Building Powerful Climates for Mathematics Teaching and Learning

inside + × = ÷ mathematics

Using Tools in Support of Mathematical Thinking
Mia Buljan, 2nd Grade

Desiree H. Pointer Mace, David Foster, and Audrey Poppers with Mia Buljan

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Appreciations

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With deep gratitude,

Desiree Pointer Mace, David Foster, and Audrey Poppers

Overview

How thoughtful teachers start their year off, and how you can do the same, no matter what day it is.

Introduction: Happy new year!

If you have already read other Mia guides, please skip to page 9.)

When is Day One for you?

Teachers have unique rhythms and timelines in their profession. There are multiple times when it makes sense to reconsider the way you approach your teaching. For you, it might be the beginning of the school year. You might have a new group of students, a new grade level or content area, or a new context. After the winter holidays might also be a time of renewal and reconsideration of your teaching practice. Or it might be at the beginning of a new semester or trimester. Or after required standardized testing is completed.

It doesn't matter when that Day One is for you-- what's important is that we all arrive at times when we resolve to try something new, to recalibrate the way we've been approaching the teaching and learning in our setting. This set of guides is for you.

In this guide, Inside Mathematics invites you to explore the teaching practice of an engaging elementary practitioner, Mia Buljan (2nd grade). Like most teachers, Mia would never describe her teaching as "best practice," but she is someone who continues to learn from and with her students each year. Through the documentation of her classroom, we open up new conversations-- around the daily work to help children begin to see themselves as mathematical thinkers who can draw on their own strategies and those of others to understand and find solution pathways in various problem settings.

Mia: I love my job! I don't know why. I think elementary school is where the action is. I think that kids are still... I think that kids are not fully formed? And so you have a high level of impact on the choices that you make, you see immediate results. They start to ... mimic you, they start to talk like you talk, they start to care about things that you care about... Teaching is the best job I ever had... I've had other jobs, they're not interesting, they're not creative. When there's a kid in front of you who doesn't understand something, the creativity and passion it takes to figure out what they need to know and help them learn it, is the single most satisfying thing about teaching. That this kid, right in front of you, needs you to work tirelessly, is very satisfying compared to other jobs that I have had.

Video reflection: Why do you love teaching?

No matter what day it is when you read this, for you, it is Day One. Today, you're deciding to explore some other teachers' practices so you can rethink your own. Welcome!

Throughout these guides, you will find occasional questions formatted like this for individual or small group reflection. We encourage you to use these questions to deepen your engagement with the video excerpts.

Connections to Research and Standards

Building Mathematically Powerful Students

Our focus in creating these guides is to invite you into classrooms so that you can consider different ways to approach your teaching. In Mia's classroom, though her school context and students may differ from yours, she and her students are engaged in practices that are strongly supported by research on teaching. Mia is working to challenge students to become mathematically powerful. Ruth Parker's landmark 1993 volume *Mathematical Power* reinforces the need for students to "do mathematics: to conjecture, invent, play, discover, represent, apply, prove, experiment, and communicate" (p.212). Our representations in these guides show second grade students doing math in just this way.

We also draw on the Teaching for Robust Understandings in Mathematics framework (aka TRU Math Dimensions, Schoenfeld & Floden 2014). In it, the authors set forth characteristics of "mathematically powerful" classrooms.

The Five Dimensions of Mathematically Powerful Classrooms:

The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.*	*The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to move forward.*

image from Schoenfeld & Floden 2014, p. 2

To be sure, daily life in classrooms is complex. No one teacher ever feels like all aspects of teaching is exactly in place. But if we think about these characteristics, we can then begin to align them with the moments when things are clicking, when students are making connections, when teachers are challenging learners to follow a line of reasoning or defend their thinking.

This framework asks us to keep in mind the math itself, how cognitively challenging the climate and task are, how learners are all equitably engaged in the task, how students balance and negotiate mathematical understandings among themselves, and how the teacher (and the students) evaluate and assess the learners' developing understandings.

Connections to Standards and Practices

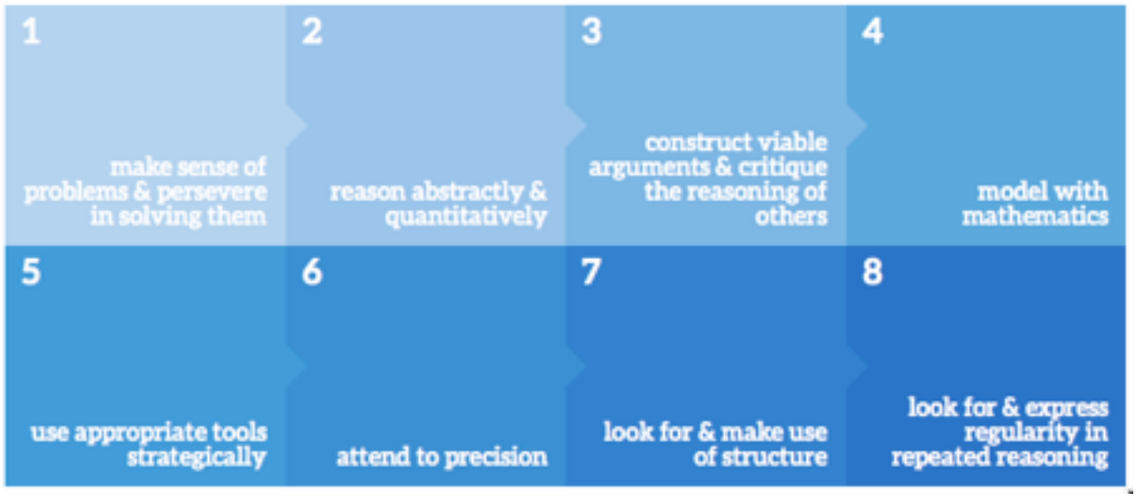
Depending on your school setting, you are also challenged to show how your instructional decision-making aligns with local or national frameworks for mathematics teaching. The National Council of Teachers of Mathematics (NCTM) has recommended eight Mathematics Teaching Practices as part of their "Principles to Actions: Ensuring Mathematical Success for All" series of documents.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

NCTM, 2014 http://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf

Like the TRU framework, the PtA practices focus on active engagement in mathematics by teachers and students alike. There is no passive or receptive stance if students are being supported in productive struggle. There's no lecturing for sustained periods if practitioners are facilitating meaningful mathematical discourse. Active exchange of ideas undergirds all of these practices.

The Common Core State Standards also highlight eight standards of mathematical practice, which Mia uses in her home state of California.



Standards of practice are distinct from content standards in that they not only address what students should *know*, but what mathematically powerful students *do* when they are engaged in thinking and reasoning. Creating the conditions for mathematical practices in students takes time and careful creation of a learning environment and interactive norms to support students in increasingly assuming responsibility for their own mathematical learning.

Building Cultures of Thinking

While we have highlighted teachers' practices during math instructional time, you don't have to be a math teacher to draw from these insights and work to engage learners. The teachers' classrooms we've documented open up conversations about what it takes to create cultures of thinking and make thinking visible. Elements of the work of Harvard University's Project Zero on Visible thinking are evident in Mia's classroom. Ron Ritchhart's 2015 book *Creating Cultures of Thinking* describes ways in which teacher expectations, language, use of time, modeling and apprenticeship, routines, structured opportunities, interactions, and environment all contribute powerfully to student learning. From day one, Mia establishes and reinforces the expectation that learning is an active process and that engagement with the math and with other learners is continuous. Ritchhart distinguishes "learning-oriented" from "work-oriented" classrooms; in the latter, teachers are concerned with compliance and completion of tasks, in the former they are "listening for the learning" (p.45) and use questioning to scaffold and extend students' understandings.

Powerful Assessment Outcomes

This is not just a story about great teaching (though the people with whom we've collaborated certainly are strong practitioners!) We want to reinforce the powerful outcomes of approaching teaching in an authentic way that develops students' agency, authority and identity as mathematicians. Each year since 1999, students in districts in the San Francisco Bay Area have taken a performance assessment test called the Mathematics Assessment Collaborative (MAC) exam. The design and architecture of these performance tasks were developed by the Shell Centre at the University of Nottingham. The exam assesses not only math content, but also the Standards of Mathematical Practice. All the tasks must be hand-scored. The test

is given in grades 2 through Algebra 2 or Integrated Course 3 in high school. Prior to adopting the Common Core State Standards in Mathematics, students in middle school underperformed on the performance assessments. In 2013, only 21% of eighth graders met standards on the MAC exams. In 2015, eighth grade students showed significant gains in student achievement. The percentage of students meeting standard almost double with 40% of the students meeting standard. This dramatic gain is due to more effective instruction and de-tracking students. By engaging all learners in interesting tasks, students' assessment data rise dramatically.

Thinking about Content, Structure, and Strategies

In all of the guides in this series, we want to underscore that teachers are considering multiple dimensions of mathematics learning as they teach. They think about the content outcomes (like understanding place value and "ten-ness"), they make explicit to students the structure of a problem (Is this a put-together or a take-apart problem? is this a constant rate of change problem?), and they present and refer back to strategies for problem solving (Do you count all? Count back? Think about a part/part/whole relationship?). Your students will benefit most if you consider how they will respond to these dimensions in any given problem or learning opportunity.

This takes time. It's important to be patient with ourselves as learners, just as it's important to be patient with our students.

Mia on "Mentor Problems"

Mia: One of the things that's really obvious when you're teaching reading is... you don't do that by getting a brand new book. You go get a book with a character that they already love, like Chrysanthemum, or Pete the Cat. Lo que sea... Whatever it is, right? You take that book that they know and love, and you say, let's look at the character. If you had two books, you could probably teach everything you need to teach, by the lens with which you're looking at the book today...When I started thinking about a math story problem as an actual story, that made a lot of sense to me, to think about it as 'This is my mentor text, and this is the problem where my kids are going to learn about combining things.'" (play video for more)

The Power of Re-engagement

The teachers represented on Inside Mathematics frequently make use of re-engagement to surface misconceptions or identify stronger approaches. Inside Mathematics has several resources related to this approach at (http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons) . This approach frequently presents two

or more different approaches to solving a problem (e.g. “Learner A” and “Learner B”) and then invites students to evaluate the learners’ approaches and make recommendations to them. Often these exemplar learners’ work are selected directly from a teacher’s own group of students. Though students may recognize work as their own, what’s critical in formative re-engagement is that the emphasis is on advising and recommending changes to the learner rather than simply engaging in peer correction of the answers.

Using Tools in Support of Mathematical Thinking

2

Access to mathematical tools informs their use in elementary school classrooms.

Using Tools in Support of Mathematical Thinking

Concepts

1. **Presenting Tools**
2. **Using Classroom Supports**

Introduction

In Mia's classroom, students and teachers have equitable access to tools, resources, and supports for learning.

Presenting Tools

The first week of school, Mia introduces her students to the concept of showing a "math idea" by using tools. She challenges her students first to "tell your hand" in a whisper voice what she's trying to show with her tool.

Mia: Today during math we're going to get a new tool. Who remembers the tool that we had yesterday? Bibi, what was it?
Student A: Counters?
Mia: It was counters! You did lots of interesting things with the counters. But today, we're going to be getting some cubes. And in order to use the cubes, we're going to finish talking about taking care of our tools.
Mia: So I have some cubes here. Do you see my cubes? And I had this math idea in my head? I had this math idea that I was trying to show with my cubes. What do you think my math idea was with these cubes? What do you think is happening with these cubes? What am I trying to explain with these cubes? Tell

Video (Day 2): Presenting new tools and modeling their use

your hand. What's going on with these cubes? I shouldn't hear you when you tell your hand! Just tell your hand.
Mia: Who has an idea? Why would I make my cubes look like this? What do you think I'm trying to show or explain with this idea? Who thinks this is shaped like a pyramid, that goes up in a point, and then down the other side?
Student B: Oh, I know that.
Mia: Trinitie, that's definitely what it looks like, do you want to add on to that? Or a different idea?
Student C: Different idea.
Mia: She wants to share a different idea. Before you share your different idea, does someone want to add on to what Mark said?
Mia: Interesting. So visually, when you look at it, it seems like a pyramid, but it's not actually a pyramid. Mark, is that right?
Student A: Yes.
Mia: Trinitie, you want to try a new idea? What do you think I'm trying to do here?
Mia: See how patient you all are while she gets her idea together? That's what mathematicians do! And you're thinking about what she might be thinking about, because you don't know if you agree with her or not until she says her idea.
Student C: You're trying to match them?
Mia: Tell me a little bit about matching them! Does anybody else see matching here?
Several students: Yeah.
Mia: Hmmm. Do these match? Let me show you. Trinitie, what matches? You said I'm trying to match them.
Student C: These two,
Mia: Ah! Does everybody agree with her that these two match?
Students: Yes.
Mia: Hmmmmm. Anybody see another one that matches?

In modeling her own thinking for her students, Mia engages the students in identifying that the tools can be used for different purposes without telling them directly that a given tool is used for a particular purpose. She praises them for showing patience for each other as they are formulating their responses - "that's what good mathematicians do!"

F *How do you model your use of tools as a means of introducing them to students?*

Mia reviews with students early on how to care for and organize their tools for learning. She has her students use an "I Found This!" bucket to gather stray cubes, counters, and other materials for learning.

Mia: Clean quietly! You've got one minute.... We're done cleaning! 4! You're in your seat!
Mia: Excellent work. One of the things that we talked about was making sure we have all of our pieces. So, quick like a bunny, look at the floor and see if there's anything we need to pick up.
Mia: If you're in group 2, come sit on the carpet quietly. Do you see what

Video (Day 2): Establishing consensus about noise level and care of materials

happened?
Student: I found another one!
Mia: So we have what we call the "I found this!" bucket. Every time you pick something up and you want to tell me you found it? If it's math. Look at all the stuff we found! I found this! Only math pieces go in here. But when you find it? You can put it in the I Found This! bucket. So go ahead and put it in there for us. And then, we can put them back where they belong. But if you find something on the floor, can you put it in the I Found This! bucket?
Students: Yes.
Mia: Is that something that you can do?
Mia: Friends, you did a really good job with your cubes. I was super interested in your ideas. I have a sad face, though. Do you want to hear about my sad face?
Students: Mmm hmm.
Mia: My sad face is that some of your great ideas, I couldn't even hear them because it was so loud. Oh my goodness, it was so loud. And I know it's exciting, and it's easy to get excited, but you're still inside and it's still math. So I'm going to ask you next time to remember that number 2 voice. (play video to continue)

Mia ensures that her elementary students understand how to put away their tool bags, creating the conditions for future access to the tools. She introduces tools one at a time, allowing for exploration and generation of ideas about how the tools could be used.

Mia practices specific procedures with her learners for putting away their math bags efficiently and quietly.

Mia: ...they put their math bags back. So what we're going to do, very quickly, by color, I'm going to send you back to your desks. You are going to come hang up your math bag-- stop, you will stop. Notice that most of them have a number on them right now. What number does this one have on it?
Students: 5!
Mia: Where does it go?
Student: Over here!
Student: On the 5.
Mia: So can you find the hook that has your number on it?
Students: Yes.
Mia: If you look down here, to number 31, 32, 33, 34, 35?
Student: 36.
Mia: Those bags don't have numbers on them. So if you don't have numbers, you're going to hang them down here. You're going to match it. Friends, who thinks they can do this?
Mia: I'm going to teach you a new game. Quick like a bunny. It's called "Silent Star." Do you know Silent Star?
Students: No.
Mia: Oh ho ho! You ready? Silent Star means that as long as I can see a star, nobody can talk, and I mean nobody. She can't talk, you can't talk, I can't talk, if the principal walks in, zip it! You got it?

Video (Day 2): Practicing putting away math bags and doing so quietly

In an early "dot talk" Mia asks students to match a configuration of dots to one that is briefly shown, encouraging students to rec-

ognize patterns and structures, replicate them, and explain the groups that they saw in the configuration of dots.

Mia: Boys and girls, today you are going to be doing another dot talk.
Mia (reflecting): Today's lesson we're doing a dot talk, and our first sort of "official" dot talk did not go that well on the second day of school, from my perspective. I mean, it was fine, we got through it, and stuff happened, but in terms of the process of a dot talk, and the focus of what we're doing during a dot talk, for me got swallowed up a little bit. Partly because of where it was in the lesson, it was later in the lesson, and partly because of -- there was a lot of moving parts at that point for them to negotiate. So my idea today was to put the dot talk at the front of the lesson, and to set out dots before we even handed out our math bags, which can be a little distracting in the beginning.

Video (Day 3): Re-introducing students to the Dot Talk routine and the use of counters

Mia continues to focus on introducing tools so that students can make good choices about their use. Mia follows up a few days after the beginning of school with a second dot talk, in which she encourages her students to match a tool to the task itself. She sets out the dots before distributing the task, challenging her students to think about the configuration of dots and then replicate it in front of them.

(Continued from the previous video)
Mia: To do the dot talk today, I'm going to show it to you, but remember, I'm only going to show it to you for a second.
Mia: Boys and girls, you're going to need some dots, but we don't know how many dots yet. I want you to look at my dots, I want your eyes up here. I'm not going to show you very long. You ready? I want to make sure you're paying attention, because you're not going to get to see it for very long. When I show it to you, I don't want you just to count. There's nothing wrong with counting. But I want you to think about the pieces, the parts that help you figure it out. So you're going to make yours look just like mine. (Play video for more)

Mia can then extend and reinforce these ideas during transitions as well, as evident in her dismissal of students to recess by having them show their dot configuration card at the classroom door. She recognizes that the students are "giving me the best they can" at the beginning of the year, noting that it's hard for seven-year-old bodies to sit and attend for 90 minutes.

F *What tools do you introduce, when, and how?*

At the beginning of the year, Mia engages her students in matching tools to best use. She asks a student using cubes to

measure classroom items “Can you test it? Could you measure this? What would it look like if you measured that?”

Mia: How could you find out? Is there a way to test it?
Mia: The New York Statue? You think it’s big enough to measure that?
Student: Mm hm!
Mia: Could you measure this?
Student: Yeah, sure!
Mia: What would you look like, if you measured that?

Video (Day 2): Activating students’ sense of wonder in the use of tools for real-life situations

In introducing additional tools (e.g. ten-sticks), Mia is strategic about connecting students to what they’ve done before, to maintain their engagement and their sense of wonder and exploration.

Mia: We’re gonna get to explore a new tool. I know! It’s so exciting! Let me show you the tools.
Student: Oh, yeah!
Mia: There’s two of them!
Mia (reflecting): Very quickly, I want them to start thinking about much bigger numbers, and the ten sticks, if they can figure out that they are in fact always ten, they can be very helpful for building larger numbers. They’re very vexing in other ways, because they can’t be broken apart. So if you have anything-- and they can’t be hooked together. So if you have anything that requires regrouping, either to subtract or to add, which comes up pretty quickly, there’s a lot to figure out. So it’s really important to me that they understand the unifix cubes first as a breakable, as a decomposable tool, and then later we add in the ten sticks as an efficient tool, but there’s the relationship there.
Mia: Anybody know what these are called?
Student: Sticks!
Student: Tens!
Student: Ten sticks!
Mia: Why are they called ten sticks if I only have one here? Do I have ten in my hand or do I have one in my hand? Do I have ten in my hand now?
Mia: What about these? Are these tens?
Students: No.
Mia: What do we call these?
Students: Ones.
Mia: Boys and girls, you are going to get some ones, and you are going to get some tens, and I want you to figure out how these work. But before I give them to you, when I call you by color, to go back to your desks, what you’re gonna do is put all those dots away, in the tray that’s in the middle, and you’re going to collect, some one in your group needs to collect all of your mats. The little papers that we were building on.
...
Mia (reflecting): I wanted to do like three things there, really, which was using the cubes-- we use them to share ideas, use the cubes

Video (Day 3): Differentiating tools from each other

to introduce the idea that ideas take time, and then use the cubes to introduce the idea that we don't take other people's ideas apart. We have to ask before we help them clean up.

Mia introduces the ten-sticks after the unifix cubes, because she wants her students to understand decomposing numbers and be able to break them apart in different ways before introducing groups of ten. In doing so, she reinforces norms about tools: that they are to share ideas, that ideas take time, and that it's important to ask before taking apart other people's ideas, as they may be mathematically important to a peer's work.

F *How can you give the cognitive demand of tool selection over to the students?*

By mid-year, students are adept in the use of tools and are able to represent their thinking using tools. Because they make their thinking visible with tools, Mia is able to simultaneously check in with individual students and also hold others accountable for sustaining independent work, getting help from peers.

Mia: Just because he thinks you're right or because you know its right?
Student A: It's right.
Mia: How do you know it's right?
Student A: Because it's 16 soldiers, plus 7
Student B: 7 plus 9.
Mia: Oh, 7 plus 9? It's a put together problem?
Student A: Yeah.
Mia: See if you can find one that's a take apart problem.
Student A: Well, there's two already here.
Mia: Which one of these is take apart?
Student A: Well, there's two already.
Student B: Okay I'll try to do the second one.
Student B: I think all of these, I think it's just more than 12.
Mia: How could you check? How could you keep track of it? Is this 17 here?
Student B: Yeah.
Mia: You sure?
Student B: Yeah.
Mia: You don't sound sure.
Student B: 1, 2,3,
Mia (to Student C): Is this independent? Who could you ask instead of me?
Student B: 4,5,6, 7
Mia (to Student C): Who can help you? Are you having a problem with someone in your group? So you're going to have to find somebody that you trust outside of your group.
Student B: ...8,9,10,11,12
Mia: Netalia, come here. It's okay, you're learning. So who do you trust that you could talk to about a problem?
Student B: 13,14,15,16...

Video (Day 110): Making student thinking visible using tools, and building student independence

Mia: Not in your group, someone outside your group. Go look at the other groups and come back with a name.
Student B: 17.
Mia: You sure?
Student B: Yeah.
Mia: Okay, so how many parachuted out?
Student B: 1,2,3,4,
Mia (to Student D): Off you go.
Student B: 5,6,7,8,9,10,11,12.
Mia: I have to say Rayhan, I found it very helpful the way you kept this, the ones that parachuted out over here and the ones that were in the helicopter here. So how many are left on the helicopter?
Student B: 1,2,3,4,5.
Mia: I'm convinced, that was really helpful for me

Mia balances one-on-one time with a student while supporting and encouraging other students to be independent. She and her students use the tools to count together and to convince others of the reasonableness of their approach.

F How can you use tools and questioning to engage your students in representing their thinking in various ways?

Using Classroom Supports

Once students have access to tools, they can then incorporate them into their reasoning. In the early days of the year, two of Mia's students are able to see that they have different answers and can then use their unifix cubes to defend their thinking.

Mia: And are you done with the problem?
Student A: No.
Mia: Oh! What does she do now? Those are her 18 stickers, and then what happens in the problem?
Student A: She takes 7 more?
Mia: Uh huh. The store! So show me that.
Mia: Sayana. Where's your 18, and where's your 7? And Ashley, do you agree? And do you have the same thing? Okay, so Sayana, when she counts hers, she gets 25 and you get 23. Who do you think is right?
Student B: Her.
Mia: Are you sure? Count it together and see if you agree.

Video (Day 3): Inviting students to compare answers and defend their thinking

How do you invite students to compare their answers, represent, and defend their thinking?

Mia also uses charts to engage students in understanding numerical relationships; in the third week of school, she engages her students in counting backwards from 99 to 0, highlighting patterns in place value.

Mia (reflecting): I'm going to teach them a new game, it's called "Build it Fast," we play it on the 0-99 chart...it's to introduce the idea of patterns in place value, so instead of using 1-100 you use 0 to 99, so the patterns are much clearer.
Mia: This game has to do with that yellow chart...These cards have numbers on them and we're going to put them inside that chart. How many cards do you think we're going to use? Tell your hand. How many cards are we going to use? Bibi?
(Most students offer different ideas: 100, 99, 40, 13, 1500, 130, 30, 100, 99; Mia responds to each with "Maybe!")
Mia: Boys and girls, how many do you think are in this row here?
Student: 10?
Mia: Maybe! Let's count!
Students: 1,2,3,4,5,6,7,8,9,10!
Mia: How many do you think are in this row?
Students: 10!
Mia: Let's count!
Students: 1,2,3,4,5,6,7,8,9,10.
Mia: How many are in this row?
Students: 10!

Video (Day 14): Helping students understand the purpose of the pocket 100 chart

Mia: You sure?
...
Mia: Who thinks they can find one that WON'T be 10? Sayana?
Student: The bottom.
Mia: Let's try. (Play video for more)

By using tools to show their thinking, students are able to make explicit their mathematical understandings and evaluate the soundness of their approach, and Mia is able to use questions to help each student take the next step.

F *How can a "game" approach engage students with tools?*

By the middle of the year, then, Mia's students are able to use the tools available to them to share and defend their thinking to themselves, to their teacher, and to each other. Diva is "very convincing!" when she shares her tray and describes how she connected the quantities in the problem to her representation using cubes.

Student: Okay, 10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57. 58, 59, 66... Wait, what?
Mia: My job today, as I walk around, is to watch them work and to keep asking them, "Are they inside, or are they...?" Because they'll keep building it.
Student: I'm supposed to take them away.
Mia: So, where do the ones that you're going to take away come from?
Student: They come from my hand.
Mia: Okay. So show me how many you have in your hand right now. How many does he start with?
Student: He starts with... 58.
Mia: Okay, so where are those? Put those in your hand.
Student (gathering ten sticks): 10, 20, 30, 40, 50, 51, 52, 53, 54...(selects more ten sticks out of tool bag) 55,56,57, 58.
Mia: RJ! Come here.

Video (Day 14): Observing students' use of tools to diagnose their thinking and strategy.

After a few months of practice, at mid-year Mia's students are able to see other students' approaches to solving the problem, and integrate those approaches into their own when needed.

F *How do you check in with children's thinking? What questions do you ask?*

They are able to explain when they went wrong and correct their thinking, using tools in support of their reflective approach.

Mia: You like the way she counted also? What were you doing before?
Student: I was... when it was up to 100 I accidentally counted by the .. um, ones.
Mia: Ah, but you think you should keep counting by ... tens instead? Diva, you've been very convincing today and very helpful! Can you guys go get whiteboards?

Video (Day 110): Praising students' explanation of thinking

Moving into the last weeks of the school year, Mia's students individually and collectively make use of tools and resources in their classroom to support their learning.

Mia: Who can make our problem match 'Four purple frogs?' Who have I not heard from? Everybody's had a chance today, because we're so little! Ariel. Can you make it match 'four purple frogs'? Four purple frogs. Here's the pond. Four purple frogs in the pond. Let's count.
All: 1, 2, 3, 4.
Mia: Did she make it match?
All: Yes!
Mia: 5 BLUE frogs come to join. Who can make it match? La'Nya.

Video (Day 158): Students make use of tools to devise their own problems

Make it match. Let's count!
All: 1,2,3,4, 5.
Mia: Did La'Nya make it match?
All: Yes!
Mia: Can you think of a question we could ask right now? What question could we ask right now?
RJ: 4+5 equals... How many...frogs in all?
Mia: How many frogs in all.. who can answer that question?
Student: 4+5 equals 9.
Mia: Interesting. What's ANOTHER question we could ask?...
Student: 4 + 5 equals...?
Mia: We're looking for a question. That's a math sentence, and it has to match, but we need a question first. The first question that RJ came up with was 'How many frogs are there in all?' What's another question we could ask?
Student: Is it a minus problem or a plus problem?
Mia: Interesting! The problem that RJ did, was it 'put together' or 'take apart?'
Students: Put together.
Mia: Listen. he said 'How many frogs in all.' Is that putting things together or taking things apart?
Students: Putting things together.
Mia: Sayana, can you think of a different question? (Play video for more)

As they prepare to enter third grade, Mia's students demonstrate agency and authority in the use of their tool bags, selecting appropriate resources to make their thinking visible and defend it to others.

F *How can you develop students' agency and authority using tools in support of mathematical thinking?*

Future Directions

03

What else might we mine from these classroom documentations?

Future Directions

Concepts

1. **Connections to Teacher Learning**
2. **Teachers as Sense-Makers**

Connections to Teacher Learning

This guide is part of a series, focusing on two teachers: Mia Buljan (2nd grade) and Patty Ferrant (8th grade).

One powerful component of these guides is the coaching conversations throughout our documentation of Mia's and Patty's classrooms. Both teachers are experienced with both sides of a coaching dynamic, and recognize that engaging with a colleague in reflection on practice is enormously generative for our development as teachers. Others can see strengths and growth areas that we miss when we're in the middle of teaching.

Reflection: "I'm a very collaborative person by nature."

Reflection: "I get that opportunity to work with my colleagues and do the math together."

The Dana Center at the University of Texas, Austin, has created helpful tools for evaluating effective coaching (Dana Center

2011). Within the dimension of **facilitating adult learning**, coaches engage in building relational trust, developing capacity to improve student achievement, providing collaborative opportunities for faculty reflection, authentic listening, and supporting teacher efforts and needs. Within the domain of **planning and collaboration**, coaches use research-based resources, support standards, encourage and advocate for collaboration, maintains collegial partnerships, and links administrators to teachers with a focus on student achievement. Within the domain of **data support and analysis**, coaches use cyclical processes embedded in collaborative planning that provide ongoing evaluation of student learning, support teachers' focus on student learning, and maintains sustainable assessment systems. Lastly, in the dimension of **strategic competence**, coaches maintain a vision of excellence in teaching, balance content and pedagogical knowledge in context, work continuously to establish routines and trust, engage teacher groups in collaboration around key outcome questions, and consistently refine her or his knowledge of and practices for facilitating adult learning (Dana Center 2011).

The coaching conversations supporting both teachers' classrooms address many of those dimensions.

Teachers as Sense-makers

It's obvious that teachers are sense-makers too, but it was powerful to accompany Mia and Patty as they sought to understand their own teaching through a documentary lens. Too often, the complexity of teaching goes un-noticed because the practitioners are in the middle of the action. Engaging with thinking partners, looking at footage, examining student work samples to try to help external audiences understand children's thinking-- all of these are powerful and deep practices. We are fortunate for the generosity, investment, and time given to this project by both teachers, and hope that these guides will help even more

practitioners deepen their own approaches to math teaching and learning!

Resources

4

Resources

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